

**Machine Elements**

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## 8.1 MECHANISM

by Heard K. Baumeister, Amended by Staff

REFERENCES: Beggs, "Mechanism," McGraw-Hill. Hrones and Nelson, "Analysis of the Four Bar Linkage," Wiley. Jones, "Ingenious Mechanisms for Designers and Inventors," 4 vols., Industrial Press. Moliam, "The Design of Cam Mechanisms and Linkages," Elsevier. Chironis, "Gear Design and Application," McGraw-Hill.

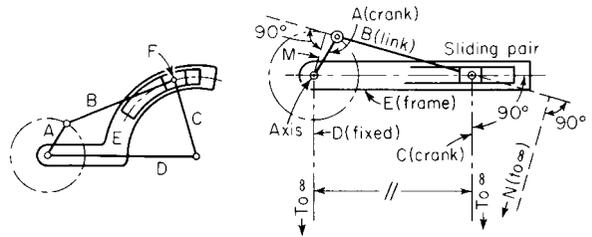
NOTE: The reader is referred to the current and near-past professional literature for extensive material on linkage mechanisms. The vast number of combinations thereof has led to the development of computer software programs to aid in the design of specific linkages.

**Definition** A mechanism is that part of a machine which contains two or more pieces so arranged that the motion of one compels the motion of the others, all in a fashion prescribed by the nature of the combination.

### LINKAGES

Links may be of any form so long as they do not interfere with the desired motion. The simplest form is four bars *A*, *B*, *C*, and *D*, fastened together at their ends by cylindrical pins, and which are all movable in parallel planes. If the links are of different lengths and each is fixed in

(Fig. 8.1.6), *A* will rotate and *E* will oscillate and the infinite links *C* and *D* may be indicated as shown. This gives the **swinging-block linkage**. When used as a quick-return motion the slotted piece and slide are usually interchanged (Fig. 8.1.7) which in no way changes the resulting motion. If the short link *A* is fixed (Fig. 8.1.8), *B* and *E* can both rotate,



Figs. 8.1.4 and 8.1.5 Sliding-block linkage.

and the mechanism known as the **turning-block linkage** is obtained. This is better known under the name of the **Whitworth quick-return motion**, and is generally constructed as in Fig. 8.1.9. The ratio of time of advance to time of return  $H/K$  of the two quick-return motions (Figs. 8.1.7 and

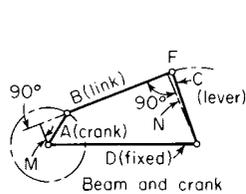


Fig. 8.1.1 Beam-and-crank mechanism.

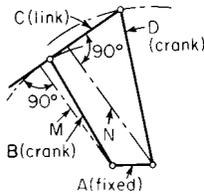


Fig. 8.1.2 Drag-link mechanism.

turn, there will be four possible combinations; but as two of these are similar there will be produced three mechanisms having distinctly different motions. Thus, in Fig. 8.1.1, if *D* is fixed *A* can rotate and *C* oscillate, giving the **beam-and-crank** mechanism, as used on side-wheel steamers. If *B* is fixed, the same motion will result; if *A* is fixed (Fig. 8.1.2), links *B* and *D* can rotate, giving the **drag-link** mechanism used to

feather the floats on paddle wheels. Fixing link *C* (Fig. 8.1.3), *D* and *B* can only oscillate, and a **rocker** mechanism sometimes used in straight-line motions is produced. It is customary to call a rotating link a **crank**; an oscillating link a **lever**, or beam; and the connecting link a **connecting rod**, or **coupler**. Discrete points on the coupler, crank, or lever can be pressed into service to provide a desired motion. The fixed link is often enlarged and used as the supporting frame.

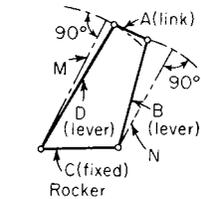


Fig. 8.1.3 Rocker mechanism.

If in the linkage (Fig. 8.1.1) the pin joint *F* is replaced by a slotted piece *E* (Fig. 8.1.4), no change will be produced in the resulting motion, and if the length of links *C* and *D* is made infinite, the slotted piece *E* will become straight and the motion of the slide will be that of pure translation, thus obtaining the engine, or **sliding-block linkage** (Fig. 8.1.5).

If in the sliding-block linkage (Fig. 8.1.5) the long link *B* is fixed

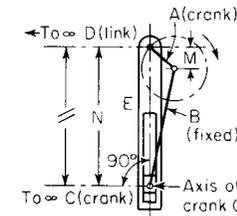


Fig. 8.1.6 Swinging-block linkage.

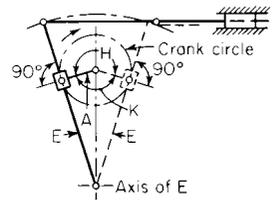


Fig. 8.1.7 Slow-advance, quick-return linkage.

8.1.9) may be found by locating, in the case of the swinging block (Fig. 8.1.7), the two tangent points (*t*) and measuring the angles  $H$  and  $K$  made by the two positions of the crank *A*. If  $H$  and  $K$  are known, the axis of *E* may be located by laying off the angles  $H$  and  $K$  on the crank circle

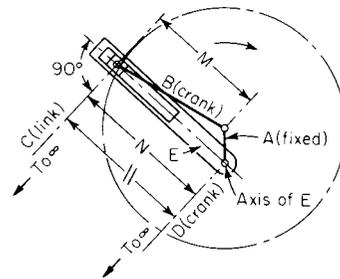


Fig. 8.1.8 Turning-block linkage.

and drawing the tangents *E*, their intersection giving the desired point. For the turning-block linkage (Fig. 8.1.9), determine the angles  $H$  and  $K$  made by the crank *B* when *E* is in the horizontal position; or, if the angles are known, the axis of *E* may be determined by drawing a hori-

zontal line through the two crankpin positions ( $S$ ) for the given angle, and the point where a line through the axis of  $B$  cuts  $E$  perpendicularly will be the axis of  $E$ .

**Velocities** of any two or more points on a link must fulfill the follow-

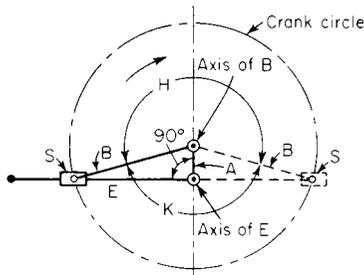


Fig. 8.1.9 Whitworth quick-return motion.

ing conditions (see Sec. 3). (1) Components along the link must be equal and in the same direction (Fig. 8.1.10):  $V_a = V_b = V_c$ . (2) Perpendiculars to  $V_A, V_B, V_C$  from the points  $A, B, C$  must intersect at a common point  $d$ , the **instant center** (or instantaneous axis). (3) The velocities of points  $A, B,$  and  $C$  are directly proportional to their distances from this center (Fig. 8.1.11):  $V_A/a = V_B/b = V_C/c$ . For a straight link the tips of

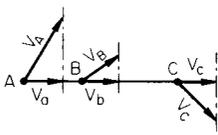


Fig. 8.1.10

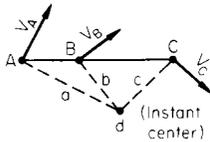


Fig. 8.1.11

the vectors representing the velocities of any number of points on the link will be on a straight line (Fig. 8.1.12);  $abc$  = a straight line. To find the velocity of any point when the velocity and direction of any two other points are known, condition 2 may be used, or a combination of conditions 1 and 3. The **linear velocity ratio** of any two points on a

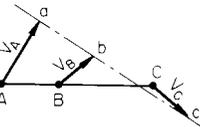


Fig. 8.1.12

linkage may be found by determining the distances  $e$  and  $f$  to the instant center (Fig. 8.1.13); then  $V_c/V_b = ef$ . This may often be simplified by noting that a line drawn parallel to  $e$  and cutting  $B$  forms two similar triangles  $efB$  and  $sAy$ , which gives  $V_c/V_b = ef = s/A$ . The **angular velocity ratio** for any position of two oscillating or rotating links  $A$  and  $C$  (Fig. 8.1.1), connected by a movable link  $B$ , may be determined by

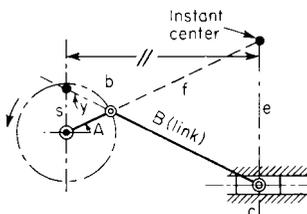


Fig. 8.1.13

scaling the length of the perpendiculars  $M$  and  $N$  from the axes of rotation to the centerline of the movable link. The angular velocity ratio is inversely proportional to these perpendiculars, or  $O_C/O_A = M/N$ . This method may be applied directly to a linkage having a sliding pair if the two infinite links are redrawn perpendicular to the sliding pair, as indicated in Fig. 8.1.14.  $M$  and  $N$  are shown also in Figs. 8.1.1, 8.1.2, 8.1.3, 8.1.5, 8.1.6, 8.1.8. In Fig. 8.1.5 one of the axes is at infinity; therefore,  $N$  is infinite, or the slide has pure translation.

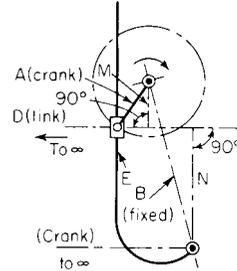


Fig. 8.1.14

**Forces** A mechanism must deliver as much work as it receives, neglecting friction; therefore, the force at any point  $F$  multiplied by the velocity  $V_F$  in the direction of the force at that point must equal the force at some other point  $P$  multiplied by the velocity  $V_P$  at that point; or the forces are inversely as their velocities and  $F/P = V_P/V_F$ . It is at times more convenient to equate the moments of the forces acting around each axis of rotation (sometimes using the instant center) to determine the force acting at some other point. In Fig. 8.1.15,  $F \times a \times c/(b \times d) = P$ .

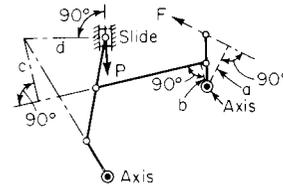


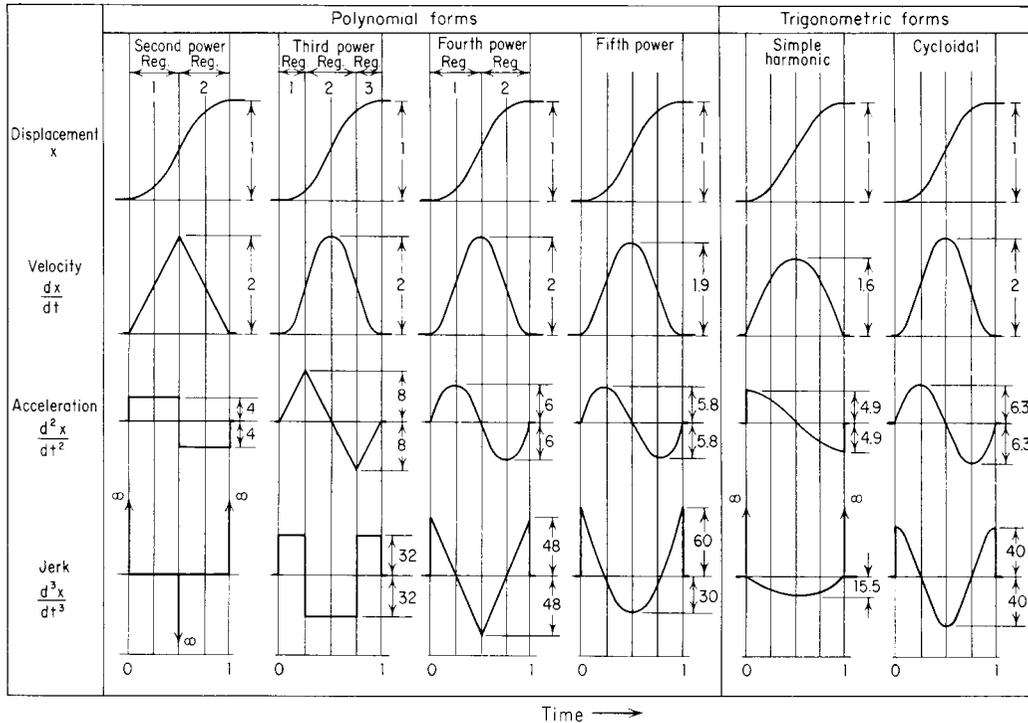
Fig. 8.1.15

### CAMS

**Cam Diagram** A cam is usually a plate or cylinder which communicates motion to a follower as dictated by the geometry of its edge or of a groove cut in its surface. In the practical design of cams, the follower (1) must assume a definite series of positions while the driver occupies a corresponding series of positions or (2) must arrive at a definite location by the time the driver arrives at a particular position. The former design may be severely limited in speed because the interrelationship between the follower and cam positions may yield a follower displacement vs. time function that involves large values for the successive time derivatives, indicating large accelerations and forces, with concomitant large impacts and accompanying noise. The second design centers about finding that particular interrelationship between the follower and cam positions that results in the minimum forces and impacts so that the speed may be made quite large. In either case, the desired interrelationship must be put into hardware as discussed below. In the case of high-speed machines, small irregularities in the cam surface or geometry may be severely detrimental.

A stepwise displacement in time for the follower running on a cam driven at constant speed is, of course, impossible because the follower would require infinite velocities. A step in velocity for the follower would result in infinite accelerations; these in turn would bring into being forces that approach infinite magnitudes which would tend to destroy the machine. A step in acceleration causes a large jerk and large

Table 8.1.1 Displacement, Velocity, Acceleration, and Jerk for Some Cams



SOURCE: Adapted from Gutman, *Mach. Des.*, Mar. 1951.

shock waves to be transmitted and reflected throughout the parts that generate noise and would tend to limit the life of the machine. A step in jerk, the third derivative of the follower displacement with respect to time, seems altogether acceptable. In those designs requiring or exhibiting clearance between the follower and cam (usually at the bottom of the stroke), as gentle and slow a ramp portion as can be tolerated must be inserted on either side of the clearance region to limit the magnitude of the acceleration and jerk to a minimum. The tolerance on the clearance adjustment must be small enough to assure that the follower will be left behind and picked up gradually by the gentle ramp portions of the cam.

Table 8.1.1 shows the comparable and relative magnitudes of velocity, acceleration, and jerk for several high-speed cam, where the displacements are all taken as 1 at time 1 without any overshoot in any of the derivatives.

The three most common forms of motion used are uniform motion (Fig. 8.1.16), harmonic motion (Fig. 8.1.17), and uniformly accelerated and retarded motion (Fig. 8.1.18). In plotting the diagrams (Fig. 8.1.18) for this last motion, divide  $ac$  into an even number of equal parts and  $bc$

into the same number of parts with lengths increasing by a constant increment to a maximum and then decreasing by the same decrement, as, for example, 1, 3, 5, 5, 3, 1, or 1, 3, 5, 7, 9, 9, 7, 5, 3, 1. In order to prevent shock when the direction of motion changes, as at  $a$  and  $b$  in the uniform motion, the harmonic motion may be used; if the cam is to be operated at high speed, the uniformly accelerated and retarded motion should preferably be employed; in either case there is a very gradual change of velocity.

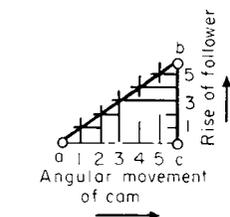


Fig. 8.1.16

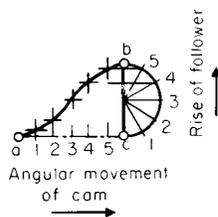


Fig. 8.1.17

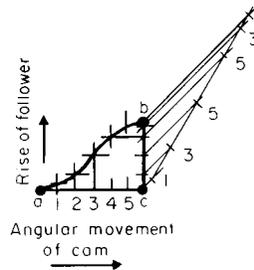


Fig. 8.1.18

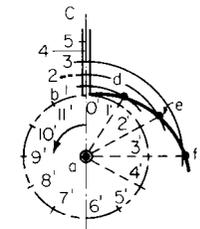


Fig. 8.1.19

**Pitch Line** The actual pitch line of a cam varies with the type of motion and with the position of the follower relative to the cam's axis. Most cams as ordinarily constructed are covered by the following four cases.

**FOLLOWER ON LINE OF AXIS.** (Fig. 8.1.19). To draw the pitch line, subdivide the motion  $bc$  of the follower in the manner indicated in Figs. 8.1.16, 8.1.17, and 8.1.18. Draw a circle with a radius equal to the smallest radius of the cam  $a0$  and subdivide it into angles  $0a1'$ ,  $0a2'$ ,

$0a3'$ , etc., corresponding with angular displacements of the cam for positions 1, 2, 3, etc., of the follower. With  $a$  as a center and radii  $a1, a2, a3$ , etc., strike arcs cutting radial lines at  $d, e, f$ , etc. Draw a smooth curve through points  $d, e, f$ , etc.

**OFFSET FOLLOWER** (Fig. 8.1.20). Divide  $bc$  as indicated in Figs. 8.1.16, 8.1.17, and 8.1.18. Draw a circle of radius  $ac$  (highest point of rise of follower) and one tangent to  $cb$  produced. Divide the outer circle into parts  $1', 2', 3'$ , etc., corresponding with the angular displacement of

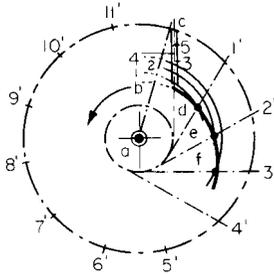


Fig. 8.1.20

the cam for positions 1, 2, 3, etc., of the follower, and draw tangents from points  $1', 2', 3'$ , etc., to the small circle. With  $a$  as a center and radii  $a1, a2, a3$ , etc., strike arcs cutting tangents at  $d, e, f$ , etc. Draw a smooth curve through  $d, e, f$ , etc.

**ROCKER FOLLOWER** (Fig. 8.1.21). Divide the stroke of the slide  $S$  in the manner indicated in Figs. 8.1.16, 8.1.17, and 8.1.18, and transfer these points to the arc  $bc$  as points 1, 2, 3, etc. Draw a circle of radius  $ak$  and divide it into parts  $1', 2', 3'$ , etc., corresponding with angular dis-

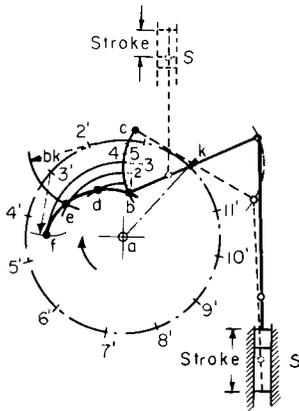


Fig. 8.1.21

placements of the cam for positions 1, 2, 3, etc., of the follower. With  $k, 1', 2', 3'$ , etc., as centers and radius  $bk$ , strike arcs  $kb, 1'd, 2'e, 3'f$ , etc., cutting at  $bdef$  arcs struck with  $a$  as a center and radii  $ab, a1, a2, a3$ , etc. Draw a smooth curve through  $b, d, e, f$ , etc.

**CYLINDRICAL CAM** (Fig. 8.1.22). In this type of cam, more than one complete turn may be obtained, provided in all cases the follower returns to its starting point. Draw rectangle  $wxyz$  (Fig. 8.1.22) representing the development of cylindrical surface of the cam. Subdivide the desired motion of the follower  $bc$  horizontally in the manner indicated in Figs. 8.1.16, 8.1.17, and 8.1.18, and plot the corresponding angular displacement  $1', 2', 3'$ , etc., of the cam vertically; then through the intersection of lines from these points draw a smooth curve. This may best be shown by an example, assuming the following data for the

diagram in Fig. 8.1.22: Total motion of follower =  $bc$ ; circumference of cam =  $2\pi r$ . Follower moves harmonically 4 units to right in 0.6 turn, then rests (or "dwells") 0.4 turn, and finishes with uniform motion 6 units to right and 10 units to left in 2 turns.

**Cam Design** In the practical design of cams the following points must be noted. If only a small force is to be transmitted, sliding contact may be used, otherwise **rolling contact**. For the latter the pitch line must

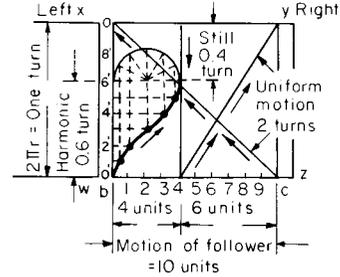


Fig. 8.1.22 Cylindrical cam.

be corrected in order to get the true slope of the cam. An approximate construction (Fig. 8.1.23) may be employed by using the pitch line as the center of a series of arcs the radii of which are equal to that of the follower roll to be used; then a smooth curve drawn tangent to the arcs will give the slope desired for a roll working on the periphery of the cam

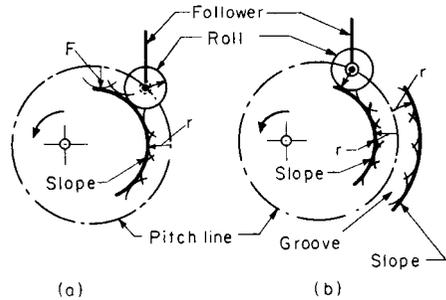


Fig. 8.1.23

(Fig. 8.1.23a) or in a groove (Fig. 8.1.23b). For plate cams the roll should be a small cylinder, as in Fig. 8.1.24a. In cylindrical cams it is usually sufficiently accurate to make the roll conical, as in Fig. 8.1.24b, in which case the taper of the roll produced should intersect the axis of the cam. If the pitch line  $abc$  is made too sharp (Fig. 8.1.25) the follower

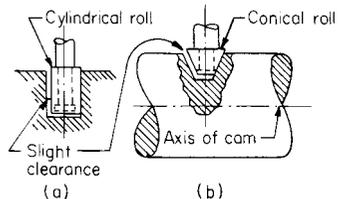


Fig. 8.1.24 Plate cam.

will not rise the full amount. In order to prevent this **loss of rise**, the pitch line should have a radius of curvature at all parts of not less than the roll's diameter plus  $\frac{1}{8}$  in. For the same rise of follower,  $a$ , the angular motion of the cam,  $O$ , the slope of the cam changes considerably, as indicated by the heavy lines  $A, B$ , and  $C$  (Fig. 8.1.26). Care should be

taken to keep a moderate slope and thereby keep down the side thrust on the follower, but this should not be carried too far, as the cam would become too large and the friction increase.

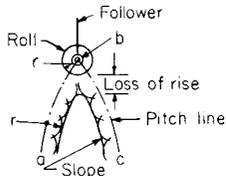


Fig. 8.1.25

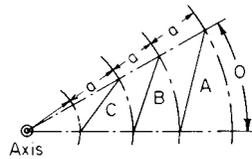


Fig. 8.1.26

**ROLLING SURFACES**

In order to connect two shafts so that they shall have a definite angular velocity ratio, rolling surfaces are often used; and in order to have no slipping between the surfaces they must fulfill the following two conditions: the line of centers must pass through the point of contact, and the arcs of contact must be of equal length. The angular velocities, expressed usually in r/min, will be inversely proportional to the radii:  $N/n = r/R$ . The two surfaces most commonly used in practice, and the only ones having a constant angular velocity ratio, are cylinders where the shafts are parallel, and cones where the shafts (projected) intersect at an angle. In either case there are

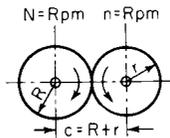


Fig. 8.1.27

two possible directions of rotation, depending upon whether the surfaces roll in opposite directions (external contact) or in the same direction (internal contact). In Fig. 8.1.27,  $R = nc/(N + n)$  and  $r = Nc/(N + n)$ ; in Fig. 8.1.28,  $R = nc/(N - n)$  and  $r = Nc/(N - n)$ . In Fig. 8.1.29,  $\tan B = \sin A/(n/N + \cos A)$  and  $\tan C = \sin A/$

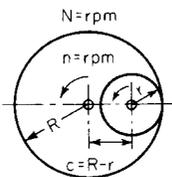


Fig. 8.1.28

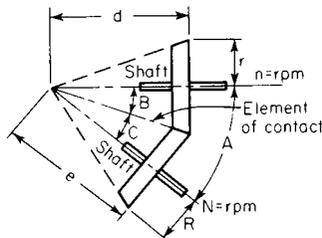


Fig. 8.1.29

$(N/n + \cos A)$ ; in Fig. 8.1.30,  $\tan B = \sin A/(N/n - \cos A)$ , and  $\tan C = \sin A/(n/N - \cos A)$ . With the above values for the angles B and C, and the length d or e of one of the cones, R and r may be calculated.

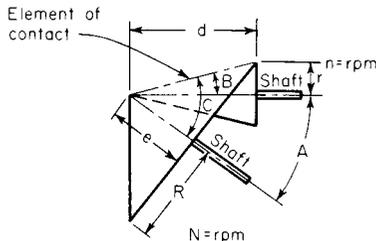


Fig. 8.1.30

The natural limitations of rolling without slip, with the use of pure rolling surfaces limited to the transmission of very small amounts of torque, led historically to the alteration of the geometric surfaces to include teeth and tooth spaces, i.e., **toothed wheels**, or simply **gears**.

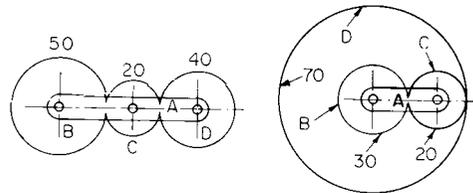
Modern gear tooth systems are described in greater detail in Sec. 8.3. This brief discussion is limited to the kinematic considerations of some common gear combinations.

**EPICYCLIC TRAINS**

Epicyclic trains are combinations of gears in which some of or all the gears have a motion compounded of rotation about an axis and a translation or revolution of that axis. The gears are usually connected by a link called an arm, which often rotates about the axis of the first gear. Such trains may be calculated by first considering all gears locked and the arm turned; then the arm locked and the gears rotated. The algebraic sum of the separate motions will give the desired result. The following examples and method of tabulation will illustrate this. The figures on each gear refer to the number of teeth for that gear.

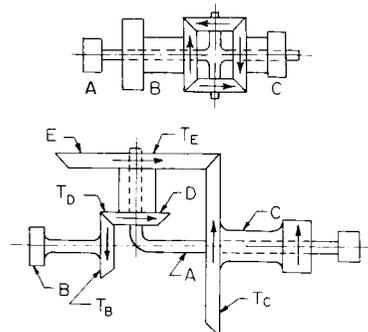
	A	B	C	D
Gear locked, Fig. 8.1.31	+1	+1	+1	+1
Arm locked, Fig. 8.1.31	0	-1	$+1 \times \frac{50}{20}$	$-1 \times \frac{50}{20} \times \frac{20}{40}$
Addition, Fig. 8.1.31	+1	0	$+3\frac{1}{2}$	$-\frac{1}{4}$
Gears locked, Fig. 8.1.32	+1	+1	+1	+1
Arm locked, Fig. 8.1.32	0	-1	$+1 \times \frac{30}{20}$	$+1 \times \frac{30}{20} \times \frac{20}{70}$
Addition, Fig. 8.1.32	+1	0	$+2\frac{1}{2}$	$+1\frac{3}{7}$

In Figs. 8.1.31 and 8.1.32 lock the gears and turn the arm A right-handed through 1 revolution (+1); then lock the arm and turn the gear B back to where it started (-1); gears C and D will have rotated the amount indicated in the tabulation. Then the algebraic sum will give the relative turns of each gear. That is, in Fig. 8.1.31, for one turn of the



Figs. 8.1.31 and 8.1.32 Epicyclic trains.

arm, B does not move and C turns in the same direction  $3\frac{1}{2}$  r, and D in the opposite direction  $\frac{1}{4}$  r; whereas in Fig. 8.1.32, for one turn of the arm, B does not turn, but C and D turn in the same direction as the arm, respectively,  $2\frac{1}{2}$  and  $1\frac{3}{7}$  r. (Note: The arm in the above case was turned +1 for convenience, but any other value might be used.)



Figs. 8.1.33 and 8.1.34 Bevel epicyclic trains.

**Bevel epicyclic trains** are epicyclic trains containing bevel gears and may be calculated by the preceding method, but it is usually simpler to use the general formula which applies to all cases of epicyclic trains:

$$\frac{\text{Turns of } C \text{ relative to arm}}{\text{Turns of } B \text{ relative to arm}} = \frac{\text{absolute turns of } C - \text{turns of arm}}{\text{absolute turns of } B - \text{turns of arm}}$$

The left-hand term gives the value of the train and can always be expressed in terms of the number of teeth ( $T$ ) on the gears. Care must be used, however, to express it as either plus (+) or minus (-), depending upon whether the gears turn in the same or opposite directions.

$$\frac{\text{Relative turns of } C}{\text{Relative turns of } B} = \frac{C - A}{B - A} = -1 \quad (\text{in Fig. 8.1.33})$$

$$= + \frac{T_E}{T_C} \times \frac{T_B}{T_D} \quad (\text{Fig. 8.1.34})$$

**HOISTING MECHANISMS**

**Pulley Block** (Fig. 8.1.35) Given the weight  $W$  to be raised, the force  $F$  necessary is  $F = V_W W / V_F = W/n$  = load/number of ropes,  $V_W$  and  $V_F$  being the respective velocities of  $W$  and  $F$ .

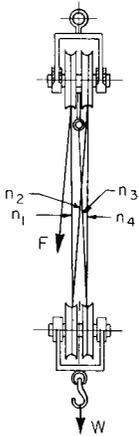


Fig. 8.1.35 Pulley block.

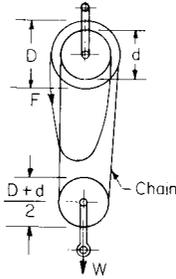


Fig. 8.1.36 Differential chain block.

**Differential Chain Block** (Fig. 8.1.36)

$$F = V_W W / V_F = W(D - d)/(2D)$$

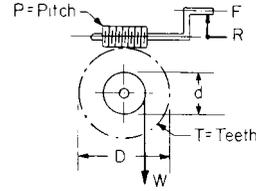


Fig. 8.1.37 Worm and worm wheel.

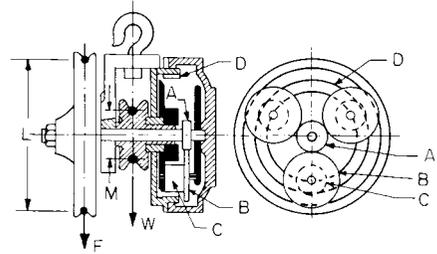


Fig. 8.1.38 Triplex chain block.

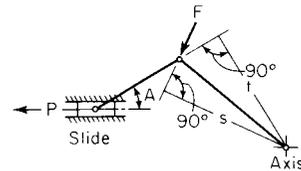


Fig. 8.1.39 Toggle joint.

**Worm and Wheel** (Fig. 8.1.37)  $F = \pi d(n/T)W/(2\pi R) = WP(d/D)/(2\pi R)$ , where  $n$  = number of threads, single, double, triple, etc.

**Triplex Chain Block** (Fig. 8.1.38) This geared hoist makes use of the epicyclic train.  $W = FL/\{M[1 + (T_D/T_C) \times (T_B/T_A)]\}$ , where  $T$  = number of teeth on gears.

**Toggle Joint** (Fig. 8.1.39)  $P = Fs (\cos A)/t$ .

**8.2 MACHINE ELEMENTS**

by Antonio F. Baldo

REFERENCES: American National Standards Institute (ANSI) Standards. International Organization for Standardization (ISO) Standards. Morden, "Industrial Fasteners Handbook," Trade and Technical Press. Parmley, "Standard Handbook of Fastening and Joining," McGraw-Hill. Bickford, "An Introduction to the Design and Behavior of Bolted Joints," Marcel Dekker. Maleev, "Machine Design," International Textbook. Shigley, "Mechanical Engineering Design," McGraw-Hill. *Machine Design* magazine, Penton/IPC. ANSI/Rubber Manufacturers Assn. (ANSI/RMA) Standards. "Handbook of Power Transmission Flat Belting," Goodyear Rubber Products Co. "Industrial V-Belting," Goodyear Rubber Products Co. Carlson, "Spring Designer's Handbook," Marcel Dekker. American Chain Assn., "Chains for Power Transmission and Material Handling—Design and Applications Handbook," Marcel Dekker. "Power Transmission Handbook," DAYCO. "Wire Rope User's Manual," American Iron and Steel Institute. Blake, "Threaded Fasteners—Materials and Design," Marcel Dekker.

NOTE: At this writing, conversion to metric hardware and machine elements continues. SI units are introduced as appropriate, but the bulk of the material is still presented in the form in which the designer or reader will find it available.

**SCREW FASTENINGS**

At present there exist two major standards for screw threads, namely Unified inch screw threads and metric screw threads. Both systems enjoy a wide application globally, but movement toward a greater use of the metric system continues.

**Unified Inch Screw Threads (or Unified Screw Threads)**

The Unified Thread Standard originated by an accord of screw thread standardization committees of Canada, the United Kingdom, and the United States in 1984. The Unified Screw-Thread Standard was published by ANSI as American Unified and American Screw Thread Publication B1.1-1974, revised in 1982 and then again in 1989. Revisions did not tamper with the basic 1974 thread forms. In conjunction with Technical Committee No. 1 of the ISO, the Unified Standard was adopted as an ISO Inch Screw Standard (ISO 5864-1978).

Of the numerous and different screw thread forms, those of greatest consequence are

- UN—unified (no mandatory radiused root)
- UNR—unified (mandatory radiused root; minimum  $0.108 = p$ )
- UNJ—unified (mandatory larger radiused root; recommended  $0.150 = p$ )
- M—metric (inherently designed and manufactured with radiused root; has  $0.125 = p$ )
- MJ—metric (mandatory larger radiused root; recommended  $0.150 = p$ )

The basic American screw thread profile was standardized in 1974, and it now carries the UN designations (UN = unified). ANSI publishes these standards and all subsequent revisions. At intervals these standards are published with a “reaffirmation date” (that is, R1988). In 1969 an *international* basic thread profile standard was established, and it is designated as M. The ISO publishes these standards with yearly updates. The UN and M profiles are the same, but UN screws are manufactured to **inch** dimensions while M screws are manufactured to **metric** dimensions.

The **metric system** has only the two thread forms: **M**, standard for commercial uses, and **MJ**, standard for aerospace use and for aerospace-quality commercial use.

Certain groups of diameter and pitch combinations have evolved over time to become those most used commercially. Such groups are called **thread series**. Currently there are 11 UN series for inch products and 13 M series for metric products.

The Unified standard comprises the following two parts:

1. **Diameter-pitch combinations.** (See Tables 8.2.1 to 8.2.5.)

- a. UN inch series:
- |                |               |
|----------------|---------------|
| Coarse         | UNC or UNRC   |
| Fine           | UNF or UNRF   |
| Extra-fine     | UNEF or UNREF |
| Constant-pitch | UN or UNR     |
- b. Metric series:
- |        |   |
|--------|---|
| Coarse | M |
| Fine   | M |

NOTE: Radiused roots apply only to external threads. The preponderance of important commercial use leans to UNC, UNF, 8UN (eight-threaded), and metric coarse M. Aerospace and aerospace-quality applications use UNJ and MJ.

2. **Tolerance classes.** The amounts of tolerance and allowance distinguish one thread class from another. Classes are designated by one of three numbers (1, 2, 3), and either letter A for external threads or letter B for internal threads. Tolerance decreases as class number increases. Allowance is specified only for classes 1A and 2A. Tolerances are based on engagement length equal to nominal diameter. 1A/1B—liberal tolerance and allowance required to permit easy assembly even with dirty or nicked threads. 2A/2B—most commonly used for general applications, including production of bolts, screws, nuts, and similar threaded fasteners. Permits external threads to be plated. 3A/3B—for closeness of fit and/or accuracy of thread applications where zero allowance is needed. 2AG—allowance for rapid assembly where high-temperature expansion prevails or where lubrication problems are important.

Unified screw threads are designated by a set of numbers and letter symbols signifying, in sequence, the nominal size, threads per inch, thread series, tolerance class, hand (only for left hand), and in some instances in parentheses a Thread Acceptability System Requirement of ANSI B1.3.

EXAMPLE.  $\frac{1}{4}$ -20 UNC-2A-LH (21), or optionally 0.250-20 UNC-2A-LH (21), where  $\frac{1}{4}$  = nominal size (fractional diameter, in, or screw number, with decimal equivalent of either being optional); 20 = number of threads per inch, *n*; UNC = thread form and series; 2A = tolerance class; LH = left hand (no symbol required for right hand); (21) = thread gaging system per ANSI B1.3.

3. **Load considerations**

a. *Static loading.* Only a slight increase in tensile strength in a

screw fastener is realized with an increase in *root* rounding radius, because minor diameter (hence cross-sectional area at the root) growth is small. Thus the basic tensile stress area formula is used in stress calculations for all thread forms. See Tables 8.2.2, 8.2.3, and 8.2.4. The designer should take into account such factors as stress concentration as applicable.

b. *Dynamic loading.* Few mechanical joints can remain absolutely free of some form of fluctuating stress, vibration, stress reversal, or impact. Metal-to-metal joints of very high-modulus materials or non-elastic-gasketed high-modulus joints plus preloading at assembly (preload to be greater than highest peak of the external fluctuating load) can realize absolute static conditions inside the screw fastener. For ordinary-modulus joints and elastic-gasketed joints, a fraction of the external fluctuating load will be transmitted to the interior of the screw fastener. Thus the fastener must be designed for fatigue according to a **static plus fluctuating load** model. See discussion under “Strength” later.

Since **fatigue failures** generally occur at locations of high **stress concentration**, screw fasteners are especially vulnerable because of the abrupt change between head and body, notchlike conditions at the thread roots, surface scratches due to manufacturing, etc. The highest stress concentrations occur at the thread roots. The stress concentration factor can be very large for nonrounded roots, amounting to about 6 for sharp or flat roots, to less than 3 for UNJ and MJ threads which are generously rounded. This can effectively double the fatigue life. UNJ and MJ threads are especially well suited for dynamic loading conditions.

### Screw Thread Profile

**Basic Profile** The basic profiles of UN and UNR are the same, and these in turn are identical to those of ISO metric threads. Basic thread shape ( $60^\circ$  thread angle) and basic dimensions (major, pitch, and minor diameters; thread height; crest, and root flats) are defined. See Fig. 8.2.1.

**Design Profile** Design profiles define the maximum material (no allowance) for external and internal threads, and they are derived from the basic profile. UN threads (external) may have either flat or rounded crests and roots. UNR threads (external) must have rounded roots, but may have flat or rounded crests. UN threads (internal) *must* have rounded roots. Any rounding must clear the basic flat roots or crests.

Basic major diameter	Largest diameter of basic screw thread.
Basic minor diameter	Smallest diameter of basic screw thread.
Basic pitch diameter	Diameter to imaginary lines through thread profile and parallel to axis so that thread and groove widths are equal. These three definitions apply to both external and internal threads.
Maximum diameters (external threads)	Basic diameters minus allowance.
Minimum diameters (internal threads)	Basic diameters.
Pitch	$1/n$ ( <i>n</i> = number of threads per inch).
Tolerance	Inward variation tolerated on maximum diameters of external threads and outward variation tolerated on minimum diameters of internal threads.

### Metric Screw Threads

Metric screw thread standardization has been under the aegis of the International Organization for Standardization (ISO). The ISO basic profile is essentially the same as the Unified screw thread basic form,

Table 8.2.1 Standard Series Threads (UN/UNR)\*

Nominal size, in		Basic major diameter, in	Threads per inch											Nominal size, in
			Series with graded pitches			Series with constant pitches								
Primary	Secondary		Coarse UNC	Fine UNF	Extra-fine UNEF	4UN	6UN	8UN	12UN	16UN	20UN	28UN	32UN	
0		0.0600	—	80	—	—	—	—	—	—	—	—	—	0
	1	0.0730	64	72	—	—	—	—	—	—	—	—	—	1
2		0.0860	56	64	—	—	—	—	—	—	—	—	—	2
	3	0.0990	48	56	—	—	—	—	—	—	—	—	—	3
4		0.1120	40	48	—	—	—	—	—	—	—	—	—	4
5		0.1250	40	44	—	—	—	—	—	—	—	—	—	5
6		0.1380	32	40	—	—	—	—	—	—	—	—	—	6
8		0.1640	32	36	—	—	—	—	—	—	—	—	—	8
10		0.1900	24	32	—	—	—	—	—	—	—	—	—	10
	12	0.2160	24	28	32	—	—	—	—	—	—	UNF	UNEF	12
¼		0.2500	20	28	32	—	—	—	—	—	UNC	UNF	UNEF	¼
⅝ <sub>16</sub>		0.3125	18	24	32	—	—	—	—	—	20	28	UNEF	⅝ <sub>16</sub>
⅜		0.3750	16	24	32	—	—	—	—	UNC	20	28	UNEF	⅜
7 <sub>16</sub>		0.4375	14	20	28	—	—	—	—	16	UNF	UNEF	32	7 <sub>16</sub>
½		0.5000	13	20	28	—	—	—	—	16	UNF	UNEF	32	½
⅝ <sub>16</sub>		0.5625	12	18	24	—	—	—	UNC	16	20	28	32	⅝ <sub>16</sub>
⅝		0.6250	11	18	24	—	—	—	12	16	20	28	32	⅝
	1 <sub>16</sub>	0.6875	—	—	24	—	—	—	12	16	20	28	32	1 <sub>16</sub>
¾		0.7500	10	16	20	—	—	—	12	UNF	UNEF	28	32	¾
	1 <sub>32</sub>	0.8125	—	—	20	—	—	—	12	16	UNEF	28	32	1 <sub>32</sub>
7 <sub>8</sub>		0.8750	9	14	20	—	—	—	12	16	UNEF	28	32	7 <sub>8</sub>
	1 <sub>8</sub>	0.9275	—	—	20	—	—	—	12	16	UNEF	28	32	1 <sub>8</sub>
1		1.0000	8	12	20	—	—	UNC	UNF	16	UNEF	28	32	1
	1 <sub>16</sub>	1.0625	—	—	18	—	—	8	12	16	20	28	—	1 <sub>16</sub>
1 <sub>8</sub>		1.1250	7	12	18	—	—	8	UNF	16	20	28	—	1 <sub>8</sub>
	1 <sub>32</sub>	1.1875	—	—	18	—	—	8	12	16	20	28	—	1 <sub>32</sub>
1 <sub>4</sub>		1.2500	7	12	18	—	—	8	UNF	16	20	28	—	1 <sub>4</sub>
	1 <sub>8</sub>	1.3125	—	—	18	—	—	8	12	16	20	28	—	1 <sub>8</sub>
1 <sub>32</sub>		1.3750	6	12	18	—	UNC	8	UNF	16	20	28	—	1 <sub>32</sub>
	1 <sub>16</sub>	1.4375	—	—	18	—	6	8	12	16	20	28	—	1 <sub>16</sub>
1 <sub>2</sub>		1.5000	6	12	18	—	UNC	8	UNF	16	20	28	—	1 <sub>2</sub>
1		1.5625	—	—	18	—	6	8	12	16	20	—	—	1 <sub>16</sub>
1 <sub>8</sub>		1.6250	—	—	18	—	6	8	12	16	20	—	—	1 <sub>8</sub>
	1 <sub>16</sub>	1.6875	—	—	18	—	6	8	12	16	20	—	—	1 <sub>16</sub>
1 <sub>4</sub>		1.7500	5	—	—	—	6	8	12	16	20	—	—	1 <sub>4</sub>
	1 <sub>32</sub>	1.8125	—	—	—	—	6	8	12	16	20	—	—	1 <sub>32</sub>
1 <sub>8</sub>		1.8750	—	—	—	—	6	8	12	16	20	—	—	1 <sub>8</sub>
	1 <sub>8</sub>	1.9375	—	—	—	—	6	8	12	16	20	—	—	1 <sub>8</sub>

**Table 8.2.1 Standard Series Threads (UN/UNR)\* (continued)**

Nominal size, in		Basic major diameter, in	Threads per inch											Nominal size, in
			Series with graded pitches			Series with constant pitches								
			Coarse UNC	Fine UNF	Extra-fine UNEF	4UN	6UN	8UN	12UN	16UN	20UN	28UN	32UN	
Primary	Secondary													
2		2.0000	4½	—	—	—	6	8	12	16	20	—	—	2
	2½	2.1250	—	—	—	—	6	8	12	16	20	—	—	2½
2¼		2.2500	4½	—	—	—	6	8	12	16	20	—	—	2¼
	2¾	2.3750	—	—	—	—	6	8	12	16	20	—	—	2¾
2½		2.5000	4	—	—	UNC	6	8	12	16	20	—	—	2½
	2⅝	2.6250	—	—	—	4	6	8	12	16	20	—	—	2⅝
2¾		2.7500	4	—	—	UNC	6	8	12	16	20	—	—	2¾
	2⅞	2.8750	—	—	—	4	6	8	12	16	20	—	—	2⅞
3		3.0000	4	—	—	UNC	6	8	12	16	20	—	—	3
	3½	3.1250	—	—	—	4	6	8	12	16	—	—	—	3½
3¼		3.2500	4	—	—	UNC	6	8	12	16	—	—	—	3¼
	3⅜	3.3750	—	—	—	4	6	8	12	16	—	—	—	3⅜
3½		3.5000	4	—	—	UNC	6	8	12	16	—	—	—	3½
	3⅝	3.6250	—	—	—	4	6	8	12	16	—	—	—	3⅝
3¾		3.7500	4	—	—	UNC	6	8	12	16	—	—	—	3¾
	3⅞	3.8750	—	—	—	4	6	8	12	16	—	—	—	3⅞
4		4.0000	4	—	—	UNC	6	8	12	16	—	—	—	4
	4½	4.1250	—	—	—	4	6	8	12	16	—	—	—	4½
4¼		4.2500	—	—	—	4	6	8	12	16	—	—	—	4¼
	4¾	4.3750	—	—	—	4	6	8	12	16	—	—	—	4¾
4½		4.5000	—	—	—	4	6	8	12	16	—	—	—	4½
	4⅝	4.6250	—	—	—	4	6	8	12	16	—	—	—	4⅝
4¾		4.7500	—	—	—	4	6	8	12	16	—	—	—	4¾
	4⅞	4.8750	—	—	—	4	6	8	12	16	—	—	—	4⅞
5		5.0000	—	—	—	4	6	8	12	16	—	—	—	5
	5½	5.1250	—	—	—	4	6	8	12	16	—	—	—	5½
5¼		5.2500	—	—	—	4	6	8	12	16	—	—	—	5¼
	5⅜	5.3750	—	—	—	4	6	8	12	16	—	—	—	5⅜
5½		5.5000	—	—	—	4	6	8	12	16	—	—	—	5½
	5⅝	5.6250	—	—	—	4	6	8	12	16	—	—	—	5⅝
5¾		5.7500	—	—	—	4	6	8	12	16	—	—	—	5¾
	5⅞	5.8750	—	—	—	4	6	8	12	16	—	—	—	5⅞
6		6.0000	—	—	—	4	6	8	12	16	—	—	—	6

\* Series designation shown indicates the UN thread form; however, the UNR thread form may be specified by substituting UNR in place of UN in all designations for external use only.  
SOURCE: ANSI B1.1-1982; reaffirmed in 1989, reproduced by permission.

Table 8.2.2 Basic Dimensions for Coarse Thread Series (UNC/UNRC)

Nominal size, in	Basic major diameter $D$ , in	Threads per inch $n$	Basic pitch diameter* $E$ , in	UNR design minor diameter external† $K_x$ , in	Basic minor diameter internal $K$ , in	Section at minor diameter at $D - 2h_b$ , in <sup>2</sup>	Tensile stress area, ‡ in <sup>2</sup>
1 (0.073)§	0.0730	64	0.0629	0.0544	0.0561	0.00218	0.00263
2 (0.086)	0.0860	56	0.0744	0.0648	0.0667	0.00310	0.00370
3 (0.099)§	0.0990	48	0.0855	0.0741	0.0764	0.00406	0.00487
4 (0.112)	0.1120	40	0.0958	0.0822	0.0849	0.00496	0.00604
5 (0.125)	0.1250	40	0.1088	0.0952	0.0979	0.00672	0.00796
6 (0.138)	0.1380	32	0.1177	0.1008	0.1042	0.00745	0.00909
8 (0.164)	0.1640	32	0.1437	0.1268	0.1302	0.01196	0.0140
10 (0.190)	0.1900	24	0.1629	0.1404	0.1449	0.01450	0.0175
12 (0.216)§	0.2160	24	0.1889	0.1664	0.1709	0.0206	0.0242
¼	0.2500	20	0.2175	0.1905	0.1959	0.0269	0.0318
⅜	0.3125	18	0.2764	0.2464	0.2524	0.0454	0.0524
½	0.3750	16	0.3344	0.3005	0.3073	0.0678	0.0775
⅝	0.4375	14	0.3911	0.3525	0.3602	0.0933	0.1063
¾	0.5000	13	0.4500	0.3334	0.4167	0.1257	0.1419
⅞	0.5625	12	0.5084	0.4633	0.4723	0.162	0.182
1	0.6250	11	0.5660	0.5168	0.5266	0.202	0.226
1¼	0.7500	10	0.6850	0.6309	0.6417	0.302	0.334
1½	0.8750	9	0.8028	0.7427	0.7547	0.419	0.462
2	1.0000	8	0.9188	0.8512	0.8647	0.551	0.606
2½	1.1250	7	1.0322	0.9549	0.9704	0.693	0.763
3	1.2500	7	1.1572	1.0799	1.0954	0.890	0.969
3½	1.3750	6	1.2667	1.1766	1.1946	1.054	1.155
4	1.5000	6	1.3917	1.3016	1.3196	1.294	1.405
5	1.7500	5	1.6201	1.5119	1.5335	1.74	1.90
6	2.0000	4½	1.8557	1.7353	1.7594	2.30	2.50
7	2.2500	4½	2.1057	1.9853	2.0094	3.02	3.25
8	2.5000	4	2.3376	2.2023	2.2294	3.72	4.00
9	2.7500	4	2.5876	2.4523	2.4794	4.62	4.93
10	3.0000	4	2.8376	2.7023	2.7294	5.62	5.97
11	3.2500	4	3.0876	2.9523	2.9794	6.72	7.10
12	3.5000	4	3.3376	3.2023	3.2294	7.92	8.33
13	3.7500	4	3.5876	3.4523	3.4794	9.21	9.66
14	4.0000	4	3.8376	3.7023	3.7294	10.61	11.08

\* British: effective diameter.  
 † See formula under definition of tensile stress area in Appendix B of ANSI B1.1-1987.  
 ‡ Design form. See Fig. 2B in ANSI B1.1-1982 or Fig. 1 in 1989 revision.  
 § Secondary sizes.  
 SOURCE: ANSI B1.1-1982, revised 1989; reproduced by permission.

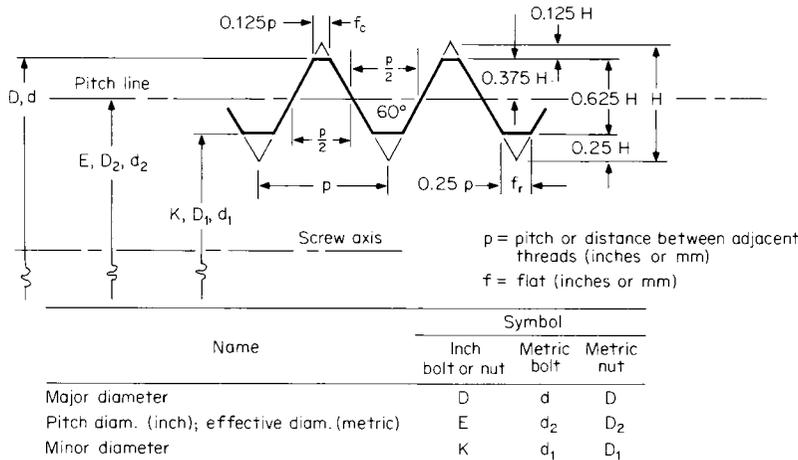


Fig. 8.2.1 Basic thread profile.

Table 8.2.3 Basic Dimensions for Fine Thread Series (UNF/UNRF)

Nominal size, in	Basic major diameter $D$ , in	Threads per inch $n$	Basic pitch diameter* $E$ , in	UNR design	Basic minor diameter internal $K$ , in	Section at minor diameter at $D - 2h_b$ , in <sup>2</sup>	Tensile stress area, ‡ in <sup>2</sup>
				minor diameter external † $K_s$ , in			
0 (0.060)	0.0600	80	0.0519	0.0451	0.0465	0.00151	0.00180
1 (0.073)§	0.0730	72	0.0640	0.0565	0.0580	0.00237	0.00278
2 (0.086)	0.0860	64	0.0759	0.0674	0.0691	0.00339	0.00394
3 (0.099)§	0.0990	56	0.0874	0.0778	0.0797	0.00451	0.00523
4 (0.112)	0.1120	48	0.0985	0.0871	0.0894	0.00566	0.00661
5 (0.125)	0.1250	44	0.1102	0.0979	0.1004	0.00716	0.00830
6 (0.138)	0.1380	40	0.1218	0.1082	0.1109	0.00874	0.01015
8 (0.164)	0.1640	36	0.1460	0.1309	0.1339	0.01285	0.01474
10 (0.190)	0.1900	32	0.1697	0.1528	0.1562	0.0175	0.0200
12 (0.216)§	0.2160	28	0.1928	0.1734	0.1773	0.0226	0.0258
¼	0.2500	28	0.2268	0.2074	0.2113	0.0326	0.0364
⅓	0.3125	24	0.2854	0.2629	0.2674	0.0524	0.0580
⅜	0.3750	24	0.3479	0.3254	0.3299	0.0809	0.0878
½	0.4375	20	0.4050	0.3780	0.3834	0.1090	0.1187
⅝	0.5000	20	0.4675	0.4405	0.4459	0.1486	0.1599
¾	0.5625	18	0.5264	0.4964	0.5024	0.189	0.203
⅞	0.6250	18	0.5889	0.5589	0.5649	0.240	0.256
1	0.7500	16	0.7094	0.6763	0.6823	0.351	0.373
1 ⅛	0.8750	14	0.8286	0.7900	0.7977	0.480	0.509
1 ¼	1.0000	12	0.9459	0.9001	0.9098	0.625	0.663
1 ⅜	1.1250	12	1.0709	1.0258	1.0348	0.812	0.856
1 ½	1.2500	12	1.1959	1.1508	1.1598	1.024	1.073
1 ⅝	1.3750	12	1.3209	1.2758	1.2848	1.260	1.315
1 ¾	1.5000	12	1.4459	1.4008	1.4098	1.521	1.581

\* British: effective diameter.

† See formula under definition of tensile stress area in Appendix B of ANSI B1.1-1982.

‡ Design form. See Fig. 2B of ANSI B1.1-1982 or Fig. 1 in 1989 revision.

§ Secondary sizes.

SOURCE: ANSI B1.1-1982, revised 1989; reproduced by permission.

Table 8.2.4 Basic Dimensions for Extra-Fine Thread Series (UNEF/UNREF)

Nominal size, in		Basic major diameter $D$ , in	Threads per inch $n$	Basic pitch diameter* $E$ , in	UNR design minor diameter external † $K_s$ , in	Basic minor diameter internal $K$ , in	Section at minor diameter at $D - 2h_b$ , in <sup>2</sup>	Tensile stress area, ‡ in <sup>2</sup>
Primary	Secondary							
	12 (0.216)	0.2160	32	0.1957	0.1788	0.1822	0.0242	0.0270
¼		0.2500	32	0.2297	0.2128	0.2162	0.0344	0.0379
⅓		0.3125	32	0.2922	0.2753	0.2787	0.0581	0.0625
⅜		0.3750	32	0.3547	0.3378	0.3412	0.0878	0.0932
½		0.4375	28	0.4143	0.3949	0.3988	0.1201	0.1274
⅝		0.5000	28	0.4768	0.4573	0.4613	0.162	0.170
¾		0.5625	24	0.5354	0.5129	0.5174	0.203	0.214
⅞		0.6250	24	0.5979	0.5754	0.5799	0.256	0.268
1		0.6875	24	0.6604	0.6379	0.6424	0.315	0.329
1 ⅛		0.7500	20	0.7175	0.6905	0.6959	0.369	0.386
1 ¼		0.8125	20	0.7800	0.7530	0.7584	0.439	0.458
1 ⅜		0.8750	20	0.8425	0.8155	0.8209	0.515	0.536
1 ½		0.9375	20	0.9050	0.8780	0.8834	0.598	0.620
1 ⅝		1.0000	20	0.9675	0.9405	0.9459	0.687	0.711
1 ¾	1 ⅛	1.0625	18	1.0264	0.9964	1.0024	0.770	0.799
1 ⅞	1 ¾	1.1250	18	1.0889	1.0589	1.0649	0.871	1.901
2	1 ⅞	1.1875	18	1.1514	1.1214	1.1274	0.977	1.009
2 ⅛		1.2500	18	1.2139	1.1839	1.1899	1.090	1.123
2 ¼	1 ⅞	1.3125	18	1.2764	1.2464	1.2524	1.208	1.244
2 ⅜	2 ¼	1.3750	18	1.3389	1.3089	1.3149	1.333	1.370
2 ½	2 ⅜	1.4375	18	1.4014	1.3714	1.3774	1.464	1.503
2 ⅝		1.5000	18	1.4639	1.4339	1.4399	1.60	1.64
2 ¾	2 ⅝	1.5625	18	1.5264	1.4964	1.5024	1.74	1.79
2 ⅞		1.6250	18	1.5889	1.5589	1.5649	1.89	1.94
3	2 ⅞	1.6875	18	1.6514	1.6214	1.6274	2.05	2.10

\* British: effective diameter.

† Design form. See Fig. 2B in ANSI B1.1-1982 or Fig. 1 in 1989 revision.

‡ See formula under definition of tensile stress area in Appendix B in ANSI B1.1-1982.

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8-14 MACHINE ELEMENTS

Table 8.2.5 ISO Metric Screw Thread Standard Series

Nominal size diam, mm			Pitches, mm														Nominal size diam, mm
			Series with graded pitches		Series with constant pitches												
Column*			Coarse	Fine	6	4	3	2	1.5	1.25	1	0.75	0.5	0.35	0.25	0.2	
0.25			0.075	—	—	—	—	—	—	—	—	—	—	—	—	—	0.25
0.3			0.8	—	—	—	—	—	—	—	—	—	—	—	—	—	0.3
	0.35		0.09	—	—	—	—	—	—	—	—	—	—	—	—	—	0.35
0.4			0.1	—	—	—	—	—	—	—	—	—	—	—	—	—	0.4
	0.45		0.1	—	—	—	—	—	—	—	—	—	—	—	—	—	0.45
0.5			0.125	—	—	—	—	—	—	—	—	—	—	—	—	—	0.5
	0.55		0.125	—	—	—	—	—	—	—	—	—	—	—	—	—	0.55
0.6			0.15	—	—	—	—	—	—	—	—	—	—	—	—	—	0.6
	0.7		0.175	—	—	—	—	—	—	—	—	—	—	—	—	—	0.7
0.8			0.2	—	—	—	—	—	—	—	—	—	—	—	—	—	0.8
	0.9		0.225	—	—	—	—	—	—	—	—	—	—	—	—	—	0.9
1			0.25	—	—	—	—	—	—	—	—	—	—	—	—	0.2	1
	1.1		0.25	—	—	—	—	—	—	—	—	—	—	—	—	0.2	1.1
1.2			0.25	—	—	—	—	—	—	—	—	—	—	—	—	0.2	1.2
	1.4		0.3	—	—	—	—	—	—	—	—	—	—	—	—	0.2	1.4
1.6			0.35	—	—	—	—	—	—	—	—	—	—	—	—	0.2	1.6
	1.8		0.35	—	—	—	—	—	—	—	—	—	—	—	—	0.2	1.8
2			0.4	—	—	—	—	—	—	—	—	—	—	—	0.25	—	2
	2.5		0.45	—	—	—	—	—	—	—	—	—	—	—	0.25	—	2.2
2.5			0.45	—	—	—	—	—	—	—	—	—	—	0.35	—	—	2.5
3			0.5	—	—	—	—	—	—	—	—	—	—	0.35	—	—	3
	3.5		0.6	—	—	—	—	—	—	—	—	—	—	0.35	—	—	3.5
4			0.7	—	—	—	—	—	—	—	—	—	0.5	—	—	—	4
	4.5		0.75	—	—	—	—	—	—	—	—	—	0.5	—	—	—	4.5
5			0.8	—	—	—	—	—	—	—	—	—	0.5	—	—	—	5
	5.5		—	—	—	—	—	—	—	—	—	—	0.5	—	—	—	5.5
6			1	—	—	—	—	—	—	—	—	0.75	—	—	—	—	6
	7		1	—	—	—	—	—	—	—	—	0.75	—	—	—	—	7
8			1.25	1	—	—	—	—	—	—	1	0.75	—	—	—	—	8
	9		1.25	—	—	—	—	—	—	—	1	0.75	—	—	—	—	9
10			1.5	1.25	—	—	—	—	—	1.25	1	0.75	—	—	—	—	10
	11		1.5	—	—	—	—	—	—	—	1	0.75	—	—	—	—	11
12			1.75	1.25	—	—	—	—	1.5	1.25	1	—	—	—	—	—	12
	14		2	1.5	—	—	—	—	1.5	1.25†	1	—	—	—	—	—	14
	15		—	—	—	—	—	—	1.5	—	1	—	—	—	—	—	15
16			2	1.5	—	—	—	—	1.5	—	1	—	—	—	—	—	16
	17		—	—	—	—	—	—	1.5	—	1	—	—	—	—	—	17
	18		2.5	1.5	—	—	—	2	1.5	—	1	—	—	—	—	—	18
20			2.5	1.5	—	—	—	2	1.5	—	1	—	—	—	—	—	20
	22		2.5	1.5	—	—	—	2	1.5	—	1	—	—	—	—	—	22
24			3	2	—	—	—	2	1.5	—	1	—	—	—	—	—	24
	25		—	—	—	—	—	2	1.5	—	1	—	—	—	—	—	25
	26		—	—	—	—	—	—	1.5	—	1	—	—	—	—	—	26
	27		3	2	—	—	—	2	1.5	—	1	—	—	—	—	—	27
	28		—	—	—	—	—	2	1.5	—	1	—	—	—	—	—	28
30			3.5	2	—	—	(3)	2	1.5	—	1	—	—	—	—	—	30
	32		—	—	—	—	—	2	1.5	—	—	—	—	—	—	—	32
	33		3.5	2	—	—	(3)	2	1.5	—	—	—	—	—	—	—	33
	35‡		—	—	—	—	—	—	1.5	—	—	—	—	—	—	—	35‡
36			4	3	—	—	—	2	1.5	—	—	—	—	—	—	—	36
	38		—	—	—	—	—	—	1.5	—	—	—	—	—	—	—	38
	39		4	3	—	—	—	2	1.5	—	—	—	—	—	—	—	39
	40		—	—	—	—	3	2	1.5	—	—	—	—	—	—	—	40
42			4.5	3	—	—	4	3	2	1.5	—	—	—	—	—	—	42
	45		4.5	3	—	—	4	3	2	1.5	—	—	—	—	—	—	45

\* Thread diameter should be selected from column 1, 2 or 3; with preference being given in that order.

† Pitch 1.25 mm in combination with diameter 14 mm has been included for spark plug applications.

‡ Diameter 35 mm has been included for bearing locknut applications.

NOTE: The use of pitches shown in parentheses should be avoided wherever possible. The pitches enclosed in the bold frame, together with the corresponding nominal diameters in columns 1 and 2, are those combinations which have been established by ISO Recommendations as a selected "coarse" and "fine" series for commercial fasteners. Sizes 0.25 mm through 1.4 mm are covered in ISO Recommendation R68 and, except for the 0.25-mm size, in ANSI B1.10.

SOURCE: ISO 261-1973, reproduced by permission.

**Table 8.2.6 Limiting Dimensions of Standard Series Threads for Commercial Screws, Bolts, and Nuts**

Nominal size diam, mm	Pitch p, mm	Basic thread designation	External thread (bolt), mm									Internal thread (nut), mm						Major diam, min
			Tol. class	Allowance	Major diameter		Pitch diameter			Minor diameter		Tol. class	Minor diameter		Pitch diameter			
					Max	Min	Max	Min	Tol.	Max*	Min†		Min	Max	Min	Max	Tol.	
1.6	0.35	M1.6	6g	0.019	1.581	1.496	1.354	1.291	0.063	1.151	1.063	6H	1.221	1.321	1.373	1.458	0.085	1.600
1.8	0.35	M1.8	6g	0.019	1.781	1.696	1.554	1.491	0.063	1.351	1.263	6H	1.421	1.521	1.573	1.568	0.085	1.800
2	0.4	M2	6g	0.019	1.981	1.886	1.721	1.654	0.067	1.490	1.394	6H	1.567	1.679	1.740	1.830	0.090	2.000
2.2	0.45	M2.2	6g	0.020	2.180	2.080	1.888	1.817	0.071	1.628	1.525	6H	1.713	1.838	1.908	2.000	0.095	2.200
2.5	0.45	M2.5	6g	0.020	2.480	2.380	2.188	2.117	0.071	1.928	1.825	6H	2.013	2.138	2.208	2.303	0.095	2.500
3	0.5	M3	6g	0.020	2.980	2.874	2.655	2.580	0.075	2.367	2.256	6H	2.459	2.599	2.675	2.775	0.100	3.000
3.5	0.6	M3.5	6g	0.021	3.479	3.354	3.089	3.004	0.085	2.742	2.614	6H	2.850	3.010	3.110	3.222	0.112	3.500
4	0.7	M4	6g	0.022	3.978	3.838	3.523	3.433	0.090	3.119	2.979	6H	3.242	3.422	3.545	3.663	0.118	4.000
4.5	0.75	M4.5	6g	0.022	4.478	4.338	3.991	3.901	0.090	3.558	3.414	6H	3.688	3.878	4.013	4.131	0.118	4.500
5	0.8	M5	6g	0.024	4.976	4.826	4.456	4.361	0.095	3.994	3.841	6H	4.134	4.334	4.480	4.605	0.125	5.000
6	1	M6	6g	0.026	5.974	5.794	5.324	5.212	0.112	4.747	4.563	6H	4.917	5.153	5.350	5.500	0.150	6.000
7	1	M7	6g	0.026	6.974	6.794	6.234	6.212	0.112	5.747	5.563	6H	5.917	6.153	6.350	6.500	0.150	7.000
8	1.25	M8	6g	0.028	7.972	7.760	7.160	7.042	0.118	6.439	6.231	6H	6.647	6.912	7.188	7.348	0.160	8.000
	1	M8 × 1	6g	0.026	7.974	7.794	7.324	7.212	0.112	6.747	6.563	6H	6.918	7.154	7.350	7.500	0.150	8.000
10	1.5	M10	6g	0.032	9.968	9.732	8.994	8.862	0.132	8.127	7.879	6H	8.376	8.676	9.026	9.206	0.180	10.000
	1.25	M10 × 1.25	6g	0.028	9.972	9.760	9.160	9.042	0.118	8.439	8.231	6H	8.646	8.911	9.188	9.348	0.160	10.000
12	1.75	M12	6g	0.034	11.966	11.701	10.829	10.679	0.150	9.819	9.543	6H	10.106	10.441	10.863	11.063	0.200	12.000
	1.25	M12 × 1.25	6g	0.028	11.972	11.760	11.160	11.028	0.118	10.439	10.217	6H	10.646	10.911	11.188	11.368	0.180	12.000
14	2	M14	6g	0.038	13.962	13.682	12.663	12.503	0.160	11.508	11.204	6H	11.835	12.210	12.701	12.913	0.212	14.000
	1.5	M14 × 1.5	6g	0.032	13.968	13.732	12.994	12.854	0.140	12.127	11.879	6H	12.376	12.676	13.026	13.216	0.190	14.000
16	2	M16	6g	0.038	15.962	15.682	14.663	14.503	0.160	13.508	13.204	6H	13.385	14.210	14.701	14.913	0.212	16.000
	1.5	M16 × 1.5	6g	0.032	15.968	15.732	14.994	14.854	0.140	14.127	13.879	6H	14.376	14.676	15.026	15.216	0.190	16.000
18	2.5	M18	6g	0.038	17.958	17.623	16.334	16.164	0.170	14.891	14.541	6H	15.294	15.744	16.376	16.600	0.224	18.000
	1.5	M18 × 1.5	6g	0.032	17.968	17.732	16.994	15.854	0.140	16.127	15.879	6H	16.376	16.676	17.026	17.216	0.190	18.000
20	2.5	M20	6g	0.042	19.958	19.623	18.334	18.164	0.170	16.891	16.541	6H	17.294	17.744	18.376	18.600	0.224	20.000
	1.5	M20 × 1.5	6g	0.032	19.968	19.732	18.994	18.854	0.140	18.127	17.879	6H	18.376	18.676	19.026	19.216	0.190	20.000
22	2.5	M22	6g	0.042	21.958	21.623	20.334	20.164	0.170	18.891	18.541	6H	19.294	19.744	20.376	20.600	0.224	22.000
	1.5	M22 × 1.5	6g	0.032	21.968	21.732	20.994	20.854	0.140	20.127	19.879	6H	20.376	20.676	21.026	21.216	0.190	22.000
24	3	M24	6g	0.048	23.952	23.577	22.003	21.803	0.200	20.271	19.855	6H	20.752	21.252	22.051	22.316	0.265	24.000
	2	M24 × 2	6g	0.038	23.962	23.682	22.663	22.493	0.170	21.508	21.194	6H	21.835	22.210	22.701	22.925	0.224	24.000
27	3	M27	6g	0.048	26.952	26.577	25.003	24.803	0.200	23.271	22.855	6H	23.752	24.252	25.051	25.316	0.265	27.000
	2	M27 × 2	6g	0.038	26.962	26.682	25.663	25.493	0.170	24.508	24.194	6H	24.835	25.210	25.701	25.925	0.224	27.000
30	3.5	M30	6g	0.053	29.947	29.522	27.674	27.462	0.212	25.653	25.189	6H	26.211	26.771	27.727	28.007	0.280	30.000
	2	M30 × 2	6g	0.038	29.962	29.682	28.663	28.493	0.170	27.508	27.194	6H	27.835	28.210	28.701	28.925	0.224	30.000
33	3.5	M33	6g	0.053	32.947	32.522	30.674	30.462	0.212	28.653	28.189	6H	29.211	29.771	30.727	31.007	0.280	33.000
	2	M33 × 2	6g	0.038	32.962	32.682	31.663	31.493	0.170	30.508	30.194	6H	30.835	31.210	31.701	31.925	0.224	33.000
36	4	M36	6g	0.060	35.940	35.465	33.342	33.118	0.224	31.033	30.521	6H	31.670	32.270	33.402	33.702	0.300	36.000
	3	M36 × 3	6g	0.048	35.952	35.577	34.003	33.803	0.200	32.271	31.855	6H	32.752	33.252	34.051	34.316	0.265	36.000
39	4	M39	6g	0.060	38.940	38.465	36.342	36.118	0.224	34.033	33.521	6H	34.670	35.270	36.402	36.702	0.300	39.000
	3	M39 × 3	6g	0.048	38.952	38.577	37.003	36.803	0.200	35.271	34.855	6H	35.752	36.252	37.051	37.316	0.265	39.000

\* Design form, see Figs. 2 and 5 of ANSI B1.13M-1979 (or Figs. 1 and 4 in 1983 revision).

† Required for high-strength applications where rounded root is specified.

SOURCE: [Appeared in ASME/SAE Interpretive document, Metric Screw Threads, B1.13 (Nov. 3, 1966), pp. 9, 10.] ISO 261-1973, reproduced by permission.

and it is shown in Fig. 8.2.1. The ISO thread series (see Table 8.2.5) are those published in ISO 261-1973. Increased overseas business sparked U.S. interest in metric screw threads, and the ANSI, through its Special Committee to Study Development of an Optimum Metric Fastener System, in joint action with an ISO working group (ISO/TC 1/TC 2), established compromise recommendations regarding metric screw threads. The approved results appear in ANSI B1.13-1979 (Table 8.2.6). This ANSI metric thread series is essentially a selected subset (boxed-in portion of Table 8.2.5) of the larger ISO 261-1973 set. The M profiles of tolerance class 6H/6g are intended for metric applications where inch class 2A/2B has been used.

**Metric Tolerance Classes for Threads** Tolerance classes are a selected combination of tolerance grades and tolerance positions applied to length-of-engagement groups.

Tolerance grades are indicated as numbers for crest diameters of nut and bolt and for pitch diameters of nut and bolt. Tolerance is the acceptable variation permitted on any such diameter.

Tolerance positions are indicated as letters, and are allowances (fundamental deviations) as dictated by field usage or conditions. Capital letters are used for internal threads (nut) and lower case for external threads (bolt).

There are three established groups of length of thread engagement, S (short), N (normal), and L (long), for various diameter-pitch combinations. Normal length of thread engagement is calculated from the formula  $N = 4.5pd^{0.2}$ , where  $p$  is pitch and  $d$  is the smallest nominal size within each of a series of groupings of nominal sizes.

In conformance with coating (or plating) requirements and demands of ease of assembly, the following tolerance positions have been established:

Bolt	Nut	
e		Large allowance
g	G	Small allowance
h	H	No allowance

See Table 8.2.7 for preferred tolerance classes.

**Table 8.2.7 Preferred Tolerance Classes**

Quality	Length of engagement														
	External threads (bolts)						Internal threads (nuts)								
	Tolerance position e (large allowance)			Tolerance position g (small allowance)			Tolerance position h (no allowance)			Tolerance position G (small allowance)			Tolerance position H (no allowance)		
	Group S	Group N	Group L	Group S	Group N	Group L	Group S	Group N	Group L	Group S	Group N	Group L	Group S	Group N	Group L
Fine							3h4h	4h	4h5h	5G	6G	7G	4H	5H	6H
Medium		6e	7e6e	5g6g	6g	7g6g	5h6h	6h	7h6h	5G	6G	7G	5H	6H	7H
Coarse					8g	9g8g					7G	8G		7H	8H

NOTE: Fine quality applies to precision threads where little variation in fit character is permissible. Coarse quality applies to those threads which present manufacturing difficulties, such as the threading of hot-rolled bars or tapping deep blind holes.

SOURCE: ISO 261-1973, reproduced by permission.

**Table 8.2.8 Acme Thread Series**

( $D$  = outside diam,  $p$  = pitch. All dimensions in inches.)  
(See Figs. 8.2.2 to 8.2.5.)

Symbols	Thread dimensions			
	29° general purpose	29° stub	60° stub	10° modified
$t$ = thickness of thread	$0.5p$	$0.5p$	$0.5p$	$0.5p$
$R$ = basic depth of thread	$0.5p$	$0.3p$	$0.433p$	$0.5p^*$
$F$ = basic width of flat	$0.3707p$	$0.4224p$	$0.250p$	$0.4563p^\dagger$
$G$ = (see Figs. 8.2.2, 8.2.3, 8.2.4)	$F - (0.52 \times \text{clearance})$	$F - (0.52 \times \text{clearance})$	$0.227p$	$F - (0.17 \times \text{clearance})$
$E$ = basic pitch diam	$D - 0.5p$	$D - 0.3p$	$D - 0.433p$	$D - 0.5p$
$K$ = basic minor diam	$D - p$	$D - 0.6p$	$D - 0.866p$	$D - p$
Range of threads, per inch	1-16	2-16	4-16	

\* A clearance of at least 0.010 in is added to  $h$  on threads of 10-pitch and coarser, and 0.005 in on finer pitches, to produce extra depth, thus avoiding interference with threads of mating parts of a minor or major diameters.

† Measured at crest of screw thread.

ISO metric screw threads are designated by a set of number and letter symbols signifying, in sequence, metric symbol, nominal size,  $\times$  (symbol), pitch, tolerance grade (on pitch diameter), tolerance position (for pitch diameter), tolerance grade (on crest diameter), and tolerance position (for crest diameter).

EXAMPLE.  $M6 \times 0.75-5g6g$ , where M = metric symbol; 6 = nominal size,  $\times$  = symbol; 0.75 = pitch-axial distance of adjacent threads measured between corresponding thread points (millimeters); 5 = tolerance grade (on pitch diameter); g = tolerance position (for pitch diameter); 6 = tolerance grade (on crest diameter); g = tolerance position (for crest diameter).

**Power Transmission Screw Threads: Forms and Proportions**

The Acme thread appears in four series [ANSI B1.8-1973 (revised 1988) and B1.5-1977]. Generalized dimensions for the series are given in Table 8.2.8.

The 29° general-purpose thread (Fig. 8.2.2) is used for all Acme thread applications outside of special design cases.

The 29° stub thread (Fig. 8.2.3) is used for heavy-loading designs and where space constraints or economic factors make a shallow thread advantageous.

The 60° stub thread (Fig. 8.2.4) finds special applications in the machine-tool industry.

The 10° modified square thread (Fig. 8.2.5) is, for all practical purposes, equivalent to a "square" thread.

For selected Acme diameter-pitch combinations, see Table 8.2.9.

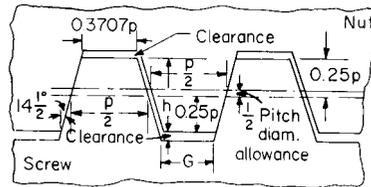
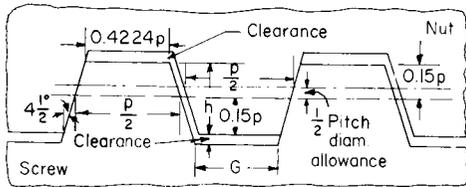


Fig. 8.2.2 29° Acme general-purpose thread.

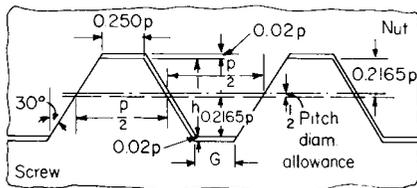
**Table 8.2.9 Acme Thread Diameter-Pitch Combinations**  
(See Figs. 8.2.2 to 8.2.5.)

Size	Threads per inch	Size	Threads per inch	Size	Threads per inch	Size	Threads per inch	Size	Threads per inch
1/4	16	3/8	8	1 1/4	5	2 1/4	3	4	2
5/16	14	3/4	6	1 3/8	4	2 1/2	3	4 1/2	2
3/8	12	7/8	6	1 1/2	4	2 3/4	3	5	2
7/16	12	1	5	1 3/4	4	3	2		
1/2	10	1 1/8	5	2	4	3 1/2	2		

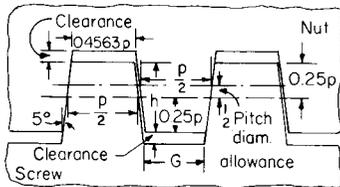
Three classes (2G, 3G, 4G) of general-purpose threads have clearances on all diameters for free movement. A fourth class (5G) of general-purpose threads has no allowance or clearance on the pitch diameter for purposes of minimum end play or backlash.



**Fig. 8.2.3** 29° stub Acme thread.



**Fig. 8.2.4** 60° stub Acme thread.



**Fig. 8.2.5** 10° modified square thread.

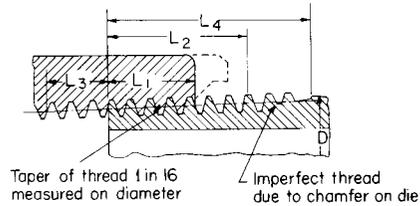
**High-Strength Bolting Screw Threads**

High-strength bolting applications include pressure vessels, steel pipe flanges, fittings, valves, and other services. They can be used for either hot or cold surfaces where high tensile stresses are produced when the joints are made up. For sizes 1 in and smaller, the ANSI coarse-thread series is used. For larger sizes, the ANSI 8-pitch thread series is used (see Table 8.2.10).

**Screw Threads for Pipes**

**American National Standard Taper Pipe Thread (ANSI/ASME B1.20.1-1983)** This thread is shown in Fig. 8.2.6. It is made to the following specifications: The taper is 1 in 16 or 0.75 in/ft. The basic length of the external taper thread is determined by  $L_2 = p(0.8D + 6.8)$ , where  $D$  is the basic outside diameter of the pipe (see Table 8.2.11). Thread designation and notation is written as: nominal size, number of threads per inch, thread series, class. For example: 3/8-18 NPT, 1/8-27 NPSC, 1/2-14 NPTR, 1/8-27 NPSM, 1/8-27 NPSL, 1-11.5 NPSH, where N = National (American) Standard, T = taper, C = coupling, S = straight, M = mechanical, L = locknut, H = hose coupling, and R = rail fittings. Where pressure-tight joints are required, it is intended that taper pipe threads be made up wrench-tight with a sealant. Descriptions of thread

series include: NPSM = free-fitting mechanical joints for fixtures, NPSL = loose-fitting mechanical joints with locknuts, NPSH = loose-fitting mechanical joints for hose coupling.



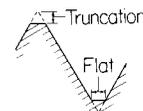
**Fig. 8.2.6** American National Standard taper pipe threads.

**American National Standard Straight Pipe Thread (ANSI/ASME B1.20.1-1983)** This thread can be used to advantage for the following: (1) pressure-tight joints with sealer; (2) pressure-tight joints without sealer for drain plugs, filler plugs, etc.; (3) free-fitting mechanical joints for fixtures; (4) loose-fitting mechanical joints with locknuts; and (5) loose-fitting mechanical joints for hose couplings. Dimensions are shown in Table 8.2.12.

**American National Standard Dry-Seal Pipe Threads (ANSI B1.20.3-1976 (inch), ANSI B1.20.4-1976 (metric translation))** Thread designation and notation include nominal size, number of threads per inch, thread series, class. For example, 1/8-27 NPTF-1, 1/8-27 NPTF-2, 1/8-27 PTF-SAE short, 1/8-27 NPSI, where N = National (American) standard, P = pipe, T = taper, S = straight, F = fuel and oil, I = intermediate. NPTF has two classes: class 1 = specific inspection of root and crest truncation *not* required; class 2 = specific inspection of root and crest truncation *is* required. The series includes: NPTF for all types of service; PTF-SAE short where clearance is not sufficient for full thread length as NPTF; NPSF, nontapered, economical to produce, and used with soft or ductile materials; NPSI nontapered, thick sections with little expansion.

Dry-seal pipe threads resemble tapered pipe threads except the form is truncated (see Fig. 8.2.7), and  $L_4 = L_2 + 1$  (see Fig. 8.2.6). Although these threads are designed for nonlubricated joints, as in automobile work, under certain conditions a lubricant is used to prevent galling. Table 8.2.13 lists truncation values.

Tap drill sizes for tapered and straight pipe threads are listed in Table 8.2.14.



**Fig. 8.2.7** American National Standard dry-seal pipe thread.

Wrench bolt heads, nuts, and wrench openings have been standardized (ANSI 18.2-1972). Wrench openings are given in Table 8.2.15; bolt head and nut dimensions are in Table 8.2.16.

**Machine Screws**

Machine screws are defined according to head types as follows:

**Flat Head** This screw has a flat surface for the top of the head with a

**Table 8.2.10 Screw Threads for High-Strength Bolting**  
(All dimensions in inches)

Size	Threads per inch	Allowance (minus)	Major diam	Major diam tolerance	Max pitch diam*	Max pitch diam tolerance	Minor diam max	Nut max minor diam	Nut max minor diam tolerance	Nut max pitch diam*	Nut max pitch diam tolerance
¼	20	0.0010	0.2490	0.0072	0.2165	0.0026	0.1877	0.2060	0.0101	0.2211	0.0036
⅜	18	0.0011	0.3114	0.0082	0.2753	0.0030	0.2432	0.2630	0.0106	0.2805	0.0041
½	16	0.0013	0.3737	0.0090	0.3331	0.0032	0.2990	0.3184	0.0111	0.3389	0.0045
¾	14	0.0013	0.4362	0.0098	0.3898	0.0036	0.3486	0.3721	0.0119	0.3960	0.0049
1	13	0.0015	0.4985	0.0104	0.4485	0.0037	0.4041	0.4290	0.0123	0.4552	0.0052
1 ⅛	12	0.0016	0.5609	0.0112	0.5068	0.0040	0.4587	0.4850	0.0127	0.5140	0.0056
1 ¼	11	0.0017	0.6233	0.0118	0.5643	0.0042	0.5118	0.5397	0.0131	0.5719	0.0059
1 ½	10	0.0019	0.7481	0.0128	0.6831	0.0045	0.6254	0.6553	0.0136	0.6914	0.0064
1 ¾	9	0.0021	0.8729	0.0140	0.8007	0.0049	0.7366	0.7689	0.0142	0.8098	0.0070
2	8	0.0022	0.9978	0.0152	0.9166	0.0054	0.8444	0.8795	0.0148	0.9264	0.0076
2 ⅛	8	0.0024	1.1226	0.0152	1.0414	0.0055	0.9692	1.0045	0.0148	1.0517	0.0079
2 ¼	8	0.0025	1.2475	0.0152	1.1663	0.0058	1.0941	1.1295	0.0148	1.1771	0.0083
2 ½	8	0.0025	1.3725	0.0152	1.2913	0.0061	1.2191	1.2545	0.0148	1.3024	0.0086
2 ¾	8	0.0027	1.4973	0.0152	1.4161	0.0063	1.3439	1.3795	0.0148	1.4278	0.0090
3	8	0.0028	1.6222	0.0152	1.5410	0.0065	1.4688	1.5045	0.0148	1.5531	0.0093
3 ⅛	8	0.0029	1.7471	0.0152	1.6659	0.0068	1.5937	1.6295	0.0148	1.6785	0.0097
3 ¼	8	0.0030	1.8720	0.0152	1.7908	0.0070	1.7186	1.7545	0.0148	1.8038	0.0100
3 ½	8	0.0031	1.9969	0.0152	1.9157	0.0073	1.8435	1.8795	0.0148	1.9294	0.0104
4	8	0.0032	2.1218	0.0152	2.0406	0.0075	1.9682	2.0045	0.0148	2.0545	0.0107
4 ⅛	8	0.0033	2.2467	0.0152	2.1655	0.0077	2.0933	2.1295	0.0148	2.1798	0.0110
4 ¼	8	0.0035	2.4965	0.0152	2.4153	0.0082	2.3431	2.3795	0.0148	2.4305	0.0117
4 ½	8	0.0037	2.7463	0.0152	2.6651	0.0087	2.5929	2.6295	0.0148	2.6812	0.0124
5	8	0.0038	2.9962	0.0152	2.9150	0.0092	2.8428	2.8795	0.0148	2.9318	0.0130
5 ⅛	8	0.0039	3.2461	0.0152	3.1649	0.0093	3.0927	3.1295	0.0148	3.1820	0.0132
5 ¼	8	0.0040	3.4960	0.0152	3.4148	0.0093	3.3426	3.3795	0.0148	3.4321	0.0133

The Unified form of thread shall be used. Pitch diameter tolerances include errors of lead and angle.

\* The maximum pitch diameters of screws are smaller than the minimum pitch diameters of nuts by these amounts.

**Table 8.2.11 ANSI Taper Pipe Thread**  
(All dimensions in inches)  
(See Fig. 8.2.6.)

Nominal pipe size	OD of pipe	Threads per inch	Pitch of thread	Hand-tight engagement length $L_1$	Effective thread external length $L_2$	Wrench makeup length for internal thread $L_3$	Overall length external thread $L_4$
⅛	0.3125	27	0.03704	0.160	0.2611	0.1111	0.3896
¼	0.405	27	0.03704	0.180	0.2639	0.1111	0.3924
⅜	0.540	18	0.05556	0.200	0.4018	0.1667	0.5946
½	0.675	18	0.05556	0.240	0.4078	0.1667	0.6006
¾	0.840	14	0.07143	0.320	0.5337	0.2143	0.7815
1	1.050	14	0.07143	0.339	0.5457	0.2143	0.7935
1 ⅛	1.315	11½	0.08696	0.400	0.6828	0.2609	0.9845
1 ¼	1.660	11½	0.08696	0.420	0.7068	0.2609	1.0085
1 ½	1.900	11½	0.08696	0.420	0.7235	0.2609	1.0252
2	2.375	11½	0.08696	0.436	0.7565	0.2609	1.0582
2 ½	2.875	8	0.12500	0.682	1.1375	0.2500	1.5712
3	3.500	8	0.12500	0.766	1.2000	0.2500	1.6337
3 ½	4.000	8	0.12500	0.821	1.2500	0.2500	1.6837
4	4.500	8	0.12500	0.844	1.3000	0.2500	1.7337
5	5.563	8	0.12500	0.937	1.4063	0.2500	1.8400
6	6.625	8	0.12500	0.958	1.5125	0.2500	1.9462
8	8.625	8	0.12500	1.063	1.7125	0.2500	2.1462
10	10.750	8	0.12500	1.210	1.9250	0.2500	2.3587
12	12.750	8	0.12500	1.360	2.1250	0.2500	2.5587
14 OD	14.000	8	0.12500	1.562	2.2500	0.2500	2.6837
16 OD	16.000	8	0.12500	1.812	2.4500	0.2500	2.8837
18 OD	18.000	8	0.12500	2.000	2.6500	0.2500	3.0837
20 OD	20.000	8	0.12500	2.125	2.8500	0.2500	3.2837
24 OD	24.000	8	0.12500	2.375	3.2500	0.2500	3.6837

**Table 8.2.12 ANSI Straight Pipe Threads**  
(All dimensions in inches)

Nominal pipe size (1)	Threads per inch (2)	Pressure-tight with seals		Pressure-tight without seals		Free-fitting (NPSM)				Loose-fitting (NPSL)			
		Pitch diam, max (3)	Minor diam, min (4)	Pitch diam, max (5)	Minor diam, min (6)	External		Internal		External		Internal	
						Pitch diam, max (7)	Major diam, max (8)	Pitch diam, max (9)	Minor diam, min (10)	Pitch diam, max (11)	Major diam, max (12)	Pitch diam, max (13)	Minor diam, min (14)
1/8	27	0.3782	0.342	0.3736	0.3415	0.3748	0.399	0.3783	0.350	0.3840	0.409	0.3989	0.362
1/4	18	0.4951	0.440	0.4916	0.4435	0.4899	0.527	0.4951	0.453	0.5038	0.541	0.5125	0.470
3/8	18	0.6322	0.577	0.6270	0.5789	0.6270	0.664	0.6322	0.590	0.6409	0.678	0.6496	0.607
1/2	14	0.7851	0.715	0.7784	0.7150	0.7784	0.826	0.7851	0.731	0.7963	0.844	0.8075	0.753
3/4	14	0.9956	0.925	0.9889	0.9255	0.9889	1.036	0.9956	0.941	1.0067	1.054	1.0179	0.964
1	11 1/2	1.2468	1.161	1.2386	1.1621	1.2386	1.296	1.2468	1.181	1.2604	1.318	1.2739	1.208
1 1/4	11 1/2	1.5915	1.506	—	—	1.5834	1.641	1.5916	1.526	1.6051	1.663	1.6187	1.553
1 1/2	11 1/2	1.8305	1.745	—	—	1.8223	1.880	1.8305	1.764	1.8441	1.902	1.8576	1.792
2	11 1/2	2.3044	2.219	—	—	2.2963	2.354	2.3044	2.238	2.3180	2.376	2.3315	2.265
2 1/2	8	2.7739	2.650	—	—	2.7622	2.846	2.7739	2.679	2.7934	2.877	2.8129	2.718
3	8	3.4002	3.277	—	—	3.3885	3.472	3.4002	3.305	3.4198	3.503	3.4393	3.344
3 1/2	8	3.9005	3.777	—	—	3.8888	3.972	3.9005	3.806	3.9201	4.003	3.9396	3.845
4	8	4.3988	4.275	—	—	4.3871	4.470	4.3988	4.304	4.4184	4.502	4.4379	4.343
5	8	—	—	—	—	5.4493	5.533	5.4610	5.366	5.4805	5.564	5.5001	5.405
6	8	—	—	—	—	6.5060	6.589	6.5177	6.423	6.5372	6.620	6.5567	6.462
8	8	—	—	—	—	—	—	—	—	8.5313	8.615	8.5508	8.456
10	8	—	—	—	—	—	—	—	—	10.6522	10.735	10.6717	10.577
12	8	—	—	—	—	—	—	—	—	12.6491	12.732	12.6686	12.574

countersink angle of 82°. It is standard for machine screws, cap screws, and wood screws.

**Round Head** This screw has a semielliptical head and is standard for machine screws, cap screws, and wood screws except that for the cap screw it is called *button head*.

**Fillister Head** This screw has a rounded surface for the top of the head, the remainder being cylindrical. The head is standard for machine screws and cap screws.

**Oval Head** This screw has a rounded surface for the top of the head and a countersink angle of 82°. It is standard for machine screws and wood screws.

**Hexagon Head** This screw has a hexagonal head for use with external wrenches. It is standard for machine screws.

**Socket Head** This screw has an internal hexagonal socket in the head for internal wrenching. It is standard for cap screws.

These screw heads are shown in Fig. 8.2.8; pertinent dimensions are in Table 8.2.17. There are many more machine screw head shapes available to the designer for special purposes, and many are found in the literature. In addition, lots of different screw head configurations have been developed to render fasteners "tamperproof"; these, too, are found in manufacturers' catalogs or the trade literature.

#### Eyebolts

Eyebolts are classified as rivet, nut, or screw, and can be had on a swivel. See Fig. 8.2.9 and Table 8.2.18. The safe working load may be obtained for each application by applying an appropriate factor of safety.

**Driving recesses** come in many forms and types and can be found in company catalogs. Figure 8.2.10 shows a representative set.

**Setscrews** are used for fastening collars, sheaves, gears, etc. to shafts to prevent relative rotation or translation. They are available in a variety

of head and point styles, as shown in Fig. 8.2.11. A complete tabulation of dimensions is found in ANSI/ASME B18.3-1982 (R86), ANSI 18.6.2-1977 (R93), and ANSI 18.6.3-1977 (R91). Holding power for various sizes is given in Table 8.2.19.

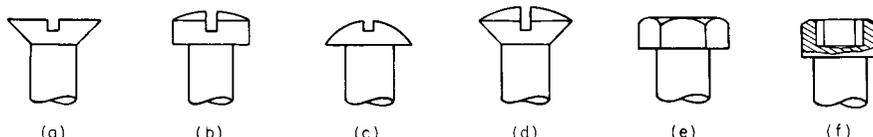
#### Locking Fasteners

Locking fasteners are used to prevent loosening of a threaded fastener in service and are available in a wide variety differing vastly in design, performance, and function. Since each has special features which may make it of particular value in the solution of a given machine problem, it is important that great care be exercised in the selection of a particular

**Table 8.2.13 ANSI Dry-Seal Pipe Threads\***  
(See Fig. 8.2.7.)

Threads per inch <i>n</i>	Truncation, in		Width of flat	
	Min	Max	Min	Max
27 Crest	0.047 <i>p</i>	0.094 <i>p</i>	0.054 <i>p</i>	0.108 <i>p</i>
Root	0.094 <i>p</i>	0.140 <i>p</i>	0.108 <i>p</i>	0.162 <i>p</i>
18 C	0.047 <i>p</i>	0.078 <i>p</i>	0.054 <i>p</i>	0.090 <i>p</i>
R	0.078 <i>p</i>	0.109 <i>p</i>	0.090 <i>p</i>	0.126 <i>p</i>
14 C	0.036 <i>p</i>	0.060 <i>p</i>	0.042 <i>p</i>	0.070 <i>p</i>
R	0.060 <i>p</i>	0.085 <i>p</i>	0.070 <i>p</i>	0.098 <i>p</i>
11 1/2 C	0.040 <i>p</i>	0.060 <i>p</i>	0.046 <i>p</i>	0.069 <i>p</i>
R	0.060 <i>p</i>	0.090 <i>p</i>	0.069 <i>p</i>	0.103 <i>p</i>
8 C	0.042 <i>p</i>	0.055 <i>p</i>	0.048 <i>p</i>	0.064 <i>p</i>
R	0.055 <i>p</i>	0.076 <i>p</i>	0.064 <i>p</i>	0.088 <i>p</i>

\* The truncation and width-of-flat proportions listed above are also valid in the metric system.



**Fig. 8.2.8** Machine screw heads. (a) Flat; (b) fillister; (c) round; (d) oval; (e) hexagonal; (f) socket.

Table 8.2.14 Suggested Tap Drill Sizes for Internal Pipe Threads

Size	Taper pipe thread										
	Minor diameter at distance			Drill for use without reamer				Straight pipe thread			
	Probable drill oversize cut (mean)	L <sub>1</sub> from large end	L <sub>1</sub> + L <sub>3</sub> from large end	Drill for use without reamer		Drill for use with reamer		Minor diameter		Theoretical drill size§	Suggested drill size†
				Theoretical drill size*	Suggested drill size†	Theoretical drill size‡	Suggested drill size†	NPSF	NPSI		
1	2	3	4	5	6	7	8	9	10	11	
Inch											
1/16-27	0.0038	0.2443	0.2374	0.2405	“.C” (0.242)	0.2336	“.A” (0.234)	0.2482	0.2505	0.2444	“.D” (0.246)
1/32-27	0.0044	0.3367	0.3298	0.3323	“.Q” (0.332)	0.3254	2 3/64 (0.328)	0.3406	0.3429	0.3362	“.R” (0.339)
1/4-18	0.0047	0.4362	0.4258	0.4315	7/17 (0.438)	0.4211	27/64 (0.422)	0.4422	0.4457	0.4375	7/16 (0.438)
3/8-18	0.0049	0.5708	0.5604	0.5659	1/16 (0.562)	0.5555	1/16 (0.563)	0.5776	0.5811	0.5727	27/64 (0.578)
1/2-14	0.0051	0.7034	0.6901	0.6983	45/64 (0.703)	0.6850	1 1/16 (0.688)	0.7133	0.7180	0.7082	45/64 (0.703)
3/4-14	0.0060	0.9127	0.8993	0.9067	29/32 (0.906)	0.8933	57/64 (0.891)	0.9238	0.9283	0.9178	59/64 (0.922)
1-11 1/2	0.0080	1.1470	1.1307	1.1390	1 9/64 (1.141)	1.1227	1 1/8 (1.125)	1.1600	1.1655	1.1520	1 1/2 (1.156)
1 1/4-11 1/2	0.0100	1.4905	1.4742	1.4805	1 31/64 (1.484)	1.4642	1 11/32 (1.469)				
1 1/2-11 1/2	0.0120	1.7295	1.7132	1.7175	1 23/32 (1.719)	1.7012	1 45/64 (1.703)				
2-11 1/2	0.0160	2.2024	2.1861	2.1864	2 3/16 (2.188)	2.1701	2 11/64 (2.172)				
2 1/2-8	0.0180	2.6234	2.6000	2.6054	2 39/64 (2.609)	2.5820	2 27/64 (2.578)				
3-8	0.0200	3.2445	3.2211	3.2245	3 15/64 (3.234)	3.2011	3 13/64 (3.203)				
Metric											
1/16-27	0.097	6.206	6.029	6.109	6.1	5.932	6.0	6.304	6.363	6.207	6.2
1/8-27	0.112	8.551	8.363	8.438	8.4	8.251	8.2	8.651	8.710	8.539	8.5
1/4-18	0.119	11.080	10.816	10.961	11.0	10.697	10.8	11.232	11.321	11.113	11.0
3/8-18	0.124	14.499	14.235	14.375	14.5	14.111	14.0	14.671	14.760	14.547	14.5
1/2-14	0.130	17.867	17.529	17.737	17.5	17.399	17.5	18.118	18.237	17.988	18.0
3/4-14	0.152	23.182	22.842	23.030	23.0	22.690	23.0	23.465	23.579	23.212	23.0
1-11 1/2	0.203	29.134	28.720	28.931	29.0	28.517	28.5	29.464	29.604	29.261	29.0
1 1/4-11 1/2	0.254	37.859	37.444	37.605	37.5	37.190	37.0				
1 1/2-11 1/2	0.305	43.929	43.514	43.624	43.5	43.209	43.5				
2-11 1/2	0.406	55.941	55.527	55.535	56.0	55.121	55.0				
2 1/2-8	0.457	66.634	66.029	66.177	66.0	65.572	65.0				
3-8	0.508	82.410	81.815	81.902	82.0	81.307	81.0				

\* Column 4 values equal column 2 values minus column 1 values.  
 † Some drill sizes listed may not be standard drills, and in some cases, standard metric drill sizes may be closer to the theoretical inch drill size and standard inch drill sizes may be closer to the theoretical metric drill size.  
 ‡ Column 6 values equal column 3 values minus column 1 values.  
 § Column 10 values equal column 8 values minus column 1 values.  
 SOURCE: ANSI B1.20.3-1976 and ANSI B1.20.4-1976, reproduced by permission.

Table 8.2.15 Wrench Bolt Heads, Nuts, and Wrench Openings

(All dimensions in inches)

Basic or max width across flats, bolt heads, and nuts	Wrench openings		Basic or max width across flats, bolt heads, and nuts	Wrench openings		Basic or max width across flats, bolt heads, and nuts	Wrench openings		Basic or max width across flats, bolt heads, and nuts	Wrench openings	
	Max	Min									
5/32	0.163	0.158	1 3/16	0.826	0.818	1 13/16	1.835	1.822	3	3.035	3.016
3/16	0.195	0.190	7/8	0.888	0.880	1 1/8	1.898	1.885	3 3/8	3.162	3.142
1/4	0.257	0.252	1 5/16	0.953	0.944	2	2.025	2.011	3 3/4	3.414	3.393
5/16	0.322	0.316	1	1.015	1.006	2 1/16	2.088	2.074	3 1/2	3.540	3.518
1 1/32	0.353	0.347	1 1/16	1.077	1.068	2 3/16	2.225	2.200	3 3/4	3.793	3.770
3/8	0.384	0.378	1 1/8	1.142	1.132	2 1/4	2.277	2.262	3 3/8	3.918	3.895
7/16	0.446	0.440	1 1/4	1.267	1.257	2 3/8	2.404	2.388	4 1/8	4.172	4.147
1/2	0.510	0.504	1 3/8	1.331	1.320	2 1/2	2.466	2.450	4 1/4	4.297	4.272
9/16	0.573	0.566	1 1/2	1.394	1.383	2 5/8	2.593	2.576	4 3/8	4.550	4.524
19/32	0.605	0.598	1 5/8	1.457	1.446	2 3/4	2.656	2.639	4 1/2	4.676	4.649
5/8	0.636	0.629	1 3/4	1.520	1.508	3	2.783	2.766	5	5.055	5.026
1 1/16	0.699	0.692	1 7/8	1.646	1.634	3 1/8	2.845	2.827	5 3/8	5.434	5.403
3/4	0.763	0.755	2	1.708	1.696	3 1/4	2.973	2.954	5 1/2	5.813	5.780
25/32	0.794	0.786				3 3/8			6	6.192	6.157

Wrenches shall be marked with the "nominal size of wrench" which is equal to the basic or maximum width across flats of the corresponding bolt head or nut. Allowance (min clearance) between maximum width across flats of nut or bolt head and jaws of wrench equals 1.005W + 0.001. Tolerance on wrench opening equals plus 0.005W + 0.004 from minimum (W equals nominal size of wrench).

**Table 8.2.16 Width Across Flats of Bolt Heads and Nuts**  
(All dimensions in inches)

Nominal size or basic major diam of thread	Dimensions of regular bolt heads unfinished, square, and hexagon		Dimensions of heavy bolt heads unfinished, square, and hexagon		Dimensions of cap-screw heads hexagon		Dimensions of setscrew heads		Dimensions of regular jam nuts, unfinished, square, and hexagon (jam nuts, hexagon only)		Dimensions of machine-screw and stove-bolt nuts, square and hexagon		Dimensions of heavy nuts and heavy jam nuts, unfinished, square, and hexagon (jam nuts, hexagon only)	
	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min
No. 0	—	—	—	—	—	—	—	—	—	—	0.1562	0.150	—	—
No. 1	—	—	—	—	—	—	—	—	—	—	0.1562	0.150	—	—
No. 2	—	—	—	—	—	—	—	—	—	—	0.1875	0.180	—	—
No. 3	—	—	—	—	—	—	—	—	—	—	0.1875	0.180	—	—
No. 4	—	—	—	—	—	—	—	—	—	—	0.2500	0.241	—	—
No. 5	—	—	—	—	—	—	—	—	—	—	0.3125	0.302	—	—
No. 6	—	—	—	—	—	—	—	—	—	—	0.3125	0.302	—	—
No. 8	—	—	—	—	—	—	—	—	—	—	0.3438	0.332	—	—
No. 10	—	—	—	—	—	—	—	—	—	—	0.3750	0.362	—	—
No. 12	—	—	—	—	—	—	—	—	—	—	0.4375	0.423	—	—
¼	0.3750	0.362	—	—	0.4375	0.428	0.2500	0.241	0.4375	0.425	0.4375	0.423	0.5000	0.488
⅜	0.5000	0.484	—	—	0.5000	0.489	0.3125	0.302	0.5625	0.547	0.5625	0.545	0.5938	0.578
½	0.6250	0.603	—	—	0.6250	0.612	0.4375	0.423	0.7500	0.728	—	—	0.7812	0.759
⅝	0.7500	0.725	0.8750	0.850	0.7500	0.736	0.5000	0.484	0.8125	0.788	—	—	0.8750	0.850
¾	0.8750	0.847	0.9375	0.909	0.8125	0.798	0.5625	0.545	0.8750	0.847	—	—	0.9375	0.909
7/8	0.9375	0.906	1.0625	1.031	0.8750	0.860	0.6250	0.606	1.0000	0.969	—	—	1.0625	1.031
1	1.1250	1.088	1.2500	1.212	1.0000	0.983	0.7500	0.729	1.1250	1.088	—	—	1.2500	1.212
1 1/8	1.3125	1.269	1.4375	1.394	1.1250	1.106	0.8750	0.852	1.3125	1.269	—	—	1.4375	1.394
1 1/4	1.5000	1.450	1.6250	1.575	1.3125	1.292	1.0000	0.974	1.5000	1.450	—	—	1.6250	1.575
1 1/2	1.6875	1.631	1.8125	1.756	1.5000	1.477	1.1250	1.096	1.6875	1.631	—	—	1.8125	1.756
1 3/4	1.8750	1.812	2.0000	1.938	1.6875	1.663	1.2500	1.219	1.8750	1.812	—	—	2.0000	1.938
1 7/8	2.0625	1.994	2.1857	2.119	—	—	1.3750	1.342	2.0625	1.994	—	—	2.1875	2.119
2	2.2500	2.175	2.3750	2.300	—	—	1.5000	1.464	2.2500	2.175	—	—	2.3750	2.300
2 1/8	2.4375	2.356	2.5625	2.481	—	—	—	—	2.4375	2.356	—	—	2.5625	2.481
2 1/4	2.6250	2.538	2.7500	2.662	—	—	—	—	2.6250	2.538	—	—	2.7500	2.662
2 1/2	2.8125	2.719	2.9375	2.844	—	—	—	—	2.8125	2.719	—	—	2.9375	2.844
2 3/4	3.0000	2.900	3.1250	3.025	—	—	—	—	3.0000	2.900	—	—	3.1250	3.025
3	3.3750	3.262	3.5000	3.388	—	—	—	—	3.3750	3.262	—	—	3.5000	3.388
3 1/2	3.7500	3.625	3.8750	3.750	—	—	—	—	3.7500	3.625	—	—	3.8750	3.750
3 3/4	4.1250	3.988	4.2500	4.112	—	—	—	—	4.1250	3.988	—	—	4.2500	4.112
4	4.5000	4.350	4.6250	4.475	—	—	—	—	4.5000	4.350	—	—	4.6250	4.475
3 1/4	—	—	—	—	—	—	—	—	—	—	—	—	5.0000	4.838
3 1/2	—	—	—	—	—	—	—	—	—	—	—	—	5.3750	5.200
3 3/4	—	—	—	—	—	—	—	—	—	—	—	—	5.7500	5.562
4	—	—	—	—	—	—	—	—	—	—	—	—	6.1250	5.925

Regular bolt heads are for general use. Unfinished bolt heads are not finished on any surface. Semifinished bolt heads are finished under head.

Regular nuts are for general use. Semifinished nuts are finished on bearing surface and threaded. Unfinished nuts are not finished on any surface but are threaded.

design in order that its properties may be fully utilized. These fasteners may be divided into six groups, as follows: seating lock, spring stop nut, interference, wedge, blind, and quick-release. The **seating-lock type** locks only when firmly seated and is therefore free-running on the bolt. The **spring stop-nut type** of fastener functions by a spring action clamping down upon the bolt. The **prevailing torque type** locks by elastic or plastic flow of a portion of the fastener material. A recent development employs an adhesive coating applied to the threads. The **wedge type** locks by relative wedging of either elements or nut and bolt. The **blind type** usually utilizes spring action of the fastener, and the **quick-release type** utilizes a quarter-turn release device. An example of each is shown in Fig. 8.2.12.

One such specification developed for prevailing torque fasteners by the Industrial Fasteners Institute is based on locking torque and may form a precedent for other types of fasteners as well.

**Coach and lag screws** find application in wood, or in masonry with an expansion anchor. Figure 8.2.13a shows two types, and Table 8.2.20 lists pertinent dimensions.

**Wood screws** [ANSI B18.22.1-1975 (R81)] are made in lengths from ¼ to 5 in for steel and from ¼ to 3½ in for brass screws, increasing by ⅛ in up to 1 in, by ¼ in up to 3 in, and by ½ in up to 5 in. Sizes are given in Table 8.2.21. Screws are made with flat, round, or oval heads. Figure 8.2.13b shows several heads.

**Washers** [ANSI B18.22.1-1975 (R81)] for bolts and lag screws, either round or square, are made to the dimensions given in Table 8.2.22. For other types of washers, see Fig. 8.2.14a and b.

**Self-tapping screws** are available in three types. **Thread-forming** tapping screws plastically displace material adjacent to the pilot hole. **Thread-cutting** tapping screws have cutting edges and chip cavities (flutes) and form a mating thread by removing material adjacent to the pilot hole. Thread-cutting screws are generally used to join thicker and harder materials and require a lower driving torque than thread-forming screws. **Metallic drive** screws are forced into the material by pressure and are intended for making permanent fastenings. These three types are further classified on the basis of thread and point form as shown in Table 8.2.23. In addition to these body forms, a number of different

Table 8.2.17 Head Diameters (Maximum), In

Machine screws						
Nominal size	Screw diam	Flat head	Round head	Fillister head	Oval head	Hexagonal head across flats
2	0.086	0.172	0.162	0.140	0.172	0.125
3	0.099	0.199	0.187	0.161	0.199	0.187
4	0.112	0.225	0.211	0.183	0.225	0.187
5	0.125	0.252	0.236	0.205	0.252	0.187
6	0.138	0.279	0.260	0.226	0.279	0.250
8	0.164	0.332	0.309	0.270	0.332	0.250
10	0.190	0.385	0.359	0.313	0.385	0.312
12	0.216	0.438	0.408	0.357	0.438	0.312
¼	0.250	0.507	0.472	0.414	0.507	0.375
⅝	0.3125	0.636	0.591	0.519	0.636	0.500
¾	0.375	0.762	0.708	0.622	0.762	0.562

Cap screws					
Nominal size	Screw diam	Flat head	Button head	Fillister head	Socket head
¼	0.250	½	⅞	⅝	⅝
⅝	0.3125	⅝	⅞	⅞	⅞
¾	0.375	¾	⅞	⅞	⅞
⅞	0.4375	⅞	⅞	⅞	⅞
1	0.500	1	1	1	1
1 ¼	0.5625	1 ¼	1 ¼	1 ¼	1 ¼
1 ½	0.625	1 ½	1 ½	1 ½	1 ½
1 ¾	0.750	1 ¾	1 ¾	1 ¾	1 ¾
2	—	—	—	—	—
2 ¼	—	—	—	—	—

head types are available. Basic dimensional data are given in Table 8.2.24.

**Carriage bolts** have been standardized in ANSI B18.5-1971, revised 1990. They come in styles shown in Fig. 8.2.15. The range of bolt diameters is no. 10 (= 0.19 in) to 1 in, no. 10 to ¾ in, no. 10 to ½ in, and no. 10 to ¼ in, respectively.

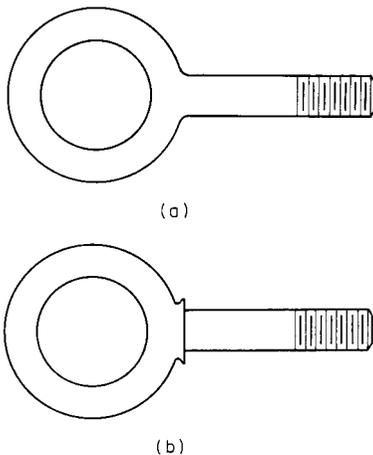


Fig. 8.2.9 Eyebolts.

### Materials, Strength, and Service Adaptability of Bolts and Screws Materials

Table 8.2.25 shows the relationship between selected metric bolt classes and SAE and ASTM grades. The first number of a metric bolt class equals the minimum tensile strength (ultimate) in megapascals (MPa) divided by 100, and the second number is the approximate ratio between minimum yield and minimum ultimate strengths.

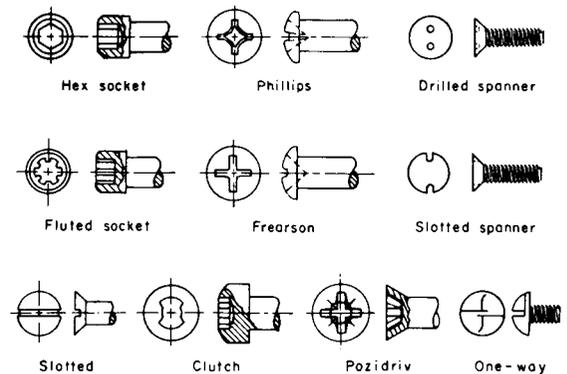


Fig. 8.2.10 Driving recesses. (Adapted, with permission, from *Machine Design*.)

**EXAMPLE.** Class 5.8 has a minimum ultimate strength of approximately 500 MPa and a minimum yield strength approximately 80 percent of minimum ultimate strength.

**Strength** The fillet between head and body, the thread runout point, and the first thread to engage the nut all create stress concentrations causing local stresses much greater than the average tensile stress in the bolt body. The complexity of the stress patterns renders ineffective the ordinary design calculations based on yield or ultimate stresses. Bolt strengths are therefore determined by laboratory tests on bolt-nut assemblies and published as **proof loads**. Fastener manufacturers are required to periodically repeat such tests to ensure that their products meet the original standards.

In order that a bolted joint remain firmly clamped while carrying its external load  $P$ , the bolt must be tightened first with sufficient torque to induce an initial tensile preload  $F_i$ . The total load  $F_B$  experienced by the bolt is then  $F_B = F_i + \epsilon P$ . The fractional multiplier  $\epsilon$  is given by  $\epsilon =$

**Table 8.2.18 Regular Nut Eyebolts—Selected Sizes**  
(Thomas Laughlin Co., Portland, Me.)  
(All dimensions in inches)  
(See Fig. 8.2.9.)

Diam and shank length	Thread length	Eye dimension		Approx breaking strength, lb	Diam and shank length	Thread length	Eye dimension		Approx breaking strength, lb
		ID	OD				ID	OD	
¼ × 2	1½	½	1	2,200	¾ × 6	3	1½	3	23,400
⅜ × 2½	1½	⅝	1¼	3,600	¾ × 10	3	1½	3	23,400
⅜ × 4½	2½	¾	1½	5,200	¾ × 15	5	1½	3	23,400
½ × 3¼	1½	1	2	9,800	⅞ × 8	4	1¾	3½	32,400
½ × 6	3	1	2	9,800	1 × 6	3	2	4	42,400
½ × 10	3	1	2	9,800	1 × 9	4	2	4	42,400
⅝ × 4	2	1¼	2½	15,800	1 × 18	7	2	4	42,400
⅝ × 6	3	1¼	2½	15,800	1¼ × 8	4	2½	5	67,800
⅝ × 10	3	1¼	2½	15,800	1¼ × 20	6	2½	5	67,800

**Table 8.2.19 Cup-Point Setscrew Holding Power**

Nominal screw size	Seating torque, lb·in	Axial holding power, lb
No. 0	0.5	50
No. 1	1.5	65
No. 2	1.5	85
No. 3	5	120
No. 4	5	160
No. 5	9	200
No. 6	9	250
No. 8	20	385
No. 10	33	540
¼ in	87	1,000
⅜ in	165	1,500
½ in	290	2,000
⅝ in	430	2,500
¾ in	620	3,000
⅞ in	620	3,500
1 in	1,225	4,000
1¼ in	2,125	5,000
1½ in	5,000	6,000
1¾ in	7,000	7,000

- NOTES: 1. Torsional holding power in inch-pounds is equal to one-half of the axial holding power times the shaft diameter in inches.  
 2. Experimental data were obtained by seating an alloy-steel cup-point setscrew against a steel shaft with a hardness of Rockwell C 15. Screw threads were class 3A, tapped holes were class 2B. Holding power was defined as the minimum load necessary to produce 0.01 in of relative movement between the shaft and the collar.  
 3. Cone points will develop a slightly greater holding power; flat, dog, and oval points, slightly less.  
 4. Shaft hardness should be at least 10 Rockwell C points less than the setscrew point.  
 5. Holding power is proportional to seating torque. Torsional holding power is increased about 6% by use of a flat on the shaft.  
 6. Data by F. R. Kull, Fasteners Book Issue, *Mach. Des.*, Mar. 11, 1965.

$K_B/(K_B + K_M)$ , where  $K_B$  = elastic constant of the bolt and  $1/K_M = 1/K_N + 1/K_W + 1/K_G + 1/K_J$ .  $K_N$  = elastic constant of the nut;  $K_W$  = elastic constant of the washer;  $K_G$  = elastic constant of the gasket;  $K_J$  = elastic constant of the clamped surfaces or joint.

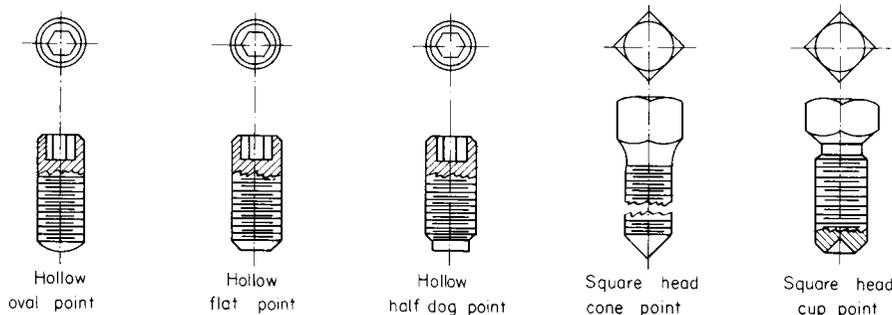
By manipulation, the fractional multiplier can be written  $\epsilon = 1/(1 + K_M/K_B)$ . When  $K_M/K_B$  approaches 0,  $\epsilon \rightarrow 1$ . When  $K_M/K_B$  approaches infinity,  $\epsilon \rightarrow 0$ . Generally,  $K_N$ ,  $K_W$ , and  $K_J$  are much stiffer than  $K_B$ , while  $K_G$  can vary from very soft to very stiff. In a metal-to-metal joint,  $K_G$  is effectively infinity, which causes  $K_M$  to approach infinity and  $\epsilon$  to approach 0. On the other hand, for a very soft gasketed joint,  $K_M \rightarrow 0$  and  $\epsilon \rightarrow 1$ . For a metal-to-metal joint, then,  $F_B = F_i + 0 \times P = F_i$ ; thus no fluctuating load component enters the bolt. In that case, the bolt remains at *static force*  $F_i$  at all times, and the static design will suffice. For a very soft gasketed joint,  $F_B = F_i + 1 \times P = F_i + P$ , which means that if  $P$  is a dynamically fluctuating load, it will be superimposed onto the static value of  $F_i$ . Accordingly, one must use the *fatigue design* for the bolt. Of course, for conditions between  $\epsilon = 0$  and  $\epsilon = 1$ , the load within the bolt body is  $F_B = F_i + \epsilon P$ , and again the *fatigue design* must be used.

In general, one wants as much preload as a bolt and joint will tolerate, without damaging the clamped parts, encouraging stress corrosion, or reducing fatigue life. For ungasketed, unpressurized joints under static loads using high-quality bolt materials, such as SAE 3 or better, the preload should be about 90 percent of proof load.

The **proof strength** is the stress obtained by dividing **proof load** by **stress area**. Stress area is somewhat larger than the root area and can be found in thread tables, or calculated approximately from a diameter which is the mean of the root and pitch diameters.

Initial sizing of bolts can be made by calculating  $\text{area} = (\% \times \text{proof load})/(\text{proof strength})$ . See Table 8.2.26 for typical physical properties.

NOTE. In European practice, proof stress of a given grade is independent of diameter and is accomplished by varying chemical composition with diameters.



**Fig. 8.2.11** Setscrews.

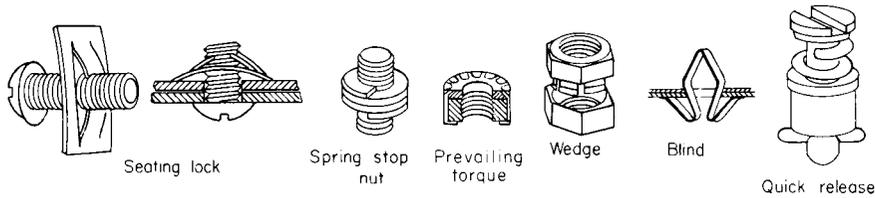


Fig. 8.2.12 Locking fasteners.

**General Notes on the Design of Bolted Joints**

**Bolts subjected to shock** and sudden change in load are found to be more serviceable when the unthreaded portion of the bolt is turned down or drilled to the area of the root of the thread. The drilled bolt is stronger in torsion than the turned-down bolt.

When a **number of bolts** are employed in fastening together two parts of a machine, such as a cylinder and cylinder head, the load carried by each bolt depends on its relative tightness, the tighter bolts carrying the

greater loads. When the conditions of assembly result in differences in tightness, lower working stresses must be used in designing the bolts than otherwise are necessary. On the other hand, it may be desirable to have the bolts the weakest part of the machine, since their breakage from overload in the machine may result in a minimum replacement cost. In such cases, the breaking load of the bolts may well be equal to the load which causes the weakest member of the machine connected to be stressed up to the elastic limit.

Table 8.2.20 Coach and Lag Screws

Diam of screw, in	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1
No. of threads per inch	10	9	7	7	6	5	4 1/2	4	3 1/2
Across flats of hexagon and square heads, in	3/8	15/32	9/16	21/32	3/4	15/16	1 1/8	1 5/16	1 1/2
Thickness of hexagon and square heads, in	3/16	1/4	5/16	3/8	7/16	17/32	5/8	3/4	7/8
Length of threads for screws of all diameters									
Length of screw, in	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2		
Length of thread, in	To head	1 1/2	2	2 1/4	2 1/2	3	3 1/2		
Length of screw, in	5	5 1/2	6	7	8	9	10-12		
Length of thread, in	4	4	4 1/2	5	6	6	7		

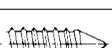
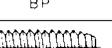
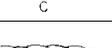
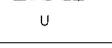
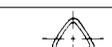
Table 8.2.21 American National Standard Wood Screws

Number	0	1	2	3	4	5	6	7	8
Threads per inch	32	28	26	24	22	20	18	16	15
Diameter, in	0.060	0.073	0.086	0.099	0.112	0.125	0.138	0.151	0.164
Number	9	10	11	12	14	16	18	20	24
Threads per inch	14	13	12	11	10	9	8	8	7
Diameter, in	0.177	0.190	0.203	0.216	0.242	0.268	0.294	0.320	0.372

Table 8.2.22 Dimensions of Steel Washers, in

Bolt size	Plain washer			Lock washer		
	Hole diam	OD	Thickness	Hole diam	OD	Thickness
3/16	1/4	5/16	3/64	0.194	0.337	0.047
1/4	5/16	3/4	1/16	0.255	0.493	0.062
5/16	3/8	7/8	1/16	0.319	0.591	0.078
3/8	7/16	1	5/64	0.382	0.688	0.094
7/16	1/2	1 1/4	5/64	0.446	0.781	0.109
1/2	9/16	1 3/8	3/32	0.509	0.879	0.125
9/16	5/8	1 1/2	3/32	0.573	0.979	0.141
5/8	11/16	1 3/4	1/8	0.636	1.086	0.156
3/4	13/16	2	1/8	0.763	1.279	0.188
7/8	15/16	2 1/4	5/32	0.890	1.474	0.219
1	1 1/16	2 1/2	5/32	1.017	1.672	0.250
1 1/8	1 1/4	2 3/4	5/32	1.144	1.865	0.281
1 1/4	1 3/8	3	5/32	1.271	2.058	0.312
1 3/8	1 1/2	3 1/4	11/64	1.398	2.253	0.344
1 1/2	1 5/8	3 1/2	11/64	1.525	2.446	0.375
1 5/8	1 3/4	3 3/4	11/64			
1 3/4	1 7/8	4	11/64			
1 7/8	2	4 1/4	11/64			
2	2 1/8	4 1/2	11/64			
2 1/4	2 3/8	4 3/4	3/16			
2 1/2	2 5/8	5	7/32			

Table 8.2.23 Tapping Screw Forms

ASA type and thread form	Description and recommendations
 <p>AB</p>	Spaced thread, with gimlet point, designed for use in sheet metal, resin-impregnated plywood, wood, and asbestos compositions. Used in pierced or punched holes where a sharp point for starting is needed.
 <p>B</p>	Type B is a blunt-point spaced-thread screw, used in heavy-gage sheet-metal and non-ferrous castings.
 <p>BP</p>	Same as type B, but has a 45 deg included angle unthreaded cone point. Used for locating and aligning holes or piercing soft materials.
 <p>C</p>	Blunt point with threads approximating machine-screw threads. For applications where a machine-screw thread is preferable to the spaced-thread form. Unlike thread-cutting screws, type C makes a chip-free assembly.
 <p>U</p>	Multiple-threaded drive screw with steep helix angle and a blunt, unthreaded starting pilot. Intended for making permanent fastenings in metals and plastics. Hammered or mechanically forced into work. Should not be used in materials less than one screw diameter thick.
 <p>D</p>	Blunt point with single narrow flute and threads approximating machine-screw threads. Flute is designed to produce a cutting edge which is radial to screw center. For low-strength metals and plastics; for high-strength brittle metals; and for rethreading clogged pretapped holes.
 <p>F</p>	Approximate machine-screw thread and blunt point.
 <p>G</p>	Approximate machine-screw thread with single through slot which forms two cutting edges. For low-strength metals and plastics.
 <p>T</p>	Same as type D with single wide flute for more chip clearance.
 <p>BF</p>	Spaced thread with blunt point and five evenly spaced cutting grooves and chip cavities. Wall thickness should be 1½ times major diameter of screw. Reduces stripping in brittle plastics and die castings.
 <p>BT</p>	Same as type BF except for single wide flute which provides room for twisted, curly chips.
 <p>TT</p>	Thread-rolling screws roll-form clean, screw threads. The plastic movement of the material it is driven into locks it in place. The Taptite form is shown here.

SOURCE: *Mach. Des.*, Mar. 11, 1965.

**Bolts screwed up tight** have an initial stress due to the tightening (pre-load) before any external load is applied to the machine member. The initial tensile load due to screwing up for a tight joint varies approximately as the diameter of the bolt, and may be estimated at 16,000 lb/in of diameter. The actual value depends upon the applied torque and the efficiency of the screw threads. Applying this rule to bolts of 1-in diam or less results in excessively high stresses, thus demonstrating why bolts of small diameter frequently fail during assembly. It is advisable to use

as large-diameter bolts as possible in pressure-tight joints requiring high tightening loads.

In pressure-tight joints without a gasket the force on the bolt under load is essentially never greater than the initial tightening load. When a gasket is used, the total bolt force is approximately equal to the initial tightening load plus the external load. In the first case, deviations from the rule are a result of elastic behavior of the joint faces without a gasket, and inelastic behavior of the gasket in the latter case. The fol-

Table 8.2.24 Self-Tapping Screws

Screw size	Basic major diam, in	Threads per inch					Type U	
		AB	B, BP	C	D, F, G, T	BF, BT	Max outside diam, in	Number of thread starts
00	—	—	—	—	—	—	0.060	6
0	0.060	40	48	—	—	48	0.075	6
1	0.073	32	42	—	—	42	—	—
2	0.086	32	32	56 and 64	56 and 64	32	0.100	8
3	0.099	28	28	48 and 56	48 and 56	28	—	—
4	0.112	24	24	40 and 48	40 and 48	24	0.116	7
5	0.125	20	20	40 and 44	40 and 44	20	—	—
6	0.138	18	20	32 and 40	32 and 40	20	0.140	7
7	0.151	16	19	—	—	19	0.154	8
8	0.164	15	18	32 and 36	32 and 36	18	0.167	8
10	0.190	12	16	24 and 32	24 and 32	16	0.182	8
12	0.216	11	14	24 and 28	24 and 28	14	0.212	8
14	0.242	10	—	—	—	—	0.242	9
¼	0.250	—	14	20 and 28	20 and 28	14	—	—
16	0.268	10	—	—	—	—	—	—
18	0.294	9	—	—	—	—	—	—
5/16	0.3125	—	12	18 and 24	18 and 24	12	0.315	14
20	0.320	9	—	—	—	—	—	—
24	0.372	9	—	—	—	—	—	—
¾	0.375	—	12	16 and 24	16 and 24	12	0.378	12
7/16	0.4375	—	10	—	—	—	—	—
½	0.500	—	10	—	—	—	—	—

lowing generalization will serve as a guide. If the bolt is more yielding than the connecting members, it should be designed simply to resist the initial tension or the external load, whichever is greater. If the probable yielding of the bolt is 50 to 100 percent of that of the connected members, take the resultant bolt load as the initial tension plus one-half the external load. If the yielding of the connected members is probably four to five times that of the bolt (as when certain packings are used), take the resultant bolt load as the initial tension plus three-fourths the external load.

In cases where bolts are subjected to cyclic loading, an increase in the initial tightening load decreases the operating stress range. In certain applications it is customary to fix the tightening load as a fraction of the yield-point load of the bolt.

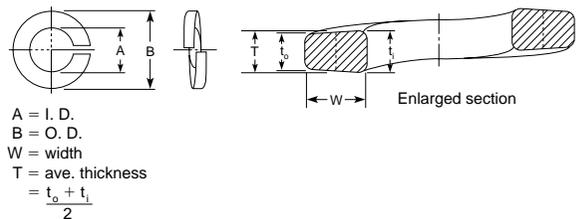


Fig. 8.2.14 a Regular helical spring lock washer.

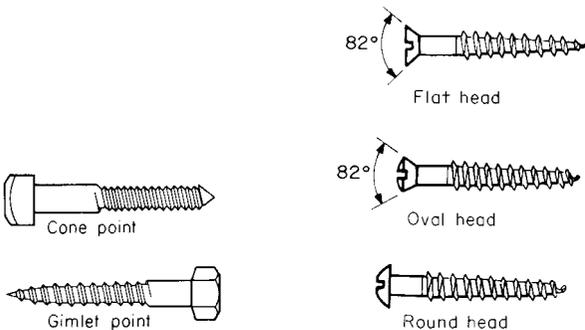


Fig. 8.2.13 a Coach and lag screws.

Fig. 8.2.13 b Wood screws.

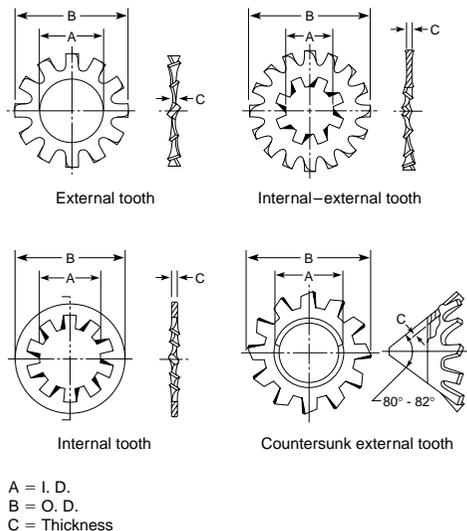


Fig. 8.2.14 b Toothed lock washers.

Table 8.2.25 ISO Metric Fastener Materials

Metric bolt	Metric nut class normally used	Roughly equivalent U.S. bolt materials	
		SAE J429 grades	ASTM grades
4.6	4 or 5	1	A193, B8; A307, grade A
4.8	4 or 5	2	
5.8	5	2	
8.8	8	5	A325, A449
9.9	9	5 +	A193, B7 and B16
10.9	10 or 12	8	A490; A354, grade 8D
12.9	10 or 12		A540; B21 through B24

SOURCE: Bickford, "An Introduction to the Design and Behavior of Bolted Joints," Marcel Dekker, 1981; reproduced by permission. See Appendix G for additional metric materials.

**Table 8.2.26a Specifications and Identification Markings for Bolts, Screws, Studs, Sems,<sup>a</sup> and U Bolts<sup>b</sup>**  
(Multiply the strengths in kpsi by 6.89 to get the strength in MPa.)

SAE grade	ASTM grade	Metric <sup>c</sup> grade	Nominal diameter, in	Proof strength, kpsi	Tensile strength, kpsi	Yield <sup>d</sup> strength, kpsi	Core hardness, Rockwell min/max	Products <sup>e</sup>
1	A307	4.6	¼ thru 1½	33	60	36	B70/B100	B, Sc, St
2		5.8	¼ thru ¾	55	74	57	B80/B100	B, Sc, St
		4.6	Over ¾ thru 1½	33	60	36	B70/B100	B, Sc, St
4	A449 or A325 type 1	8.9	¼ thru 1½	65 <sup>f</sup>	115	100	C22/C32	St
5		8.8	¼ thru 1	85	120	92	C25/C34	B, Sc, St
		7.8	Over 1 thru 1½	74	105	81	C19/C30	B, Sc, St
5.1		8.6	Over 1½ to 3	55	90	58	B, Sc, St	B, Sc, St
		8.8	No. 6 thru ⅝	85	120		C25/C40	Se
		8.8	No. 6 thru ½	85	120		C25/C40	B, Sc, St
5.2	A325 type 2	8.8	¼ thru 1	85	120	92	C26/C36	B, Sc
7 <sup>g</sup>	A354 Grade BD	10.9	¾ thru 1½	105	133	115	C28/C34	B, Sc
8		10.9	¼ thru 1½	120	150	130	C33/C39	B, Sc, St
8.1	A574	10.9	¼ thru 1½	120	150	130	C32/C38	St
8.2		10.9	¼ thru 1	120	150	130	C35/C42	B, Sc
		12.9	0 thru ½	140	180	160	C39/C45	SHCS
		12.9	⅝ thru 1½	135	170	160	C37/C45	SHCS

NOTE: Company catalogs should be consulted regarding *proof loads*. However, approximate values of proof loads may be calculated from: proof load = proof strength × stress area.

<sup>a</sup> Sems are screw and washer assemblies.

<sup>b</sup> Compiled from ANSI/SAE J429j; ANSI B18.3.1-1978; and ASTM A307, A325, A354, A449, and A574.

<sup>c</sup> Metric grade is xx.x where xx is approximately 0.01  $S_{ut}$  in MPa and .x is the ratio of the minimum  $S_y$  to  $S_{ut}$ .

<sup>d</sup> Yield strength is stress at which a permanent set of 0.2% of gage length occurs.

<sup>e</sup> B = bolt, Sc = Screws, St = studs, Se = sems, and SHCS = socket head cap screws.

<sup>f</sup> Entry appears to be in error but conforms to the standard, ANSI/SAE J429j.

<sup>g</sup> Grade 7 bolts and screws are roll-threaded after heat treatment.

SOURCE: Shigley and Mitchell, "Mechanical Engineering Design," 4th ed., McGraw-Hill, 1983, by permission.

In order to avoid the possibility of bolt failure in pressure-tight joints and to obtain uniformity in bolt loads, some means of determining initial bolt load (**preload**) is desirable. Calibrated torque wrenches are available for this purpose, reading directly in inch-pounds or inch-ounces. Inaccuracies in initial bolt load are possible even when using a torque wrench, owing to variations in the coefficient of friction between the nut and the bolt and, further, between the nut or bolthead and the abutting surface.

An exact method to determine the preload in a bolt requires that the bolt elongation be measured. For a through bolt in which both ends are accessible, the elongation is measured, and the preload force  $P$  is obtained from the relationship

$$P = AEe \div l$$

where  $E$  = modulus of elasticity,  $l$  = original length,  $A$  = cross-sectional area,  $e$  = elongation. In cases where both ends of the bolt are not accessible, strain-gage techniques may be employed to determine the strain in the bolt, and thence the preload.

High-strength bolts designated ASTM A325 and A490 are almost exclusively employed in the assembly of structural steel members, but they are applied in mechanical assemblies such as flanged joints. The direct tension indicator (DTI) (Fig. 8.2.16), for use with high-strength bolts, allows **bolt preload** to be applied rapidly and simply. The device is a hardened washer with embossed protrusions (Fig. 8.2.16a). Tightening the bolt causes the protrusions to flatten and results in a decrease in the gap between washer and bolthead. The prescribed degree of bolt-tightening load, or preload, is obtained when the gap is reduced to a predetermined amount (Fig. 8.2.16c). A feeler gage of a given thickness is used to determine when the gap has been closed to the prescribed amount (Fig. 8.2.16b and c). With a paired bolt and DTI, the degree of gap closure is proportional to bolt preload. The system is reported to provide bolt-preload force accuracy within +15 percent of that prescribed (Fig. 8.2.16d). The devices are available in both inch and metric series and are covered under ASTM F959 and F959M.

**Preload-indicating bolts and nuts** provide visual assurance of preload in that tightening to the desired preload causes the wavy flange to flatten flush with the clamped assembly (Fig. 8.2.17).

**In drilling and tapping cast iron for steel studs.** It is necessary to tap to a

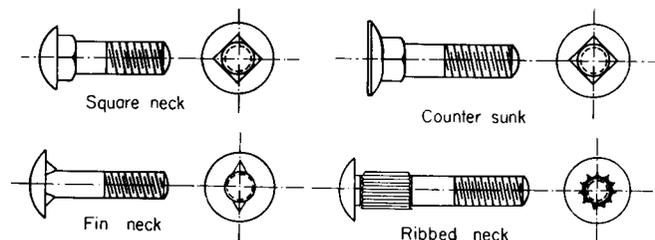


Fig. 8.2.15 Carriage bolts.

depth equal to 1½ times the stud diameter so that the strength of the cast-iron threads in shear may equal the tensile strength of the stud. Drill sizes and depths of hole and thread are given in Table 8.2.27.

It is not good practice to drill holes to be tapped through the metal into pressure spaces, for even though the bolt fits tightly, leakage will result that is difficult to eliminate.

**Screw thread inserts** made of high-strength material (Fig. 8.2.18) are useful in many cases to provide increased thread strength and life. Soft or ductile materials tapped to receive thread inserts exhibit improved load-carrying capacity under static and dynamic loading conditions. Holes in which threads have been stripped or otherwise damaged can be restored through the use of thread inserts.

Holes for thread inserts are drilled oversize and specially tapped to receive the insert selected to mate with the threaded fastener used. The standard material for inserts is 18-8 stainless steel, but other materials are available, such as phosphor bronze and Inconel. Recommended insert lengths are given in Table 8.2.28.

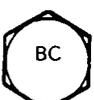
**Drill Sizes** Unified thread taps are listed in Table 8.2.29.

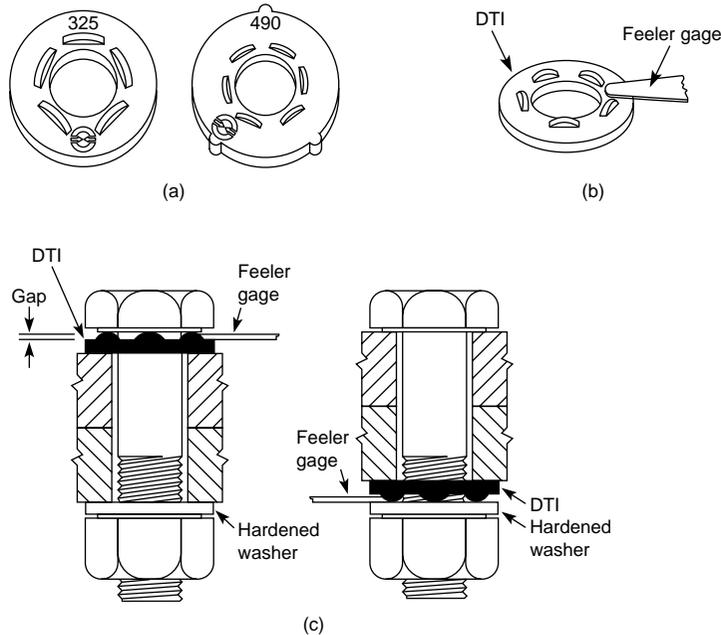
## RIVET FASTENINGS

**Forms and Proportion of Rivets** The forms and proportions of small and large rivets have been standardized and conform to ANSI B18.1.1-1972 (R89) and B18.1.2-1972 (R89) (Figs. 8.2.19a and b).

**Materials Specifications for Rivets and Plates** See Sec. 6 and 12.2. **Conventional signs** to indicate the form of the head to be used and

Table 8.2.26b ASTM and SAE Grade Head Markings for Steel Bolts and Screws

Grade marking	Specification	Material
	SAE grade 1 ASTM A307 SAE grade 2	Low- or medium-carbon steel Low-carbon steel Low- or medium-carbon steel
No mark		
	SAE grade 5 ASTM A449	Medium-carbon steel, quenched and tempered
	SAE grade 5.2	Low-carbon martensite steel, quenched and tempered
	ASTM A325 type 1	Medium-carbon steel, quenched and tempered; radial dashes optional
	ASTM A325 type 2	Low-carbon martensite steel, quenched and tempered
	ASTM A325 type 3	Atmospheric corrosion (weathering) steel, quenched and tempered
	ASTM A354 grade BC	Alloy steel, quenched and tempered
	SAE grade 7	Medium-carbon alloy steel, quenched and tempered, roll-threaded after heat treatment
	SAE grade 8 ASTM A354 grade BD	Medium-carbon alloy steel, quenched and tempered Alloy steel, quenched and tempered
	SAE grade 8.2	Low-carbon martensite steel, quenched and tempered
	ASTM A490 type 1	Alloy steel, quenched and tempered
	ASTM A490 type 3	Atmospheric corrosion (weathering) steel, quenched and tempered



**DTI Gaps To Give Required Minimum Bolt Tension**

DTI Fitting	325	490
<b>Under bolt head</b>		
Plain finish DTIs	0.015" (.40 mm)	0.015"
Mechanically galvanized DTIs	0.005" (.125 mm)	—
Epoxy coated on mechanically galvanized DTIs	0.005" (.125 mm)	—
<b>Under turned Element</b>		
With hardened washers (plain finish)	0.005" (.125 mm)	0.005"

With average gaps equal or less than above, bolt tensions will be greater than in adjacent listing

**Minimum Bolt Tensions**  
In thousands of pounds (Kips)

Bolt dia.	A325	A490
1/2"	12	—
5/8"	19	—
3/4"	28	35
7/8"	39	49
1 "	51	64
1 1/8"	56	80
1 1/4"	71	102
1 3/8"	85	121
1 1/2"	103	—

**Fig. 8.2.16** Direct tension indicators; gap and tension data. (Source: J & M Turner Inc.)

whether the rivet is to be driven in the shop or the field at the time of erection are given in Fig. 8.2.20. Rivet lengths and grips are shown in Fig. 8.2.19b.

For structural riveting, see Sec. 12.2.

**Punched vs. Drilled Plates** Holes in plates forming parts of riveted structures are punched, punched and reamed, or drilled. Punching, while

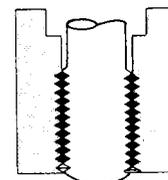


**Fig. 8.2.17** Load-indicating wavy-flange bolt (or nut).

cheaper, is objectionable. The holes in different plates cannot be spaced with sufficient accuracy to register perfectly on being assembled. If the hole is punched out, say 1/16 in smaller than is required and then reamed to size, the metal injury by cold flow during punching will be removed. Drilling, while more expensive, is more accurate and does not injure the metal

**Tubular Rivets**

In tubular rivets, the end opposite the head is made with an axial hole (partway) to form a thin-walled, easily upsettable end. As the material at the edge of the rivet hole is rolled over against the surface of the joint, a **clinch** is formed (Fig. 8.2.21a).



**Fig. 8.2.18** Screw thread insert.

Two-part tubular rivets have a thin-walled head with attached thin-walled rivet body and a separate thin-walled expandable plug. The head-body is inserted through a hole in the joint from one side, and the plug from the other. By holding an anvil against the plug bottom and

**Table 8.2.27** Depths to Drill and Tap Cast Iron for Studs

Diam of stud, in	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
Diam of drill, in	13/64	17/64	5/16	3/8	27/64	31/64	17/32	41/64	3/4	55/64
Depth of thread, in	3/8	15/32	9/16	21/32	3/4	27/32	15/16	1 1/8	1 1/16	1 1/2
Depth to drill, in	7/16	17/32	5/8	23/32	27/32	15/16	1 1/32	1 1/4	1 7/16	1 5/8

**Table 8.2.28** Screw-Thread Insert Lengths (Heli-Coil Corp.)

Shear strength of parent material, lb/in <sup>2</sup>	Bolt material ultimate tensile strength, lb/in <sup>2</sup>				
	60,000	90,000	125,000	170,000	220,000
	Length in terms of nominal insert diameter				
15,000	1 1/2	2	2 1/2	3	
20,000	1	1 1/2	2	2 1/2	3
25,000	1	1 1/2	2	2	2 1/2
30,000	1	1	1 1/2	2	2
40,000	1	1	1 1/2	1 1/2	2
50,000	1	1	1	1 1/2	1 1/2

hammering on the head, the plug is caused to expand within the head, thus locking both parts together (Fig. 8.2.21a).

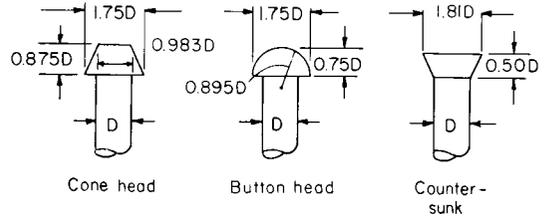
**Blind Rivets**

Blind rivets are inserted and set all from one side of a structure. This is accomplished by mechanically expanding, through the use of the rivet's built-in mandrel, the back (blind side) of the rivet into a bulb or upset head after insertion. Blind rivets include the *pull type* and *drive-pin type*.

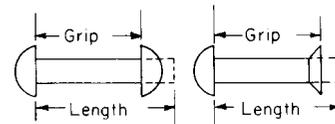
The pull-type rivet is available in two configurations: a self-plugging type and a pull-through type. In the self-plugging type, part of the

mandrel remains permanently in the rivet body after setting, contributing additional shear strength to the fastener. In the pull-through type, the entire mandrel is pulled through, leaving the installed rivet empty.

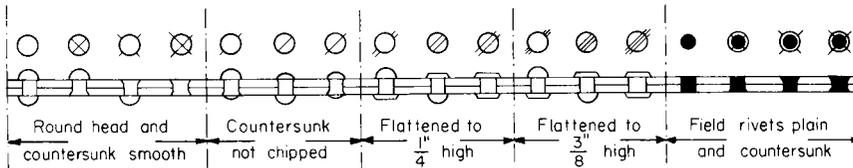
In a drive-pin rivet, the rivet body is slotted. A pin is driven forward into the rivet, causing both flaring of the rivet body and upset of the blind side (Fig. 8.2.21b).



**Fig. 8.2.19a** Rivet heads.



**Fig. 8.2.19b** Rivet length and grip.



**Fig. 8.2.20** Conventional signs for rivets.

**Table 8.2.29** Tap-Drill Sizes for American National Standard Screw Threads (The sizes listed are the commercial tap drills to produce approx 75% full thread)

Size	Coarse-thread series		Fine-thread series		Size	Coarse-thread series		Fine-thread series	
	Threads per inch	Tap drill size	Threads per inch	Tap drill size		Threads per inch	Tap drill size	Threads per inch	Tap drill size
No. 0	—	—	80	3/64	3/4	10	2 1/32	16	1 1/16
No. 1	64	No. 53	72	No. 53	7/8	9	49/64	14	13/16
No. 2	56	No. 50	64	No. 50	1	8	7/8	14	15/16
No. 3	48	No. 47	56	No. 45	1 1/8	7	63/64	12	13/64
No. 4	40	No. 43	48	No. 42	1 1/4	7	17/64	12	1 11/64
No. 5	40	No. 38	44	No. 37	1 3/8	6	17/32	12	1 19/64
No. 6	32	No. 36	40	No. 33	1 1/2	6	1 21/64	12	1 27/64
No. 8	32	No. 29	36	No. 29	1 3/4	5	1 35/64		
No. 10	24	No. 25	32	No. 21	2	4 1/2	1 25/32		
No. 12	24	No. 16	28	No. 14	2 1/4	4 1/2	2 1/32		
1/4	20	No. 7	28	No. 3	2 1/2	4	2 1/4		
5/16	18	F	24	I	2 3/4	4	2 1/2		
3/8	16	5/16	24	Q	3	4	2 3/4		
7/16	14	U	20	25/64	3 1/4	4	3		
1/2	13	27/64	20	29/64	3 1/2	4	3 1/4		
5/8	12	31/64	18	33/64	3 3/4	4	3 1/2		
3/4	11	17/32	18	37/64	4	4	3 3/4		

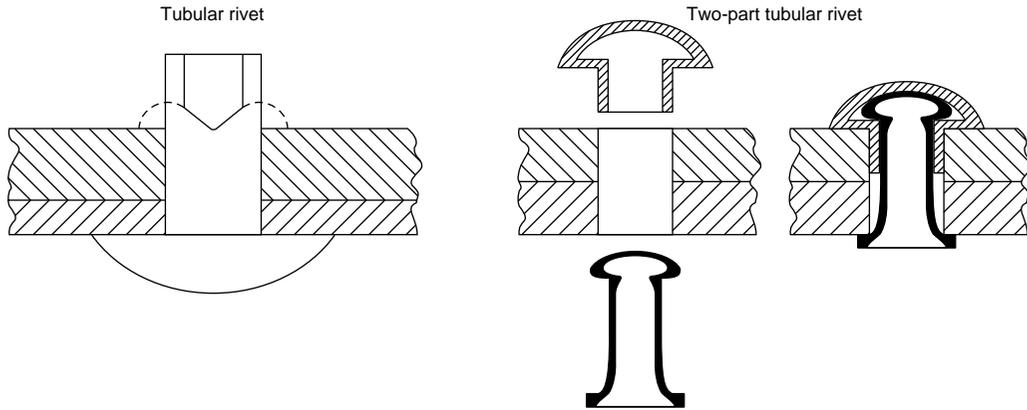


Fig. 8.2.21a Tubular rivets.

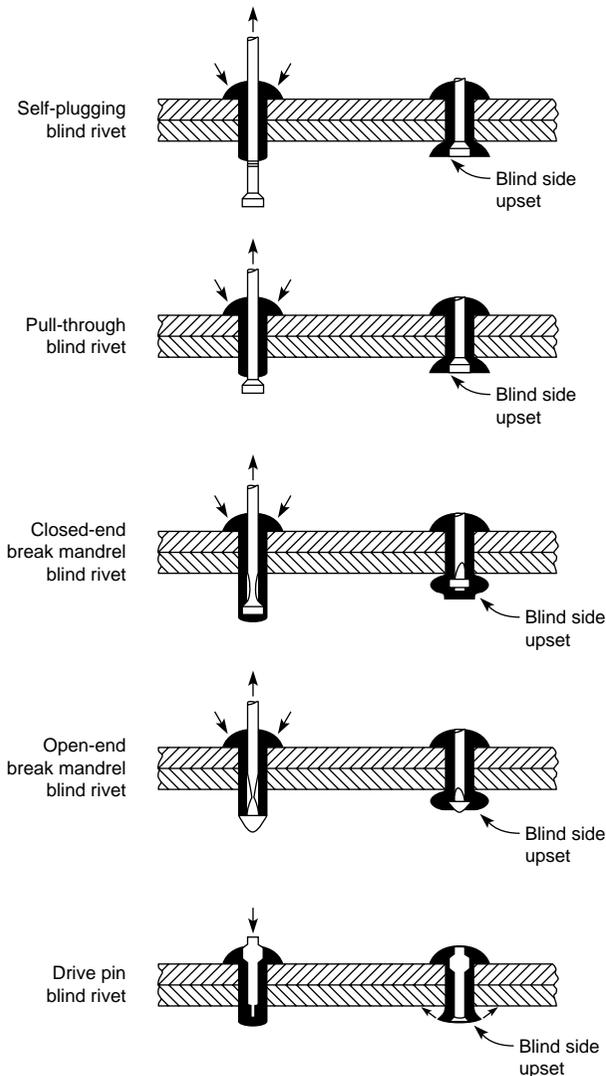


Fig. 8.2.21b Blind rivets.

## KEYS, PINS, AND COTTERS

Keys and key seats have been standardized and are listed in ANSI B17.1-1967 (R89). Descriptions of the principal key types follow.

**Woodruff keys** [ANSI B17.2-1967 (R90)] are made to facilitate removal of pulleys from shafts. They should not be used as sliding keys. Cutters for milling out the key seats, as well as special machines for using the cutters, are to be had from the manufacturer. Where the hub of the gear or pulley is relatively long, two keys should be used. Slightly rounding the corners or ends of these keys will obviate any difficulty met with in removing pulleys from shafts. The key is shown in Fig. 8.2.22 and the dimensions in Table 8.2.30.

**Square and flat plain taper keys** have the same dimensions as gib-head keys (Table 8.2.31) up to the dotted line of Fig. 8.2.23. **Gib-head keys** (Fig. 8.2.23) are necessary when the smaller end is inaccessible for drifting out and the larger end is accessible. It can be used, with care,

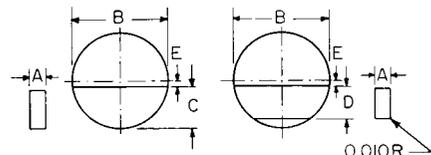


Fig. 8.2.22 Woodruff key.

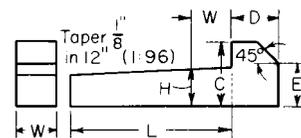


Fig. 8.2.23 Gib-head taper stock key.

with all sizes of shafts. Its use is forbidden in certain jobs and places for safety reasons. Proportions are given in Table 8.2.31.

The minimum stock length of keys is 4 times the key width, and maximum stock length of keys is 16 times the key width. The increments of increase of length are 2 times the width.

**Sunk keys** are made to the form and dimensions given in Fig. 8.2.24 and Table 8.2.32. These keys are adapted particularly to the case of fitting adjacent parts with neither end of the key accessible. **Feather keys** prevent parts from turning on a shaft while allowing them to move in a lengthwise direction. They are of the forms shown in Fig. 8.2.25 with dimensions as given in Table 8.2.32.

In **transmitting large torques**, it is customary to use two or more keys.

**Table 8.2.30 Woodruff Key Dimensions [ANSI B17.2-1967 (R90)]**  
(All dimensions in inches)

Key no.	Nominal key size, $A \times B$	Width of key $A$		Diam of key $B$		Height of key				Distance below $E$
		Max	Min	Max	Min	$C$		$D$		
						Max	Min	Max	Min	
204	$\frac{1}{16} \times \frac{1}{2}$	0.0635	0.0625	0.500	0.490	0.203	0.198	0.194	0.188	$\frac{3}{64}$
304	$\frac{3}{32} \times \frac{1}{2}$	0.0948	0.0928	0.500	0.490	0.203	0.198	0.194	0.188	$\frac{3}{64}$
305	$\frac{3}{32} \times \frac{5}{8}$	0.0948	0.0938	0.625	0.615	0.250	0.245	0.240	0.234	$\frac{1}{16}$
404	$\frac{1}{8} \times \frac{1}{2}$	0.1260	0.1250	0.500	0.490	0.203	0.198	0.194	0.188	$\frac{3}{64}$
405	$\frac{1}{8} \times \frac{5}{8}$	0.1260	0.1250	0.625	0.615	0.250	0.245	0.240	0.234	$\frac{1}{16}$
406	$\frac{1}{8} \times \frac{3}{4}$	0.1260	0.1250	0.750	0.740	0.313	0.308	0.303	0.297	$\frac{1}{16}$
505	$\frac{5}{32} \times \frac{5}{8}$	0.1573	0.1563	0.625	0.615	0.250	0.245	0.240	0.234	$\frac{1}{16}$
506	$\frac{5}{32} \times \frac{3}{4}$	0.1573	0.1563	0.750	0.740	0.313	0.308	0.303	0.297	$\frac{1}{16}$
507	$\frac{5}{32} \times \frac{7}{8}$	0.1573	0.1563	0.875	0.865	0.375	0.370	0.365	0.359	$\frac{1}{16}$
606	$\frac{3}{16} \times \frac{3}{4}$	0.1885	0.1875	0.750	0.740	0.313	0.308	0.303	0.297	$\frac{1}{16}$
607	$\frac{3}{16} \times \frac{7}{8}$	0.1885	0.1875	0.875	0.865	0.375	0.370	0.365	0.359	$\frac{1}{16}$
608	$\frac{3}{16} \times 1$	0.1885	0.1875	1.000	0.990	0.438	0.433	0.428	0.422	$\frac{1}{16}$
609	$\frac{3}{16} \times 1\frac{1}{8}$	0.1885	0.1875	1.125	1.115	0.484	0.479	0.475	0.469	$\frac{3}{64}$
807	$\frac{1}{4} \times \frac{7}{8}$	0.2510	0.2500	0.875	0.865	0.375	0.370	0.365	0.359	$\frac{1}{16}$
808	$\frac{1}{4} \times 1$	0.2510	0.2500	1.000	0.990	0.438	0.433	0.428	0.422	$\frac{1}{16}$
809	$\frac{1}{4} \times 1\frac{1}{8}$	0.2510	0.2500	1.125	1.115	0.484	0.479	0.475	0.469	$\frac{3}{64}$
810	$\frac{1}{4} \times 1\frac{1}{4}$	0.2510	0.2500	1.250	1.240	0.547	0.542	0.537	0.531	$\frac{5}{64}$
811	$\frac{1}{4} \times 1\frac{3}{8}$	0.2510	0.2500	1.375	1.365	0.594	0.589	0.584	0.578	$\frac{3}{32}$
812	$\frac{1}{4} \times 1\frac{1}{2}$	0.2510	0.2500	1.500	1.490	0.641	0.636	0.631	0.625	$\frac{7}{64}$
1008	$\frac{5}{16} \times 1$	0.3135	0.3125	1.000	0.990	0.438	0.433	0.428	0.422	$\frac{1}{16}$
1009	$\frac{5}{16} \times 1\frac{1}{8}$	0.3135	0.3125	1.125	1.115	0.484	0.479	0.475	0.469	$\frac{5}{64}$
1010	$\frac{5}{16} \times 1\frac{1}{4}$	0.3135	0.3125	1.250	1.240	0.547	0.542	0.537	0.531	$\frac{5}{64}$
1011	$\frac{5}{16} \times 1\frac{3}{8}$	0.3135	0.3125	1.375	1.365	0.594	0.589	0.584	0.578	$\frac{3}{32}$
1012	$\frac{5}{16} \times 1\frac{1}{2}$	0.3135	0.3125	1.500	1.490	0.641	0.636	0.631	0.625	$\frac{7}{64}$
1210	$\frac{3}{8} \times 1\frac{1}{4}$	0.3760	0.3750	1.250	1.240	0.547	0.542	0.537	0.531	$\frac{5}{64}$
1211	$\frac{3}{8} \times 1\frac{3}{8}$	0.3760	0.3750	1.375	1.365	0.594	0.589	0.584	0.578	$\frac{3}{32}$
1212	$\frac{3}{8} \times 1\frac{1}{2}$	0.3760	0.3750	1.500	1.490	0.641	0.636	0.631	0.625	$\frac{7}{64}$

Numbers indicate the nominal key dimensions. The last two digits give the nominal diameter ( $B$ ) in eighths of an inch, and the digits preceding the last two give the nominal width ( $A$ ) in thirty-seconds of an inch. Thus, 204 indicates a key  $\frac{3}{32} \times \frac{5}{8}$  or  $\frac{1}{16} \times \frac{1}{2}$  in; 1210 indicates a key  $1\frac{1}{2} \times 1\frac{1}{4}$  or  $\frac{3}{8} \times 1\frac{1}{4}$  in.

Another means for fastening gears, pulleys flanges, etc., to shafts is through the use of mating pairs of tapered sleeves known as **grip springs**. A set of sleeves is shown in Fig. 8.2.26. For further references see data issued by the Ringfeder Corp., Westwood, NJ.

**Tapered pins** (Fig. 8.2.27) can be used to transmit very small torques or for positioning. They should be fitted so that the parts are drawn together to prevent their working loose when the pin is driven home. Table 8.2.33 gives dimensions of Morse tapered pins.

The Groov-Pin Corp., New Jersey, has developed a special **grooved**

**pin** (Fig. 8.2.28) which may be used instead of smooth taper pins in certain cases.

**Straight pins**, likewise, are used for transmission of light torques or for positioning. **Spring pins** have come into wide use recently. Two types shown in Figs. 8.2.29 and 8.2.30 deform elastically in the radial direction when driven; the resiliency of the pin material locks the pin in place. They can replace straight and taper pins and combine the advantages of both, i.e., simple tooling, ease of removal, reusability, ability to be driven from either side.

**Table 8.2.31 Dimensions of Square and Flat Gib-Head Taper Stock Keys, in**

Shaft diam	Square type						Flat type					Tolerance	
	Key		Gib head			Max width $W$	Height at large end, $\dagger H$	Key		Gib head		On width (-)	On height (+)
	Max width $W$	Height at large end, $\dagger H$	Height $C$	Length $D$	Height edge of chamfer $E$			Height $C$	Length $D$	Height edge of chamfer $E$			
$\frac{1}{2}-\frac{9}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{7}{32}$	$\frac{5}{32}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	0.0020	0.0020	
$\frac{5}{8}-\frac{7}{8}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{9}{32}$	$\frac{7}{32}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{5}{32}$	0.0020	0.0020	
$1\frac{1}{16}-1\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{7}{16}$	$1\frac{1}{32}$	$1\frac{1}{32}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	0.0020	0.0020	
$1\frac{3}{16}-1\frac{3}{8}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{13}{32}$	$1\frac{3}{32}$	$1\frac{3}{32}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	0.0020	0.0020	
$1\frac{7}{16}-1\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$1\frac{1}{16}$	$1\frac{5}{32}$	$1\frac{5}{32}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{5}{16}$	0.0020	0.0020	
$1\frac{13}{16}-2\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{8}$	$1\frac{9}{32}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{7}{16}$	0.0025	0.0025	
$2\frac{1}{16}-2\frac{3}{4}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{16}$	$2\frac{3}{32}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	0.0025	0.0025	
$2\frac{7}{8}-3\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	0.0025	0.0025	
$3\frac{3}{8}-3\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	1	1	$\frac{7}{8}$	$\frac{5}{8}$	$1\frac{1}{16}$	$\frac{7}{8}$	$\frac{3}{4}$	0.0030	0.0030	
$3\frac{7}{8}-4\frac{1}{2}$	1	1	$1\frac{3}{4}$	$1\frac{3}{16}$	$1\frac{3}{16}$	1	$\frac{3}{4}$	$1\frac{1}{4}$	1	$1\frac{13}{16}$	0.0030	0.0030	
$4\frac{3}{4}-5\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4}$	2	$1\frac{7}{16}$	$1\frac{7}{16}$	$1\frac{1}{4}$	$\frac{7}{8}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	0.0030	0.0030	
$5\frac{3}{4}-6$	$1\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{1}{2}$	1	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	0.0030	0.0030	

\* Stock keys are applicable to the general run of work and the tolerances have been set accordingly. They are not intended to cover the finer applications where a closer fit may be required.

† This height of the key is measured at the distance  $W$  equal to the width of the key, from the gib head.



**Table 8.2.33 Morse Standard Taper Pins**  
(Taper, 1/8 in/ft. Lengths increase by 1/4 in. Dimensions in inches)

Size no.	0	1	2	3	4	5	6	7	8	9	10
Diam at large end	0.156	0.172	0.193	0.219	0.250	0.289	0.341	0.409	0.492	0.591	0.706
Length	0.5–3	0.5–3	0.75–3.5	0.75–3.5	0.75–4	0.75–4	0.75–5	1–5	1.25–5	1.5–6	1.5–6

There are 17 series, as follows: 2.5/5, 3/6, 4/8, 5/10, 6/12, 8/16, 10/20, 12/24, 16/32, 20/40, 24/48, 32/64, 40/80, 48/96, 64/128, 80/160, 128/256. The number of teeth within each series varies from 6 to 50. Both a flat-root and a fillet-root type are provided. There are three **types of fits**: (1) **major diameter**—fit controlled by varying the major diameter of the external spline; (2) **sides of teeth**—fit controlled by varying tooth thickness and customarily used for fillet-root splines; (3) **minor diameter**—fit controlled by varying the minor diameter of the internal spline. Each type of fit is further divided into three classes: (a) **sliding**—clearance at all points; (b) **close**—close on either major diameter, sides of teeth, or minor diameter; (c) **press**—interference on either the major diameter, sides of teeth, or minor diameter. Important basic formulas for tooth proportions are:

$$\begin{aligned}
 D &= \text{pitch diam} \\
 N &= \text{number of teeth} \\
 P &= \text{diametral pitch} \\
 p &= \text{circular pitch} \\
 t &= \text{circular tooth thickness} \\
 a &= \text{addendum} \\
 b &= \text{dedendum} \\
 D_O &= \text{major diam} \\
 \text{TIF} &= \text{true involute form diam} \\
 D_R &= \text{minor diam}
 \end{aligned}$$

#### Flat and Fillet Roots

$$\begin{aligned}
 D &= N/P \\
 p &= \pi/P \\
 t &= p/2 \\
 a &= 0.5000/P \\
 D_O(\text{external}) &= \frac{N+1}{P} \\
 \text{TIF}(\text{internal}) &= \frac{N+1}{P} \\
 D_R &= \frac{N+1}{P} \quad (\text{minor-diameter fits only}) \\
 \text{TIF}(\text{external}) &= \frac{N-1}{P}
 \end{aligned}$$

$b = 0.600/P + 0.002$  (For major-diameter fits, the internal spline dedendum is the same as the addendum; for minor-diameter fits, the dedendum of the external spline is the same as the addendum.)

#### Fillet Root Only

$$\begin{aligned}
 & \frac{1}{2} \text{ through } 1\frac{1}{24} & 1\frac{1}{32} \text{ through } 4\frac{5}{16} \\
 D_O(\text{internal}) &= \frac{N+1.8}{P} & D_O(\text{internal}) &= \frac{N+1.8}{P} \\
 D_R(\text{external}) &= \frac{N-1.8}{P} & D_R(\text{external}) &= \frac{N-2}{P} \\
 b(\text{internal}) &= 0.900/P & b(\text{internal}) &= 0.900/P \\
 b(\text{external}) &= 0.900/P & b(\text{external}) &= 1.000/P
 \end{aligned}$$

Internal spline dimensions are basic while external spline dimensions are varied to control fit.

The advantages of involute splines are: (1) maximum strength at the minor diameter, (2) self-centering equalizes bearing and stresses among all teeth, and (3) ease of manufacture through the use of standard gear-cutting tools and methods.

The design of involute splines is critical in shear. The torque capacity may be determined by the formula  $T = LD^2S_s/1.2732$ , where  $L$  = spline length,  $D$  = pitch diam,  $S_s$  = allowable shear stress.

**Parallel-side splines** have been standardized by the SAE for 4, 6, 10, and 16 spline fittings. They are shown in Fig. 8.2.34; pertinent data are in Tables 8.2.34 and 8.2.35.

#### DRY AND VISCOUS COUPLINGS

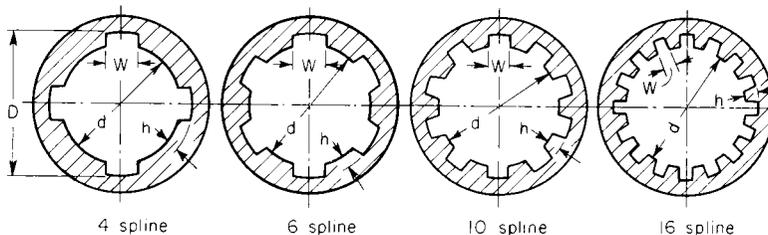
A **coupling** makes a semipermanent connection between two shafts. They are of three main types: **rigid**, **flexible**, and **fluid**.

#### Rigid Couplings

Rigid couplings are used only on shafts which are perfectly aligned. The **flanged-face coupling** (Fig. 8.2.35) is the simplest of these. The flanges must be keyed to the shafts. The **keyless compression coupling** (Fig. 8.2.36) affords a simple means for connecting abutting shafts without the necessity of key seats on the shafts. When drawn over the slotted tapered sleeve the two flanges automatically center the shafts and provide sufficient contact pressure to transmit medium or light loads. **Ribbed-clamp couplings** (Fig. 8.2.37) are split longitudinally and are bored to the shaft diameter with a shim separating the two halves. It is necessary to key the shafts to the coupling.

#### Flexible Couplings

Flexible couplings are designed to connect shafts which are misaligned either laterally or angularly. A secondary benefit is the absorption of



**Fig. 8.2.34** Parallel-sided splines.

**Table 8.2.34 Dimensions of Spline Fittings, in (SAE Standard)**

Nominal diam	4-spline for all fits		6-spline for all fits		10-spline for all fits		16-spline for all fits	
	<i>D</i> max*	<i>W</i> max†	<i>D</i> max*	<i>W</i> max†	<i>D</i> max*	<i>W</i> max†	<i>D</i> max*	<i>W</i> max†
¾	0.750	0.181	0.750	0.188	0.750	0.117		
⅞	0.875	0.211	0.875	0.219	0.875	0.137		
1	1.000	0.241	1.000	0.250	1.000	0.156		
1¼	1.125	0.271	1.125	0.281	1.125	0.176		
1½	1.250	0.301	1.250	0.313	1.250	0.195		
1¾	1.375	0.331	1.375	0.344	1.375	0.215		
1½	1.500	0.361	1.500	0.375	1.500	0.234		
1¾	1.625	0.391	1.625	0.406	1.625	0.254		
1¾	1.750	0.422	1.750	0.438	1.750	0.273		
2	2.000	0.482	2.000	0.500	2.000	0.312	2.000	0.196
2¼	2.250	0.542	2.250	0.563	2.250	0.351		
2½	2.500	0.602	2.500	0.625	2.500	0.390	2.500	0.245
3	3.000	0.723	3.000	0.750	3.000	0.468	3.000	0.294
3½	—	—	—	—	3.500	0.546	3.500	0.343
4	—	—	—	—	4.000	0.624	4.000	0.392
4½	—	—	—	—	4.500	0.702	4.500	0.441
5	—	—	—	—	5.000	0.780	5.000	0.490
5½	—	—	—	—	5.500	0.858	5.500	0.539
6	—	—	—	—	6.000	0.936	6.000	0.588

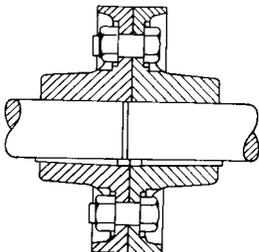
\* Tolerance allowed of -0.001 in for shafts ¾ to 1¼ in, inclusive; of -0.002 for shafts 2 to 3 in, inclusive; -0.003 in for shafts 3½ to 6 in, inclusive, for 4-, 6-, and 10-spline fittings; tolerance of -0.003 in allowed for all sizes of 16-spline fittings.

† Tolerance allowed of -0.002 in for shafts ¾ in to 1¼ in, inclusive; of -0.003 in for shafts 2 to 6 in, inclusive, for 4-, 6-, and 10-spline fittings; tolerance of -0.003 allowed for all sizes of 16-spline fittings.

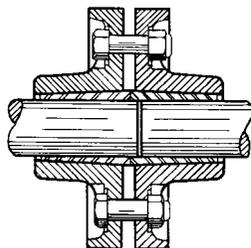
**Table 8.2.35 Spline Proportions**

No. of splines	<i>W</i> for all fits	Permanent fit		To slide when not under load		To slide under load	
		<i>h</i>	<i>d</i>	<i>h</i>	<i>d</i>	<i>h</i>	<i>d</i>
4	0.241 <i>D</i>	0.075 <i>D</i>	0.850 <i>D</i>	0.125 <i>D</i>	0.750 <i>D</i>		
6	0.250 <i>D</i>	0.050 <i>D</i>	0.900 <i>D</i>	0.075 <i>D</i>	0.850 <i>D</i>	0.100 <i>D</i>	0.800 <i>D</i>
10	0.156 <i>D</i>	0.045 <i>D</i>	0.910 <i>D</i>	0.070 <i>D</i>	0.860 <i>D</i>	0.095 <i>D</i>	0.810 <i>D</i>
16	0.098 <i>D</i>	0.045 <i>D</i>	0.910 <i>D</i>	0.070 <i>D</i>	0.860 <i>D</i>	0.095 <i>D</i>	0.810 <i>D</i>

impacts due to fluctuations in shaft torque or angular speed. The **Oldham, or double-slider, coupling** (Fig. 8.2.38) may be used to connect shafts which have only lateral misalignment. The **“Fast” flexible coupling** (Fig. 8.2.39) consists of two hubs each keyed to its respective shaft. Each hub has generated splines cut at the maximum possible



**Fig. 8.2.35** Flanged face coupling.



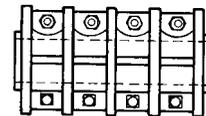
**Fig. 8.2.36** Keyless compression coupling.

distance from the shaft end. Surrounding the hubs is a casing or sleeve which is split transversely and bolted by means of flanges. Each half of this sleeve has generated internal splines cut on its bore at the end opposite to the flange. These internal splines permit a definite error of alignment between the two shafts.

Another type, the Waldron coupling (Midland-Ross Corp.), is shown in Fig. 8.2.40.

The chain coupling shown in Fig. 8.2.41 uses silent chain, but stan-

dard roller chain can be used with the proper mating sprockets. Nylon links enveloping the sprockets are another variation of the chain coupling.



**Fig. 8.2.37** Ribbed-clamp coupling.

**Steefflex couplings** (Fig. 8.2.42) are made with two grooved steel hubs keyed to their respective shafts. Connection between the two halves is secured by a specially tempered alloy-steel member called the “grid.”



**Fig. 8.2.38** Double-slider coupling.

In the rubber flexible coupling shown in Fig. 8.2.43, the torque is transmitted through a comparatively soft rubber section acting in shear. The type in Fig. 8.2.44 loads the intermediate rubber member in compression. Both types permit reasonable shaft misalignment and are recommended for light loads only.

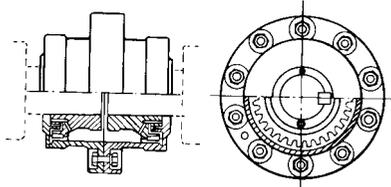


Fig. 8.2.39 "Fast" flexible coupling.

**Universal joints** are used to connect shafts with much larger values of misalignment than can be tolerated by the other types of flexible couplings. Shaft angles up to 30° may be used. The Hooke's-type joint (Fig. 8.2.45) suffers a loss in efficiency with increasing angle which

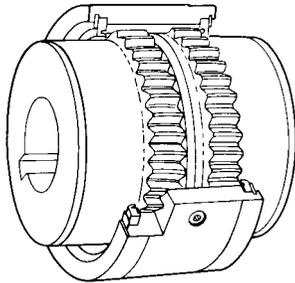


Fig. 8.2.40 Waldron coupling.

may be approximated for angles up to 15° by the following relation: efficiency = 100(1 - 0.003θ), where θ is the angle between the shafts. The velocity ratio between input and output shafts with a single universal joint is equal to

$$\omega_2/\omega_1 = \cos \theta / 1 - \sin^2 \theta \sin^2 (\alpha + 90^\circ)$$

where ω<sub>2</sub> and ω<sub>1</sub> are the angular velocities of the driven and driving shafts respectively, θ is the angle between the shafts, and α is the angu-

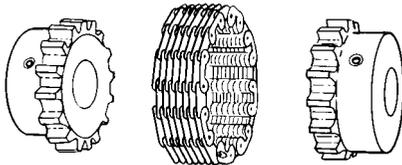


Fig. 8.2.41 Chain coupling.

lar displacement of the driving shaft from the position where the pins on the drive-shaft yoke lie in the plane of the two shafts. A velocity ratio of 1 may be obtained at any angle using two Hooke's-type joints and an intermediate shaft. The intermediate shaft must make equal angles with the main shafts, and the driving pins on the yokes attached to the intermediate shaft must be set parallel to each other.

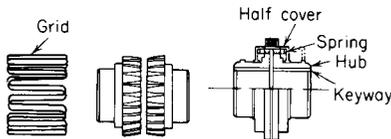


Fig. 8.2.42 Falk Steelflex coupling.

The Bendix-Weiss "rolling-ball" universal joint provides constant angular velocity. Torque is transmitted between two yokes through a set of four balls such that the centers of all four balls lie in a plane which

bisects the angle between the shafts. Other variations of constant velocity universal joints are found in the Rzeppa, Tracta, and double Cardan types.

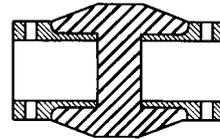


Fig. 8.2.43 Rubber flexible coupling, shear type.

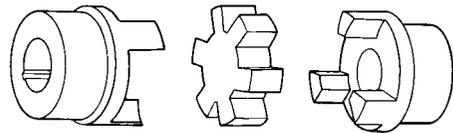


Fig. 8.2.44 Rubber flexible coupling, compression type.

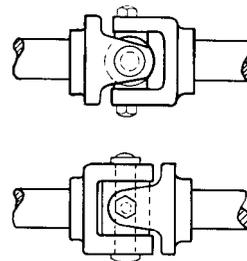


Fig. 8.2.45 Hooke's universal joint.

### Fluid Couplings

(See also Sec. 11.)

Fluid couplings (Fig. 8.2.46) have two basic parts—the input member, or impeller, and the output member, or runner. There is no mechanical connection between the two shafts, power being transmitted by kinetic energy in the operating fluid. The impeller *B* is fastened to the flywheel *A* and turns at engine speed. As this speed increases, fluid within the impeller moves toward the outer periphery because of centrifugal force. The circular shape of the impeller directs the fluid toward the runner *C*, where its kinetic energy is absorbed as torque delivered by shaft *D*. The positive pressure behind the fluid causes flow to continue toward the hub and back through the impeller. The toroidal space in both the impeller and runner is divided into compartments by a series of flat radial vanes.

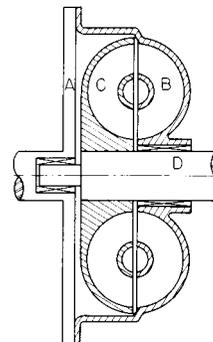


Fig. 8.2.46 a Fluid coupling. (A) Flywheel; (B) impeller; (C) runner; (D) output shaft.

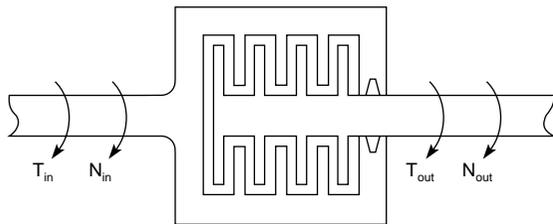


Fig. 8.2.46b Schematic of viscous coupling.

The torque capacity of a fluid coupling with a full-load slip of about 2.5 percent is  $T = 0.09n^2D^5$ , where  $n$  is the impeller speed, hundreds of r/min, and  $D$  is the outside diameter, ft. The output torque is equal to the input torque over the entire range of input-output speed ratios. Thus the prime mover can be operated at its most effective speed regardless of the speed of the output shaft. Other advantages are that the prime mover cannot be stalled by application of load and that there is no transmission of shock loads or torsional vibration between the connected shafts.

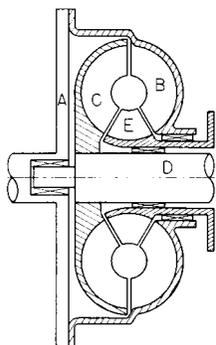


Fig. 8.2.47 Hydraulic torque converter. (A) Flywheel; (B) impeller; (C) runner; (D) output shaft; (E) reactor.

Each element is designed independently of each other according to the load requirements. Other refinements include the addition of multiple stages in the runner and reactor stages as in steam reaction turbines (see Sec. 9). The advantages of a torque converter are the ability to multiply starting torque 5 to 6 times and to serve as a stepless transmission. As in the coupling, torque varies as the square of speed and the fifth power of diameter.

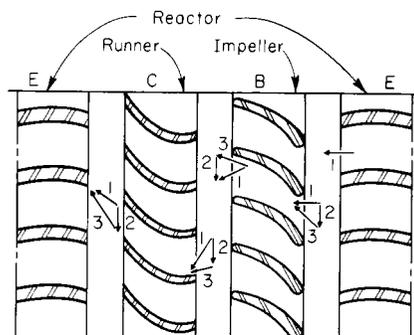


Fig. 8.2.48 Schematic of converter blading. (1) Absolute fluid velocity; (2) velocity vector—converter elements; (3) fluid velocity relative to converter elements.

A **hydraulic torque converter** (Fig. 8.2.47) is similar in form to the hydraulic coupling, with the addition of a set of stationary guide vans, the reactor, interspersed between the runner and the impeller. All blades in a converter have compound curvature. This curvature is designed to control the direction of fluid flow. Kinetic energy is therefore transferred as a result of both a scalar and vectorial change in fluid velocity. The blades are designed such that the fluid will be moving in a direction parallel to the blade surface at the entrance (Fig. 8.2.48) to each section. With a design having fixed blading, this can be true at only one value of runner and impeller velocity, called the design point. Several design modifications are possible to overcome this difficulty. The angle of the blades can be made adjustable, and the elements can be divided into sections operating independently

Optimum efficiency (Fig. 8.2.49) over the range of input-output speed ratios is obtained by a combination converter coupling. When the output speed rises to the point where the torque multiplication factor is 1.0, the clutch point, the torque reaction on the reactor element reverses direction. If the reactor is mounted to freewheel in this opposite direction, the unit will act as a coupling over the higher speed ranges. An automatic friction clutch (see "Clutches," below) set to engage at or near the clutch point will also eliminate the poor efficiency of the converter at high output speeds.

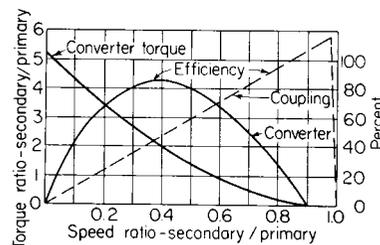


Fig. 8.2.49 Hydraulic coupling characteristic curves. (Heldt, "Torque Converters and Transmissions," Chilton.)

**Viscous couplings** are becoming major players in mainstream front-wheel-drive applications and are already used in four-wheel-drive vehicles.

Torque transmission in a viscous coupling relies on shearing forces in an entrapped fluid between axially positioned disks rotating at different angular velocities (Fig. 8.2.46b), all encased in a lifetime leakproof housing. A hub carries the so-called inner disks while the housing carries the so-called outer disks. Silicone is the working fluid.

Operation of the coupling is in normal (slipping) mode when torque is being generated by viscous shear. However, prolonged slipping under severe starting conditions causes heat-up, which in turn causes the fluid, which has a high coefficient of thermal expansion, to expand considerably with increasing temperature. It then fills the entire available space, causing a rapid pressure increase, which in turn forces the disks together into metal-to-metal frictional contact. Torque transmission now increases substantially. This self-induced torque amplification is known as the **hump effect**. The point at which the hump occurs can be set by the design and coupling setup. Under extreme conditions, 100 percent lockup occurs, thus providing a self-protecting relief from overheating as fluid shear vanishes. This effect is especially useful in autos using viscous couplings in their limited-slip differentials, when one wheel is on low-friction surfaces such as ice. The viscous coupling transfers torque to the other gripping wheel. This effect is also useful when one is driving up slopes with uneven surface conditions, such as rain or snow, or on very rough surfaces. Such viscous coupling differentials have allowed a weight and cost reduction of about 60 percent. A fuller account can be found in Barlage, *Viscous Couplings Enter Main Stream Vehicles*, *Mech. Eng.*, Oct. 1993.

## CLUTCHES

Clutches are couplings which permit the disengagement of the coupled shafts during rotation.

**Positive clutches** are designed to transmit torque without slip. The **jaw clutch** is the most common type of positive clutch. These are made with **square jaws** (Fig. 8.2.50) for driving in both directions or **spiral jaws** (Fig. 8.2.51) for unidirectional drive. Engagement speed should be limited to 10 r/min for square jaws and 150 r/min for spiral jaws. If disengagement under load is required, the jaws should be finish-machined and lubricated.

**Friction clutches** are designed to reduce coupling shock by slipping during the engagement period. They also serve as safety devices by slipping when the torque exceeds their maximum rating. They may be

divided into two main groups, **axial** and **rim** clutches, according to the direction of contact pressure.

The **cone clutch** (Fig. 8.2.52) and the **disk clutch** (Fig. 8.2.53) are examples of axial clutches. The disk clutch may consist of either a

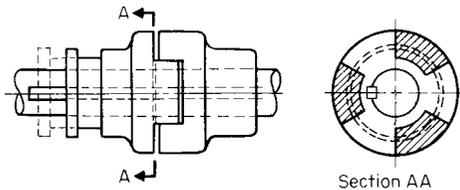


Fig. 8.2.50 Square-jaw clutch.

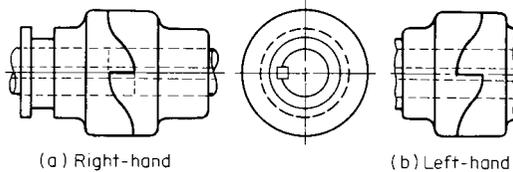


Fig. 8.2.51 Spiral-jaw clutch.

single plate or multiple disks. Table 8.2.36 lists typical friction materials and important design data. The torque capacity of a disk clutch is given by  $T = 0.5ifF_aD_m$ , where  $T$  is the torque,  $i$  the number of pairs of contact surfaces,  $f$  the applicable coefficient of friction,  $F_a$  the axial

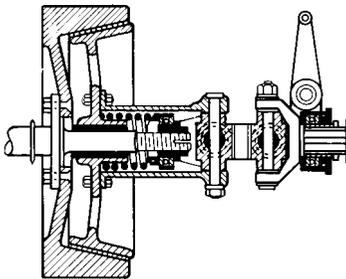


Fig. 8.2.52 Cone clutch.

engaging force, and  $D_m$  the mean diameter of the clutch facing. The spring forces holding a disk clutch in engagement are usually of relatively high value, as given by the allowable contact pressures. In order to lower the force required at the operating lever, elaborate linkages are required, usually having lever ratios in the range of 10 to 12. As these linkages must rotate with the clutch, they must be adequately balanced and the effect of centrifugal forces must be considered. Disk clutches

are often run wet, either immersed in oil or in a spray. The advantages are reduced wear, smoother action, and lower operating temperatures. Disk clutches are often operated automatically by either air or hydraulic cylinders as, for examples, in automobile automatic transmissions.

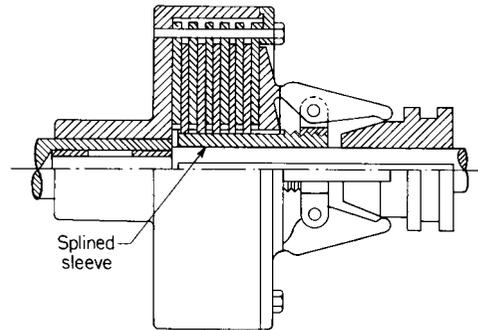


Fig. 8.2.53 Multidisk clutch.

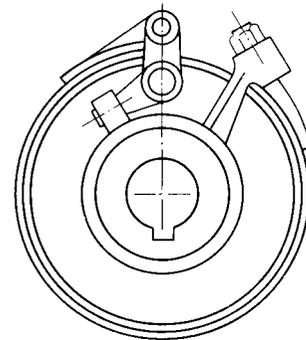


Fig. 8.2.54 Band clutch.

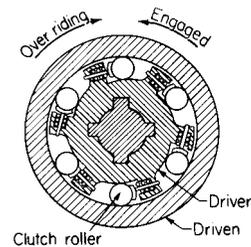


Fig. 8.2.55 Overrunning clutch.

Table 8.2.36 Friction Coefficients and Allowable Pressures

Materials in contact	Friction coefficient $f$			Allowable pressure, lb/in <sup>2</sup>
	Dry	Greasy	Lubricated	
Cast iron on cast iron	0.2–0.15	0.10–0.06	0.10–0.05	150–250
Bronze on cast iron	—	0.10–0.05	0.10–0.05	80–120
Steel on cast iron	0.30–0.20	0.12–0.07	0.10–0.06	120–200
Wood on cast iron	0.25–0.20	0.12–0.08	—	60–90
Fiber on metal	—	0.20–0.10	—	10–30
Cork on metal	0.35	0.30–0.25	0.25–0.22	8–15
Leather on metal	0.5–0.3	0.20–0.15	0.15–0.12	10–30
Wire asbestos on metal	0.5–0.35	0.30–0.25	0.25–0.20	40–80
Asbestos blocks on metal	0.48–0.40	0.30–0.25	—	40–160
Asbestos on metal, short action	—	—	0.25–0.20	200–300
Metal on cast iron, short action	—	—	0.10–0.05	200–300

SOURCE: Maleev, *Machine Design*, International Textbook, by permission.

**Rim clutches** may be subdivided into two groups: (1) those employing either a band or block (Fig. 8.2.54) in contact with the rim and (2) overrunning clutches (Fig. 8.2.55) employing the wedging action of a roller or sprag. Clutches in the second category will automatically engage in one direction and freewheel in the other.

**HYDRAULIC POWER TRANSMISSION**

Hydraulic power transmission systems comprise machinery and auxiliary components which function to generate, transmit, control, and utilize hydraulic power. The **working fluid**, a pressurized incompressible liquid, is usually either a petroleum base or a fire-resistant type. The latter are water and oil emulsions, glycol-water mixtures, or synthetic liquids such as silicones or phosphate esters.

Liquid is pressurized in a **pump** by virtue of its resistance to flow; the pressure difference between pump inlet and outlet results in flow. Most hydraulic applications employ positive-displacement pumps of the gear, vane, screw, or piston type; piston pumps are axial, radial, or reciprocating (see Sec. 14).

Power is transmitted from pump to controls and point of application through a combination of **conduit and fittings** appropriate to the particular application. Flow characteristics of hydraulic circuits take into account fluid properties, pressure drop, flow rate, and pressure-surfing tendencies. Conduit systems must be designed to minimize changes in flow velocity, velocity distribution, and random fluid eddies, all of which dissipate energy and result in pressure drops in the circuit (see Sec. 3). Pipe, tubing, and flexible hose are used as hydraulic power conduits; suitable fittings are available for all types and for transition from one type to another.

**Controls** are generally interposed along the conduit between the pump and point of application (i.e., an actuator or motor), and act to control pressure, volume, or flow direction.

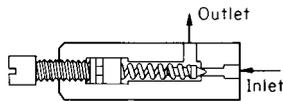


Fig. 8.2.56 Relief valve.

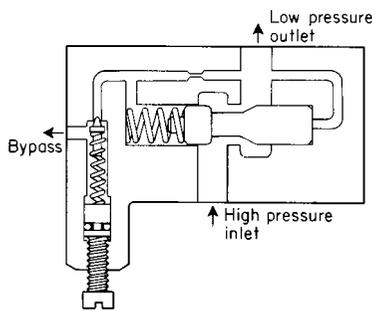


Fig. 8.2.57 Reducing valve.

**Pressure control valves**, of which an ordinary safety valve is a common type (normally closed), include relief and reducing valves and pressure switches (Figs. 8.2.56 and 8.2.57). Pressure valves, normally closed, can be used to control sequential operations in a hydraulic circuit. **Flow control valves** throttle flow to or bypass flow around the unit being controlled, resulting in pressure drop and temperature increase as pressure energy is dissipated. Figure 8.2.58 shows a simple needle valve with variable orifice usable as a flow control valve. **Directional control valves** serve primarily to direct hydraulic fluid to the point of application. Directional control valves with rotary and sliding spools are shown in Figs. 8.2.59 and 8.2.60.

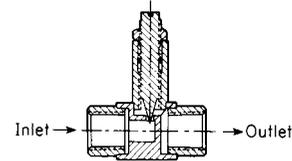


Fig. 8.2.58 Needle valve.

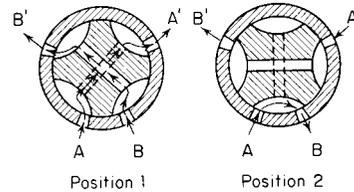


Fig. 8.2.59 Rotary-spool directional flow valve.

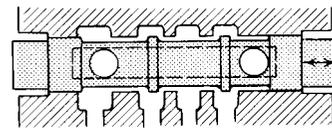


Fig. 8.2.60 Sliding-spool directional flow valve.

A poppet (valve) mechanism is shown in Fig. 8.2.61, a diaphragm valve in Fig. 8.2.62, and a shear valve in Fig. 8.2.63.

**Accumulators** are effectively "hydraulic flywheels" which store potential energy by accumulating a quantity of pressurized hydraulic fluid

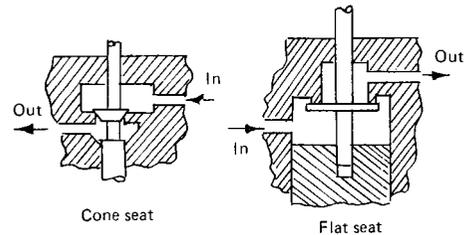


Fig. 8.2.61 Poppet valve.

in a suitable enclosed vessel. The bag type shown in Fig. 8.2.64 uses pressurized gas inside the bag working against the hydraulic fluid outside the bag. Figure 8.2.65 shows a piston accumulator.

Pressurized hydraulic fluid acting against an **actuator** or motor converts fluid pressure energy into mechanical energy. Motors providing

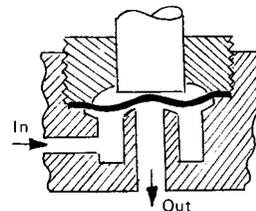


Fig. 8.2.62 Diaphragm valve.

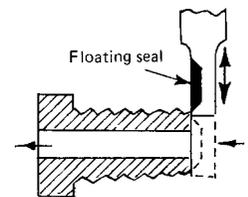


Fig. 8.2.63 Shear valve.

continuous rotation have operating characteristics closely related to their pump counterparts. A linear actuator, or cylinder (Fig. 8.2.66), provides straight-line reciprocating motion; a rotary actuator (Fig. 8.2.67) provides arcuate oscillatory motion. Figure 8.2.68 shows a one-shot booster (linear motion) which can be used to deliver sprays through a nozzle.

**Hydraulic fluids** (liquids and air) are conducted in **pipe, tubing, or flexible hose**. Hose is used when the lines must flex or in applications in which fixed, rigid conduit is unsuitable. Table 8.2.37 lists SAE standard hoses. Maximum recommended operating pressure for a broad range of industrial applications is approximately 25 percent of rated bursting pressure. Due consideration must be given to the operating-temperature range; most applications fall in the range from  $-40$  to  $200^{\circ}\text{F}$  ( $-40$  to  $95^{\circ}\text{C}$ ). Higher operating temperatures can be accommodated with appropriate materials.

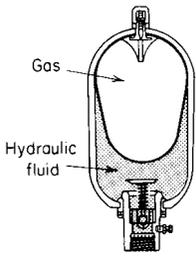


Fig. 8.2.64 Bag accumulator.

**Hose fittings** are of the screw-type or swaged, depending on the particular application and operating pressure and temperature. A broad variety of hose-end fittings is available from the industry.

**Pipe** has the advantage of being relatively cheap, is applied mainly in straight runs, and is usually of steel. Fittings for pipe are either standard pipe fittings for fairly low pressures or more elaborate ones suited to leak-proof high-pressure operation.

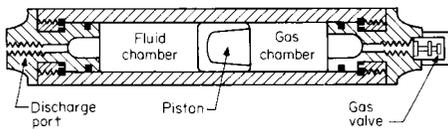


Fig. 8.2.65 Piston accumulator.

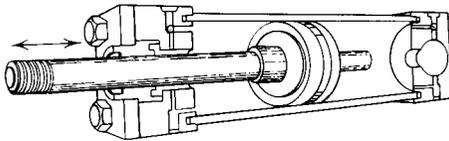


Fig. 8.2.66 Linear actuator or hydraulic cylinder.

**Tubing** is more easily bent into neat forms to fit between inlet and outlet connections. Steel and stainless-steel tubing is used for the highest-pressure applications; aluminum, plastic, and copper tubing is also used as appropriate for the operating conditions of pressure and temperature. Copper tubing hastens the oxidation of oil-base hydraulic fluids; accordingly, its use should be restricted either to air lines or with liquids which will not be affected by copper in the operating range.

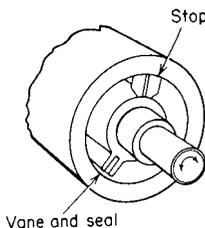


Fig. 8.2.67 Rotary actuator.

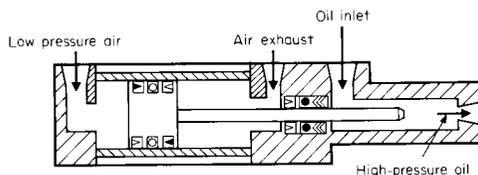


Fig. 8.2.68 One-shot booster.

Table 8.2.37 SAE Standard Hoses

100R1A	One-wire-braid reinforcement, synthetic rubber cover
100R1T	Same as R1A except with a thin, nonskive cover
100R2A	Two-wire-braid reinforcement, synthetic rubber cover
100R2B	Two spiral wire plus one wire-braid reinforcement, synthetic rubber cover
100R2AT	Same as R2A except with a thin, nonskive cover
100R2BT	Same as R2B except with a thin, nonskive cover
100R3	Two rayon-braid reinforcement, synthetic rubber cover
100R5	One textile braid plus one wire-braid reinforcement, textile braid cover
100R7	Thermoplastic tube, synthetic fiber reinforcement, thermoplastic cover (thermoplastic equivalent to SAE 100R1A)
100R8	Thermoplastic tube, synthetic fiber reinforcement, thermoplastic cover (thermoplastic equivalent to SAE 100R2A)
100R9	Four-ply, light-spiral-wire reinforcement, synthetic rubber cover
100R9T	Same as R9 except with a thin, nonskive cover
100R10	Four-ply, heavy-spiral-wire reinforcement, synthetic rubber cover
100R11	Six-ply, heavy-spiral-wire reinforcement, synthetic rubber cover

**Tube fittings** for permanent connections allow for brazed or welded joints. For temporary or separable applications, **flared** or **flareless fittings** are employed (Figs. 8.2.69 and 8.2.70). ANSI B116.1-1974 and B116.2-1974 pertain to tube fittings. The variety of fittings available is vast; the designer is advised to refer to manufacturers' literature for specifics.

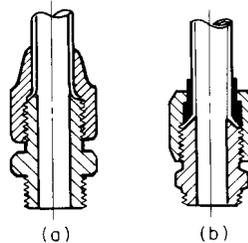


Fig. 8.2.69 Flared tube fittings.  
(a) A  $45^{\circ}$  flared fitting; (b) Triple-lok flared fitting. (Parker-Hannafin Co.)

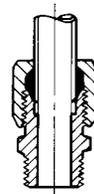


Fig. 8.2.70 Ferulok flareless tube fitting. (Parker-Hannafin Co.)

Parameters entering into the design of a hydraulic system are volume of flow per unit time, operating pressure and temperature, viscosity characteristics of the fluid within the operating range, and compatibility of the fluid/conduit material.

**Flow velocity** in suction lines is generally in the range of 1 to 5 ft/s (0.3 to 1.5 m/s); in discharge lines it ranges from 10 to 25 ft/s (3 to 8 m/s).

The pipe or tubing is under internal pressure. Selection of material and wall thickness follows from suitable equations (see Sec. 5). Safety factors range from 6 to 10 or higher, depending on the severity of the application (i.e., vibration, shock, pressure surges, possibility of physical abuse, etc.). JIC specifications provide a guide to the designer of hydraulic systems.

## BRAKES

Brakes may be classified as: (1) rim type—internally expanding or externally contracting shoes, (2) band type, (3) cone type, (4) disk or axial type, (5) miscellaneous.

**Rim Type—Internal Shoe(s)** (Fig. 8.2.71)

$$F = \begin{cases} \frac{M_N - M\mu}{d} & \text{clockwise rotation} \\ \frac{M_N + M\mu}{d} & \text{counterclockwise rotation} \end{cases}$$

where

$$M\mu = \frac{\mu P_a Br}{\sin \theta_a} \int_{\theta_1}^{\theta_2} (\sin \theta)(r - d \cos \theta) d\theta$$

$B$  = face width of frictional material;  $P_a$  = maximum pressure;  $\theta_a$  = angle to point of maximum pressure (if  $\theta_2 > 90^\circ$ ; then  $\theta_a = 90^\circ$ ;  $\theta_2 < 90^\circ$ , then  $\theta_a = \theta_2$ );  $\mu$  = coefficient of friction;  $r$  = radius of drum;  $d$  = distance from drum center to brake pivot;

$$M_N = \frac{P_a Brd}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin^2 \theta d\theta$$

and torque on drum is

$$T = \frac{\mu P_a Br^2 (\cos \theta_1 - \cos \theta_2)}{\sin \theta_a}$$

**Self-locking of the brake** ( $F = 0$ ) will occur for clockwise rotation when  $M_N = M\mu$ . This self-energizing phenomenon can be used to advantage without actual locking if  $\mu$  is replaced by a larger value  $\mu'$  so that  $1.25 \mu \leq \mu' \leq 1.50$ , from which the pivot position  $a$  can be solved.

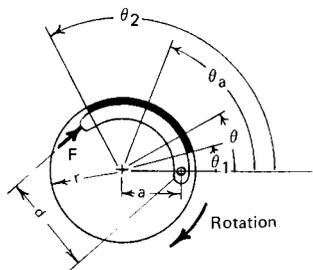


Fig. 8.2.71 Rim brake: internal friction shoe.

In automotive use there are two shoes made to pivot in opposition, so that self-energization is present and can be used to great advantage (Fig. 8.2.72).

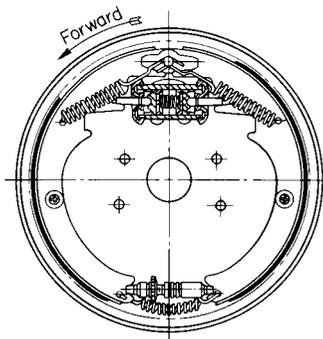


Fig. 8.2.72 Internal brake.

**Rim Type—External Shoe(s)** (Fig. 8.2.73)

The equations for  $M_N$  and  $M\mu$  are the same as above:

$$F = \begin{cases} \frac{M_N + M\mu}{d} & \text{clockwise rotation} \\ \frac{M_N - M\mu}{d} & \text{counterclockwise rotation} \end{cases}$$

Self-locking ( $F = 0$ ) can occur for counterclockwise rotation at  $M_N = M\mu$ .

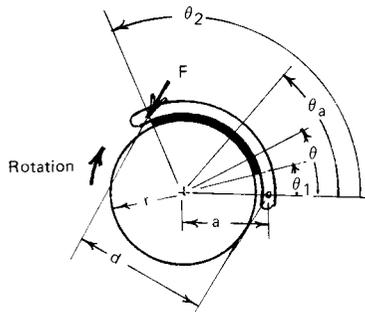


Fig. 8.2.73 Rim brake: external friction shoe.

**Band Type** (Fig. 8.2.74a, b, and c)

Flexible band brakes are used in power excavators and in hoisting. The bands are usually of an asbestos fabric, sometimes reinforced with copper wire and impregnated with asphalt.

In Fig. 8.2.74a,  $F$  = force at end of brake handle;  $P$  = tangential force at rim of wheel;  $f$  = coefficient of friction of materials in contact;

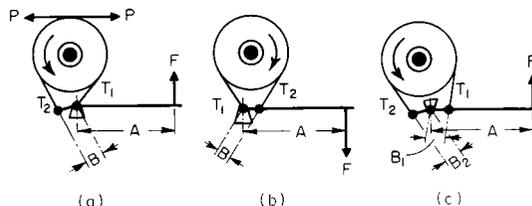


Fig. 8.2.74 Band brakes.

$a$  = angle of wrap of band, deg;  $T_1$  = total tension in band on tight side;  $T_2$  = total tension in band on slack side. Then  $T_1 - T_2 = P$  and  $T_1/T_2 = 10^{0.0076fa} = 10^b$ , where  $b = 0.0076fa$ . Also,  $T_2 = P/(10^b - 1)$  and  $T_1 = P \times 10^b/(10^b - 1)$ . The values of  $10^{0.0076fa}$  are given in Fig. 8.2.90 for  $a$  in radians.

For the arrangement shown in Fig. 8.2.74a,

$$FA = T_2 B = PB/(10^b - 1)$$

and  $F = PB/[A(10^b - 1)]$

For the construction illustrated in Fig. 8.2.74b,

$$F = PB/[A\{10^b/(10^b - 1)\}]$$

For the differential brake shown in Fig. 8.2.74c,

$$F = (P/A)[(B_2 - 10^b B_1)/(10^b - 1)]$$

In this arrangement, the quantity  $10^b B_1$  must always be less than  $B_2$ , or the band will grip the wheel and the brake, or part of the mechanism to which it is attached, will rupture.

It is usual in practice to have the leverage ratio  $A/B$  for band brakes about 10:1.

If  $f$  for wood on iron is taken at 0.3 and the angle of wrap for the band is  $270^\circ$ , i.e., subtends three-fourths of the circumference, then  $10^b = 4$  approx; the loads required for a given torque will be as follows for the cases just considered and for the leverage ratios stated above:

Band brake, Fig. 8.2.74a	$F = 0.033P$
Band brake, Fig. 8.2.74b	$F = 0.133P$
Band brake, Fig. 8.2.74c	$F = 0.016P$

In the case of Fig. 8.2.74c, the dimension  $B_2$  must be greater than  $B_1 \times 10^b$ . Accordingly,  $B_1$  is taken as  $1/4$ ,  $A$  as 10, and, since  $10^b = 4$ ,  $B_2$  is taken as  $1 1/2$ .

The principal function of a brake is to absorb energy. This energy appears at the surface of the brake as heat, which must be carried away

at a sufficiently rapid rate to prevent burning of the wooden blocks. Suitable proportions may be arrived at as follows:

Let  $p$  = unit pressure on brake surface,  $\text{lb/in}^2 = R$  (reaction against block)/area of block;  $v$  = velocity of brake rim surface,  $\text{ft/s} = 2\pi rn/60$ , where  $n$  = speed of brake wheel,  $\text{r/min}$ . Then  $pv$  = work absorbed per  $\text{in}^2$  of brake surface per second, and  $pv \leq 1,000$  for intermittent applications of load with comparatively long periods of rest and poor means for carrying away heat (wooden blocks);  $pv \leq 500$  for continuous application of load and poor means for carrying away heat (wooden blocks);  $pv \leq 1,400$  for continuous application of load with effective means for carrying away heat (oil bath).

**Cone Brake** (see Fig. 8.2.75)

**Uniform Wear**

$$F = \frac{\pi P_a d}{2} (D - d)$$

$$T = \frac{\pi \mu P_a d}{8 \sin \alpha} (D^2 - d^2)$$

where  $P_a$  = maximum pressure occurring at  $d/2$ .

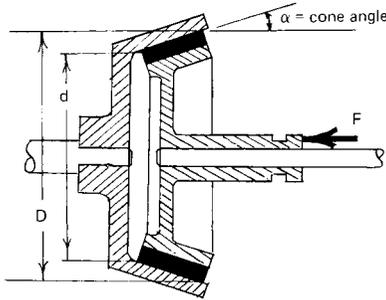


Fig. 8.2.75 Cone brake.

**Uniform Pressure**

$$F = \frac{\pi P_a}{4} (D^2 - d^2)$$

$$T = \frac{\pi \mu P_a}{12 \sin \alpha} (D^3 - d^3) = \frac{F \mu}{3 \sin \alpha} \times \frac{D^3 - d^3}{D^2 - d^2}$$

Figure 8.2.76 shows a cone brake arrangement used for lowering heavy loads.

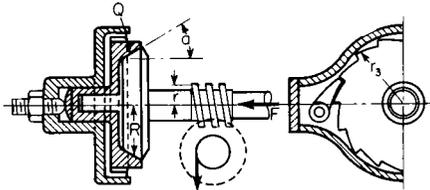


Fig. 8.2.76 Cone brake for lowering loads.

**Disk Brakes** (see Fig. 8.2.77)

Disk brakes are free from "centrifugal" effects, can have large frictional areas contained in small space, have better heat dissipation qualities than the rim type, and enjoy a favorable pressure distribution.

**Uniform Wear**

$$F = \frac{\pi P_a d}{2} (D - d)$$

$$T = \frac{F \mu}{4} (D + d)$$

**Uniform Pressure**

$$F = \frac{\pi P_a}{4} (D^2 - d^2)$$

$$T = \frac{F \mu}{3} \times \frac{D^3 - d^3}{D^2 - d^2}$$

These relations apply to a single surface of contact. For caliper disk, or multidisk brakes, the above relations are applied for each surface of contact.

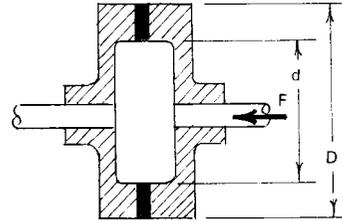


Fig. 8.2.77 Disk brake.

Selected friction materials and properties are listed in Table 8.2.38.

Frequently disk brakes are made as shown in Fig. 8.2.78. The pinion  $Q$  engages the gear in the drum (not shown). When the load is to be raised, power is applied through the gear and the connection between  $B$  and  $C$  is accomplished by the advancing of  $B$  along  $A$  and the clamping of the friction disks  $D$  and  $D$  and the ratchet wheel  $E$ . The reversal of the motor disconnects  $B$  and  $C$ . In lowering the load, only as much reversal of rotation of the gear is given as is needed to reduce the force in the friction disks so that the load may be lowered under control.

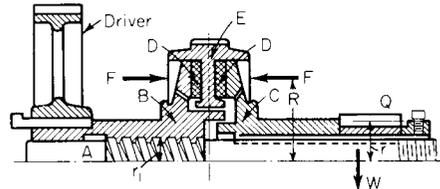


Fig. 8.2.78 Disk brake.

A **multidisk brake** is shown in Fig. 8.2.79. This type of construction results in an increase in the number of friction faces. The drum shaft is geared to the pinion  $A$ , while the motive power for driving comes through the gear  $G$ . In raising the load, direct connection is had between  $G$ ,  $B$ , and  $A$ . In lowering,  $B$  moves relatively to  $G$  and forces the friction plates together, those plates fast to  $E$  being held stationary by the pawl on  $E$ . In the figure, there are three plates fast to  $E$ , one fast to  $G$ , and one fast to  $C$ .

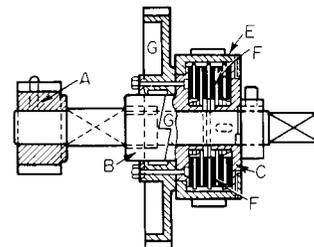


Fig. 8.2.79 Multidisk brake.

**Eddy-current brakes** (Fig. 8.2.80) are used with flywheels where quick braking is essential, and where large kinetic energy of the rotating

Table 8.2.38 Selected Friction Materials and Properties

Material	Opposing material	Friction coefficient			Max pressure		Max temperature	
		Dry	Wet	In oil	lb/in <sup>2</sup>	kPa	°F	°C
Sintered metal	Cast iron or steel	0.1–0.4	0.05–.01	0.05–0.08	150–250	1,000–1,720	450–1,250	232–677
Wood	Cast iron or steel	0.2–0.35	0.16	0.12–0.16	60–90	400–620	300	149
Leather	Cast iron or steel	0.3–0.5	0.12		10–40	70–280	200	93
Cork	Cast iron or steel	0.3–0.5	0.15–0.25	0.15–0.25	8–14	55–95	180	82
Felt	Cast iron or steel	0.22	0.18		5–10	35–70	280	138
Asbestos-woven	Cast iron or steel	0.3–0.6	0.1–0.2	0.08–0.10	50–100	350–700	400–500	204–260
Asbestos-molded	Cast iron or steel	0.2–0.5	0.08–0.12	0.06–0.09	50–150	350–1,000	400–500	204–260
Asbestos-impregnated	Cast iron or steel	0.32	0.12					
Cast iron	Cast iron	0.15–0.20	0.05	0.03–0.06	150–250	1,000–1,720	500	260
Cast iron	Steel			0.03–0.06	100–250	690–1,720	500	260
Graphite	Steel	0.25	0.05–0.1	0.12 (av)	300	2,100	370–540	188–282

masses precludes the use of block brakes due to excessive heating, as in reversible rolling mills. A number of poles  $a$  are electrically excited (north and south in turn) and create a magnetic flux which permeates the gap and the iron of the rim, causing eddy current. The flywheel energy

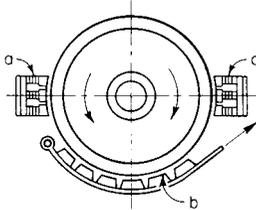


Fig. 8.2.80 Eddy-current brake.

is converted through these currents into heat. The hand brake  $b$  may be used for quicker stopping when the speed of the wheel is considerably decreased; i.e., when the eddy-current brake is inefficient. Two brakes are provided to avoid bending forces on the shaft.

**Electric brakes** are often used in cranes, bridges, turntables, and machine tools, where an automatic application of the brake is important as soon as power is cut off. The brake force is supplied by an adjustable spring which is counteracted by the force of a solenoid or a centrifugal thruster. Interruption of current automatically applies the spring-activated brake shoes. Figures 8.2.81 and 8.2.82 show these types of electric brake.

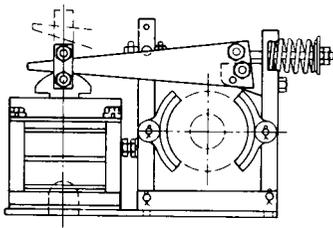


Fig. 8.2.81 Solenoid-type electric brake.

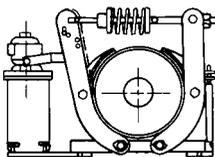


Fig. 8.2.82 Thruster-type electric brake.

## SHRINK, PRESS, DRIVE, AND RUNNING FITS

**Inch Systems** ANSI B4.1-1967 (R87) recommends preferred sizes, allowances, and tolerances for fits between plain cylindrical parts. Such fits include bearing, shrink and drive fits, etc. Terms used in describing fits are defined as follows: **Allowance:** minimum clearance (positive allowance) or maximum interference (negative allowance) between mating parts. **Tolerance:** total permissible variation of size. **Limits of size:** applicable maximum and minimum sizes. **Clearance fit:** one having limits of size so prescribed that a clearance always results when mating parts are assembled. **Interference fit:** In this case, limits are so prescribed that interference always results on assembly. **Transition fit:** This may have either a clearance or an interference on assembly. **Basic size:** one from which limits of size are derived by the application of allowances and tolerances. **Unilateral tolerance:** In this case a variation in size is permitted in only one direction from the basic size.

Fits are divided into the following general classifications: (1) running and sliding fits, (2) locational clearance fits, (3) transition fits, (4) locational interference fits, and (5) force or shrink fits.

1. **Running and sliding fits.** These are intended to provide similar running performance with suitable lubrication allowance throughout the range of sizes. These fits are further subdivided into the following classes:

**Class RC1: close-sliding fits.** Intended for accurate location of parts which must assemble without perceptible play.

**Class RC2: sliding fits.** Parts made with this fit move and turn easily but are not intended to run freely; also, in larger sizes they may seize under small temperature changes.

**Class RC3: precision-running fits.** These are intended for precision work at slow speeds and light journal pressures but are not suitable where appreciable temperature differences are encountered.

**Class RC4: close-running fits.** For running fits on accurate machinery with moderate surface speeds and journal pressures, where accurate location and minimum play is desired.

**Classes RC5 and RC6: medium-running fits.** For higher running speeds or heavy journal pressures.

**Class RC7: free-running fits.** For use where accuracy is not essential, or where large temperature variations are likely to be present, or both.

**Classes RC8 and RC9: loose-running fits.** For use with materials such as cold-rolled shafting or tubing made to commercial tolerances.

**Limits of clearance** given in ANSI B4.1-1967 (R87) for each of these classes are given in Table 8.2.39. Hole and shaft tolerances are listed on a unilateral tolerance basis in this reference to give the clearance limits of Table 8.2.39, the hole size being the basic size.

2. **Locational clearance fits.** These are intended for normally stationary parts which can, however, be freely assembled or disassembled. These are subdivided into various classes which run from snug fits for parts requiring accuracy of location, through medium clearance fits (spigots) to the looser fastener fits where freedom of assembly is of prime importance.

**Table 8.2.39 Limits of Clearance for Running and Sliding Fits (Basic Hole)**  
(Limits are in thousandths of an inch on diameter)

Nominal size range, in	Class								
	RC1	RC2	RC3	RC4	RC5	RC6	RC7	RC8	RC9
0–0.12	0.1	0.1	0.3	0.3	0.6	0.6	1.0	2.5	4.0
	0.45	0.55	0.95	1.3	1.6	2.2	2.6	5.1	8.1
0.12–0.24	1.5	0.15	0.4	0.4	0.8	0.8	1.2	2.8	4.5
	0.5	0.65	1.2	1.6	2.0	2.7	3.1	5.8	9.0
0.24–0.40	0.2	0.2	0.5	0.5	1.0	1.0	1.6	3.0	5.0
	0.6	0.85	1.5	2.0	2.5	3.3	3.9	6.6	10.7
0.40–0.71	0.25	0.25	0.6	0.6	1.2	1.2	2.0	3.5	6.0
	0.75	0.95	1.7	2.3	2.9	3.8	4.6	7.9	12.8
0.71–1.19	0.3	0.3	0.8	0.8	1.6	1.6	2.5	4.5	7.0
	0.95	1.2	2.1	2.8	3.6	4.8	5.7	10.0	15.5
1.19–1.97	0.4	0.4	1.0	1.0	2.0	2.0	3.0	5.0	8.0
	1.1	1.4	2.6	3.6	4.6	6.1	7.1	11.5	18.0
1.97–3.15	0.4	0.4	1.2	1.2	2.5	2.5	4.0	6.0	9.0
	1.2	1.6	3.1	4.2	5.5	7.3	8.8	13.5	20.5
3.15–4.73	0.5	0.5	1.4	1.4	3.0	3.0	5.0	7.0	10.0
	1.5	2.0	3.7	5.0	6.6	8.7	10.7	15.5	24.0

3. **Transition fits.** These are for applications where accuracy of location is important, but a small amount of either clearance or interference is permissible.

4. **Locational interference fits.** Used where accuracy of location is of prime importance and for parts requiring rigidity and alignment with no special requirements for bore pressure.

Data on clearance limits, interference limits, and hole and shaft diameter tolerances for locational clearance fits, transition fits, and locational interference fits are given in ANSI B4.1-1967 (R87).

5. **Force or shrink fits.** These are characterized by approximately constant bore pressures throughout the range of sizes; interference varies almost directly as the diameter, and the differences between maximum and minimum values of interference are small. These are divided into the following classes:

**Class FN1: light-drive fits.** For applications requiring light assembly pressures (thin sections, long fits, cast-iron external members).

**Class FN2: medium-drive fits.** Suitable for ordinary steel parts or for shrink fits on light sections. These are about the tightest fits that can be used on high-grade cast-iron external members.

**Class FN3: heavy-drive fits.** For heavier steel parts or shrink fits in medium sections.

**Classes FN4 and FN5: force fits.** These are suitable for parts which can be highly stressed. Shrink fits are used instead of press fits in cases where the heavy pressing forces required for mounting are impractical.

In Table 8.2.40 are listed the limits of interference (maximum and minimum values) for the above classes of force or shrink fits for various diameters, as given in ANSI B4.1-1967 (R87). Hole and shaft tolerances to give these interference limits are also listed in this reference.

**Metric System** ANSI B4.2-1978 (R94) and ANSI B4.3-1978 (R94) define limits and fits for cylindrical parts, and provide tables listing preferred values.

The standard ANSI B4.2-1978 (R94) is essentially in accord with ISO R286.

ANSI B4.2 provides 22 basic deviations, each for the shaft (a to z plus js), and the hole (A to Z plus Js). International has 18 tolerance grades: IT 01, IT 0, and IT 1 through 16.

IT grades are roughly applied as follows: measuring tools, 01 to 7; fits, 5 to 11; material, 8 to 14; and large manufacturing tolerances, 12 to 16. See Table 8.2.42 for metric preferred fits.

**Basic size**—The basic size is the same for both members of a fit, and is the size to which limits or deviations are assigned. It is designated by 40 in 40H7.

**Table 8.2.40 Limits of Interference for Force and Shrink Fits**  
(Limits are in thousandths of an inch on diameter)

Nominal size range, in	Class				
	FN 1	FN 2	FN 3	FN 4	FN 5
0.04–0.12	0.05	0.2		0.3	0.3
	0.5	0.85		0.95	1.3
0.12–0.24	0.1	0.2		0.95	1.3
	0.6	1.0		1.2	1.7
0.24–0.40	0.1	0.4		0.6	0.5
	0.75	1.4		1.6	2.0
0.40–0.56	0.1	0.5		0.7	0.6
	0.8	1.6		1.8	2.3
0.56–0.71	0.2	0.5		0.7	0.8
	0.9	1.6		1.8	2.5
0.71–0.95	0.2	0.6		0.8	1.0
	1.1	1.9		2.1	3.0
0.95–1.19	0.3	0.6	0.8	1.0	1.3
	1.2	1.9	2.1	2.3	3.3
1.19–1.58	0.3	0.8	1.0	1.5	1.4
	1.3	2.4	2.6	3.1	4.0
1.58–1.97	0.4	0.8	1.2	1.8	2.4
	1.4	2.4	2.8	3.4	5.0
1.97–2.56	0.6	0.8	1.3	2.3	3.2
	1.8	2.7	3.2	4.2	6.2
2.56–3.15	0.7	1.0	1.8	2.8	4.2
	1.9	2.9	3.7	4.7	7.2
3.15–3.94	0.9	1.4	2.1	3.6	4.8
	2.4	3.7	4.4	5.9	8.4
3.94–4.73	1.1	1.6	2.6	4.6	5.8
	2.6	3.9	4.9	6.9	9.4
4.73–5.52	1.2	1.9	3.4	5.4	7.5
	2.9	4.5	6.0	8.0	11.6
5.52–6.30	1.5	2.4	3.4	5.4	9.5
	3.2	5.0	6.0	8.0	13.6
6.30–7.09	1.8	2.9	4.4	6.4	9.5
	3.5	5.5	7.0	9.0	13.6

**Deviation**—The algebraic difference between a size and the corresponding basic size.

**Upper deviation**—The algebraic difference between the maximum limit of size and the corresponding basic size.

**Lower deviation**—The algebraic difference between the minimum limit of size and the corresponding basic size.

**Fundamental deviation**—That one of the two deviations closest to the basic size. It is designated by the letter H in 40H7.

**Tolerance**—The difference between the maximum and minimum size limits on a part.

**International tolerance grade (IT)**—A group of tolerances which vary depending on the basic size, but which provide the same relative level of accuracy within a grade. It is designated by 7 in 40H7 (IT 7).

**Tolerance zone**—A zone representing the tolerance and its position in relation to the basic size. The symbol consists of the fundamental deviation letter and the tolerance grade number (i.e., H7).

**Hole basis**—The system of fits where the minimum hole size is basic. The fundamental deviation for a hole basis system is H.

**Shaft basis**—Maximum shaft size is basic in this system. Fundamental deviation is h. NOTE: Capital letters refer to the hole and lowercase letters to the shaft.

**Clearance fit**—A fit in which there is clearance in the assembly for all tolerance conditions.

**Interference fit**—A fit in which there is interference for all tolerance conditions. Table 8.2.41 lists preferred metric sizes. Table 8.2.42 lists preferred tolerance zone combinations for clearance, transition and interference fits. Table 8.2.43 lists dimensions for the grades corresponding to preferred fits. Table 8.2.44a and b lists limits (numerical) of preferred hole-basis clearance, transitions, and interference fits.

#### Stresses Produced by Shrink or Press Fit

**STEEL HUB ON STEEL SHAFT.** The maximum equivalent stress, pounds per square inch, set up by a given press-fit allowance (in inches per inch of shaft diameter) is equal to  $3x \times 10^7$ , where  $x$  is the allowance per inch of shaft diameter (Baugher, *Trans. ASME*, 1931, p. 85). The press-fit pressures set up between a steel hub and shaft, for various ratios  $d/D$  between shaft and hub outside diameters, are given in Fig. 8.2.83. These

Table 8.2.41 Preferred Sizes (Metric)

Basic size, mm		Basic size, mm		Basic size, mm	
First choice	Second choice	First choice	Second choice	First choice	Second choice
1		10		100	
	1.1		11		110
1.2		12		120	
	1.4		14		140
1.6		16		160	
	1.8		18		180
2		20		200	
	2.2		22		220
2.5		25		250	
	2.8		28		280
3		30		300	
	3.5		35		350
4		40		400	
	4.5		45		450
5		50		500	
	5.5		55		550
6		60		600	
	7		70		700
8		80		800	
	9		90		900
				1000	

SOURCE: ANSI B 4.2-1978 (R94), reproduced by permission.

curves are accurate to 5 percent even if the shaft is hollow, provided the inside shaft diameter is not over 25 percent of the outside. The equivalent stress given above is based on the maximum shear theory and is numerically equal to the radial-fit pressure added to the tangential tension in the hub. Where the shaft is hollow, with an inside diameter equal to more than about 25 percent of the outside diameter, the allowance in inches per inch to obtain an equivalent hub stress of 30,000 lb/in<sup>2</sup> may be determined by using Lamé's thick-cylinder formulas (*Jour. Appl.*

Table 8.2.42 Description of Preferred Fits (Metric)

ISO symbol		Description	
Hole basis	Shaft basis		
Clearance fits			
H11/c11	C11/h11	<i>Loose-running</i> fit for wide commercial tolerances or allowances on external members.	↑ More clearance
H9/d9	D9/h9	<i>Free-running</i> fit not for use where accuracy is essential, but good for large temperature variations, high running speeds, or heavy journal pressures.	
H8/f7	F8/h7	<i>Close-running</i> fit for running on accurate machines and for accurate location at moderate speeds and journal pressures.	
H7/g6	G7/h6	<i>Sliding fit</i> not intended to run freely, but to move and turn freely and locate accurately.	
H7/h6	H7/h6	<i>Locational clearance</i> fit provides snug fit for locating stationary parts; but can be freely assembled and disassembled.	
Transition fits			
H7/k6	K7/h6	<i>Locational transition</i> fit for accurate location, a compromise between clearance and interference.	↓ More interference
H7/n6	N7/h6	<i>Locational transition</i> fit for more accurate location where greater interference is permissible.	
Interference fits			
H7/p6*	P7/h6	<i>Locational interference</i> fit for parts requiring rigidity and alignment with prime accuracy of location but without special bore pressure requirements.	
H7/s6	S7/h6	<i>Medium-drive</i> fit for ordinary steel parts or shrink fits on light sections, the tightest fit usable with cast iron.	
H7/u6	U7/h6	<i>Force fit</i> suitable for parts which can be highly stressed or for shrink fits where the heavy pressing forces required are impractical.	

\* Transition fit for basic sizes in range from 0 through 3 mm.

SOURCE: ANSI B4.2-1978 (R94), reproduced by permission.

Table 8.2.43 International Tolerance Grades

Basic sizes		Tolerance grades, mm					
Over	Up to and including	IT6	IT7	IT8	IT9	IT10	IT11
0	3	0.006	0.010	0.014	0.025	0.040	0.060
3	6	0.008	0.012	0.018	0.030	0.048	0.075
6	10	0.009	0.015	0.022	0.036	0.058	0.090
10	18	0.011	0.018	0.027	0.043	0.070	0.110
18	30	0.013	0.021	0.033	0.052	0.084	0.130
30	50	0.016	0.025	0.039	0.062	0.100	0.160
50	80	0.019	0.030	0.046	0.074	0.120	0.190
80	120	0.022	0.035	0.054	0.087	0.140	0.220
120	180	0.025	0.040	0.063	0.100	0.160	0.250
180	250	0.029	0.046	0.072	0.115	0.185	0.290
250	315	0.032	0.052	0.081	0.130	0.210	0.320
315	400	0.036	0.057	0.089	0.140	0.230	0.360
400	500	0.040	0.063	0.097	0.155	0.250	0.400
500	630	0.044	0.070	0.110	0.175	0.280	0.440
630	800	0.050	0.080	0.125	0.200	0.320	0.500
800	1,000	0.056	0.090	0.140	0.230	0.360	0.560
1,000	1,250	0.066	0.105	0.165	0.260	0.420	0.660
1,250	1,600	0.078	0.125	0.195	0.310	0.500	0.780
1,600	2,000	0.092	0.150	0.230	0.370	0.600	0.920
2,000	2,500	0.110	0.175	0.280	0.440	0.700	1.100
2,500	3,150	0.135	0.210	0.330	0.540	0.860	1.350

SOURCE: ANSI B4.2-1978 (R94), reproduced by permission.

*Mech.*, 1937, p. A-185). It should be noted that these curves hold only when the maximum equivalent stress is below the yield point; above the yield point, plastic flow occurs and the stresses are less than calculated.

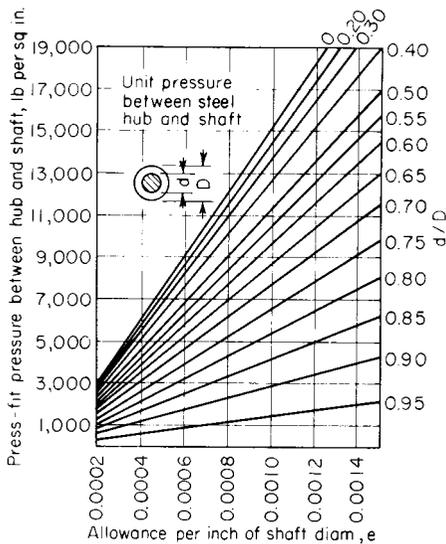


Fig. 8.2.83 Press-fit pressures between steel hub and shaft.

**Cast-Iron Hub on Steel Shaft** Where the shaft is solid, or hollow with an inside diameter not over 25 percent of the outside diameter, Fig. 8.2.84 may be used to determine maximum tensile stresses in the cast-iron hub, resulting from the press-fit allowance; for various ratios  $d/D$ , Fig. 8.2.85 gives the press-fit pressures. These curves are based on a modulus of elasticity of  $30 \times 10^6$  lb/in<sup>2</sup> for steel and  $15 \times 10^6$  for cast iron. For a hollow shaft with an inside diameter more than about  $1/4$  the outside, the Lamé formulas may be used.

**Pressure Required in Making Press Fits** The force required to press a hub on the shaft is given by  $\pi f p d l$ , where  $l$  is length of fit,  $p$  the unit press-fit pressure between shaft and hub,  $f$  the coefficient of fric-

tion, and  $d$  the shaft diameter. Values of  $f$  varying from 0.03 to 0.33 have been reported, the lower values being due to yielding of the hub as a consequence of too high a fit allowance; the average is around 0.10 to 0.15. (For additional data see Horgor and Nelson, "Design Data and Methods," ASME, 1953, pp. 87-91.)

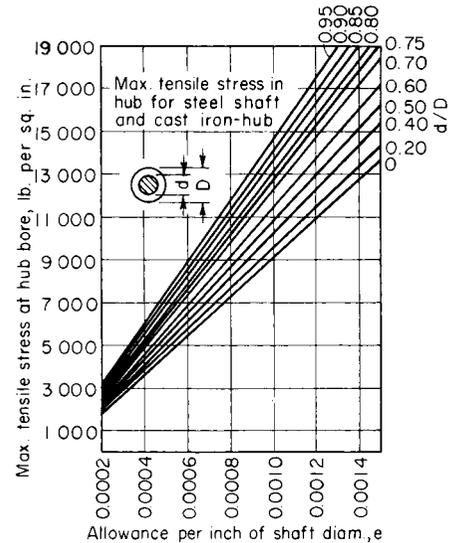


Fig. 8.2.84 Variation of tensile stress in cast-iron hub in press-fit allowance.

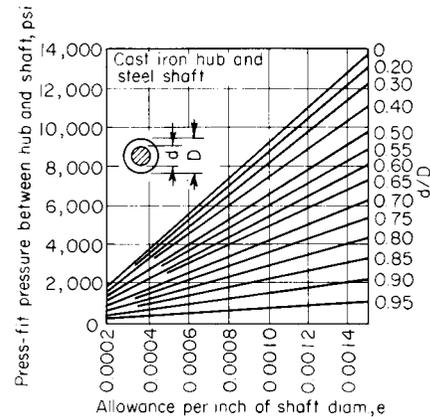


Fig. 8.2.85 Press-fit pressures between cast-iron hub and shaft.

**Torsional Holding Ability** The torque required to cause complete slippage of a press fit is given by  $T = \frac{1}{2} \pi f p l d^2$ . Local slippage will usually occur near the end of the fit at much lower torques. If the torque is alternating, stress concentration and rubbing corrosion will occur at the hub face so that, eventually, fatigue failure may occur at considerably lower torques. Only in cases of static torque application is it justifiable to use ultimate torque as a basis for design.

A designer can often improve shrink-, press-, and slip-fit cylindrical assemblies with adhesives. When applied, adhesives can achieve high frictional force with attendant greater torque transmission without extra bulk, and thus augment or even replace press fits, compensate for differential thermal expansion, make fits with leakproof seals, eliminate backlash and clearance, etc.

The adhesives used are the anaerobic (see Sec. 6.8, "Adhesives") variety, such as Loctite products. Such adhesives destabilize and tend to harden when deprived of oxygen. Design suggestions on the use of such adhesives appear in industrial catalogs.

## SHAFTS, AXLES, AND CRANKS

Most shafts are subject to combined bending and torsion, either of which may be steady or variable. Impact conditions, such as sudden starting and stopping, will cause momentary peak stresses greater than those related to the steady or variable portions of operation.

Design of shafts requires a theory of failure to express a stress in terms of loads and shaft dimensions, and an allowable stress as fixed by material strength and safety factor. Maximum shear theory of failure and distortion energy theory of failure are the two most commonly used in shaft design. Material strengths can be estimated from any one of several analytic representations of combined-load fatigue test data, starting from the linear (Soderberg, modified Goodman) which tend to give conservative designs to the nonlinear (Gerber parabolic, quadratic, Kececioglu, Bagci) which tend to give less conservative designs.

When linear representations of material strengths are used, and where both bending and torsion stresses have steady and variable components, the maximum shear theory and the distortion energy theory lead to somewhat similar formulations:

$$d = \left\{ \varepsilon \frac{n}{\pi} \left[ \left( \frac{T_a}{S_{se}} + \frac{T_m}{S_{sy}} \right)^2 + \left( \frac{M_a}{S_{se}} + \frac{M_m}{S_{sy}} \right)^2 \right]^{1/2} \right\}^{1/3}$$

where  $\varepsilon = 32$  (maximum shear theory) or 48 (distortion energy theory);  $d$  = shaft diameter;  $n$  = safety factor;  $T_a$  = amplitude torque =  $(T_{\max} - T_{\min})/2$ ;  $T_m$  = mean torque =  $(T_{\max} + T_{\min})/2$ ;  $M_a$  = amplitude bending moment =  $(M_{\max} - M_{\min})/2$ ;  $M_m$  = mean bending moment =  $(M_{\max} + M_{\min})/2$ ;  $S_{sy}$  = yield point in shear;  $S_{se}$  = completely corrected shear endurance limit =  $S'_e k_a k_b k_c k_d / K_f$ ;  $S'_e$  = statistical average endurance limit of mirror finish, standard size, laboratory test specimen at standard room temperature;  $k_a$  = surface factor, a decimal to adjust  $S'_e$  for other than mirror finish;  $k_b$  = size factor, a decimal to adjust  $S'_e$  for other than standard test size;  $k_c$  = reliability factor, a decimal to adjust  $S'_e$  to other than its implied statistical average of 50 percent safe, 50 percent fail rate (50 percent reliability);  $k_d$  = temperature factor, decimal, to adjust  $S'_e$  to other than room temperature;  $K_f = 1 + q(K_t - 1)$  = actual or fatigue stress concentration factor, a number greater than unity to adjust the nominal stress implied by  $T_a$  and  $M_a$  to a peak stress as induced by stress-raising conditions such as holes, fillets, keyways, pins, etc. ( $K_f$  for  $T_a$  need not necessarily be the same as  $K_f$  for  $M_a$ );  $q$  = notch sensitivity;  $K_t$  = theoretical or geometric stress concentration factor.

For specific values of endurance limits and various factors the reader is referred to the technical literature (e.g., ASTM, NASA technical reports, ASME technical papers) or various books [e.g., "Machinery's Handbook" (Industrial Press, New York) and machine design textbooks].

If one allows for a variation of at least 15 percent, the following approximation is useful for the endurance limit in bending:  $S'_e = 0.5S_{ut}$ . This becomes for maximum shear theory  $S'_{se} = 0.5(0.5S_{ut})$  and for distortion energy theory  $S'_{se} = 0.577(0.5S_{ut})$ .

Representative or approximate values for the various factors mentioned above were abstracted from Shigley, "Mechanical Engineering Design," McGraw-Hill, and appear below with permission.

Surface Factor  $k_a$ 

Surface condition	$S_{ut}$ , kips			
	60	120	180	240
Polished	1.00	1.00	1.00	1.00
Ground	0.89	0.89	0.89	0.89
Machined or cold-drawn	0.84	0.71	0.66	0.63
Hot-rolled	0.70	0.50	0.39	0.31
As forged	0.54	0.36	0.27	0.20

Size Factor  $k_b$ 

$$k_b = \begin{cases} 0.869d^{-0.097} & 0.3 \text{ in} < d \leq 10 \text{ in} \\ 1 & d \leq 0.3 \text{ in or } d \leq 8 \text{ mm} \\ 1.189d^{-0.097} & 8 \text{ mm} < d < 250 \text{ mm} \end{cases}$$

Reliability Factor  $k_c$ 

Reliability, %	$k_c$
50	1.00
90	0.89
95	0.87
99	0.81

Temperature Factor  $k_d$ 

Temperature		$k_d$
°F	°C	
840	450	1.00
940	482	0.71
1,020	550	0.42

Notch Sensitivity  $q$ 

$S_{ut}$ , kips	Notch radius $r$ , in			
	0.02	0.06	0.10	0.14
60	0.56	0.70	0.74	0.78
100	0.68	0.79	0.83	0.85
150	0.80	0.90	0.91	0.92
200	0.90	0.95	0.96	0.96

See Table 8.2.45 for fatigue stress concentration factors for plain press fits.

A general representation of material strengths (Marin, Design for Fatigue Loading, *Mach. Des.* 29, no. 4, Feb. 21, 1957, pp. 128–131, and series of the same title) is given as

$$\left( \frac{S_a}{S_e} \right)^m + \left( \frac{K S_m}{S_{ut}} \right)^p = 1$$

where  $S_a$  = variable portion of material strength;  $S_m$  = mean portion of material strength;  $S_e$  = adjusted endurance limit;  $S_{ut}$  = ultimate strength. Table 8.2.46 lists the constants  $m$ ,  $K$ , and  $p$  for various failure criteria.

For purposes of design, safety factors are introduced into the equation resulting in:

$$\left( \frac{\sigma'_{a,p}}{S_e/n_{se}} \right)^m + \left( \frac{K \sigma'_{m,p}}{S_{ut}/n_{ut}} \right)^p = 1$$

where

$$\sigma'_{a,p} = \frac{1}{\pi d^3} \sqrt{(32n_{Ma}M_a)^2 + 3(16n_{\tau a}T_a)^2}$$

$$\sigma'_{m,p} = \frac{1}{\pi d^3} \sqrt{(32n_{Mm}M_m)^2 + 3(16n_{\tau m}T_m)^2}$$

and  $n_{ij}$  = safety factor pertaining to a particular stress (that is,  $n_a$  = safety factor for amplitude shear stress).

**Stiffness of shafting** may become important where critical speeds, vibration, etc., may occur. Also, the lack of sufficient stiffness in shafts may give rise to bearing troubles. Critical speeds of shafts in torsion or bending and shaft deflections may be calculated using the methods of Sec. 5. For shafts of variable diameter see Spotts, "Design of Machine Elements," Prentice-Hall. In order to avoid trouble where sleeve bearings are used, the angular deflections at the bearings in general must be kept within certain limits. One rule is to make the shaft deflection over the bearing width equal to a small fraction of the oil-film thickness. Note that since stiffness is proportional to the modulus of elasticity, alloy-steel shafts are no stiffer than carbon-steel shafts of the same diameter.

**Crankshafts** For calculating the torsional stiffness of crankshafts, the formulas given in Sec. 5 may be used.

**Table 8.2.44a Limits of Fits**  
(Dimensions in mm)

Basic size		Preferred hole basis clearance fits														
		Loose-running			Free-running			Close-running			Sliding			Locational clearance		
		Hole H11	Shaft c11	Fit	Hole H9	Shaft d9	Fit	Hole H8	Shaft f7	Fit	Hole H7	g6	Fit	Hole H7	Shaft h6	Fit
1	max	1.060	0.940	0.180	1.025	0.980	0.070	1.014	0.994	0.030	1.010	0.998	0.018	1.010	1.000	0.016
	min	1.000	0.880	0.060	1.000	0.955	0.020	1.000	0.984	0.006	1.000	0.992	0.002	1.000	0.994	0.000
1.2	max	1.260	1.140	0.180	1.225	1.180	0.070	1.214	1.194	0.030	1.210	1.198	0.018	1.210	1.200	0.016
	min	1.200	1.080	0.060	1.200	1.155	0.020	1.200	1.184	0.006	1.200	1.192	0.002	1.200	1.194	0.000
1.6	max	1.660	1.540	0.180	1.625	1.580	0.070	1.614	1.594	0.030	1.610	1.598	0.018	1.610	1.600	0.016
	min	1.600	1.480	0.060	1.600	1.555	0.020	1.600	1.584	0.006	1.600	1.592	0.002	1.600	1.594	0.000
2	max	2.060	1.940	0.180	2.025	1.980	0.070	2.014	1.994	0.030	2.010	1.998	0.018	2.010	2.000	0.016
	min	2.000	1.880	0.060	2.000	1.955	0.020	2.000	1.984	0.006	2.000	1.992	0.002	2.000	1.994	0.000
2.5	max	2.560	2.440	0.180	2.525	2.480	0.070	2.514	2.494	0.030	2.510	2.498	0.018	2.510	2.500	0.016
	min	2.500	2.380	0.060	2.500	2.455	0.020	2.500	2.484	0.006	2.500	2.492	0.002	2.500	2.494	0.000
3	max	3.060	2.940	0.180	3.025	2.980	0.070	3.014	2.994	0.030	3.010	2.998	0.018	3.010	3.000	0.016
	min	3.000	2.880	0.060	3.000	2.955	0.020	3.000	2.984	0.006	3.000	2.992	0.002	3.000	2.994	0.000
4	max	4.075	3.930	0.220	4.030	3.970	0.090	4.018	3.990	0.040	4.012	3.996	0.024	4.012	3.000	0.020
	min	4.000	3.855	0.070	4.000	3.940	0.030	4.000	3.978	0.010	4.000	3.988	0.004	4.000	3.992	0.000
5	max	5.075	4.930	0.220	5.030	4.970	0.090	5.018	4.990	0.040	5.012	4.996	0.024	5.012	5.000	0.020
	min	5.000	4.855	0.070	5.000	4.940	0.030	5.000	4.978	0.010	5.000	4.988	0.004	5.000	4.992	0.000
6	max	6.075	5.930	0.220	6.030	5.970	0.090	6.018	5.990	0.040	6.012	5.996	0.024	6.012	6.000	0.020
	min	6.000	5.855	0.070	6.000	5.940	0.030	6.000	5.978	0.010	6.000	5.988	0.004	6.000	5.992	0.000
8	max	8.090	7.920	0.260	8.036	7.960	0.112	8.022	7.987	0.050	8.015	7.995	0.029	8.015	8.000	0.024
	min	8.000	7.830	0.080	8.000	7.924	0.040	8.000	7.972	0.013	8.000	7.986	0.005	8.000	7.991	0.000
10	max	10.090	9.920	0.260	10.036	9.960	0.112	10.022	9.987	0.050	10.015	9.995	0.029	10.015	10.000	0.024
	min	10.000	9.830	0.080	10.000	9.924	0.040	10.000	9.972	0.013	10.000	9.986	0.005	10.000	9.991	0.000
12	max	12.110	11.905	0.315	12.043	11.950	0.136	12.027	11.984	0.061	12.018	11.994	0.035	12.018	12.000	0.029
	min	12.000	11.795	0.095	12.000	11.907	0.050	12.000	11.966	0.016	12.000	11.983	0.006	12.000	11.989	0.000
16	max	16.110	15.905	0.315	16.043	15.950	0.136	16.027	15.984	0.061	16.018	15.994	0.035	16.018	16.000	0.029
	min	16.000	15.795	0.095	16.000	15.907	0.050	16.000	15.966	0.016	16.000	15.983	0.006	16.000	15.989	0.000
20	max	20.130	19.890	0.370	20.052	19.935	0.169	20.033	19.980	0.074	20.021	19.993	0.041	20.021	20.000	0.034
	min	20.000	19.760	0.110	20.000	19.883	0.065	20.000	19.959	0.020	20.000	19.980	0.007	20.000	19.987	0.000
25	max	25.130	24.890	0.370	25.052	24.935	0.169	25.033	24.980	0.074	25.021	24.993	0.041	25.021	25.000	0.034
	min	25.000	24.760	0.110	25.000	24.883	0.065	25.000	24.959	0.020	25.000	24.980	0.007	25.000	24.987	0.000
30	max	30.130	29.890	0.370	30.052	29.935	0.169	30.033	29.980	0.074	30.021	29.993	0.041	30.021	30.000	0.034
	min	30.000	29.760	0.110	30.000	29.883	0.065	30.000	29.959	0.020	30.000	29.980	0.007	30.000	29.987	0.000
40	max	40.160	39.880	0.440	40.062	39.920	0.204	40.039	39.975	0.089	40.025	39.991	0.050	40.025	40.000	0.041
	min	40.000	39.720	0.120	40.000	39.858	0.080	40.000	39.950	0.025	40.000	39.975	0.009	40.000	39.984	0.000
50	max	50.160	49.870	0.450	50.062	49.920	0.204	50.039	49.975	0.089	50.025	49.991	0.050	50.025	50.000	0.041
	min	50.000	49.710	0.130	50.000	49.858	0.080	50.000	49.950	0.025	50.000	49.975	0.009	50.000	49.984	0.000
60	max	60.190	59.860	0.520	60.074	59.900	0.248	60.046	59.970	0.106	60.030	59.990	0.059	60.030	60.000	0.049
	min	60.000	59.670	0.140	60.000	59.826	0.100	60.000	59.940	0.030	60.000	59.971	0.010	60.000	59.981	0.000
80	max	80.190	79.850	0.530	80.074	79.900	0.248	80.046	79.970	0.106	80.030	79.990	0.059	80.030	80.000	0.049
	min	80.000	79.660	0.150	80.000	79.826	0.100	80.000	79.940	0.030	80.000	79.971	0.010	80.000	79.981	0.000
100	max	100.220	99.830	0.610	100.087	99.880	0.294	100.054	99.964	0.125	100.035	99.988	0.069	100.035	100.000	0.057
	min	100.000	99.610	0.170	100.000	99.793	0.120	100.000	99.929	0.036	100.000	99.966	0.012	100.000	99.978	0.000
120	max	120.220	119.820	0.620	120.087	119.880	0.294	120.054	119.964	0.125	120.035	119.988	0.069	120.035	120.000	0.057
	min	120.000	119.600	0.180	120.000	119.793	0.120	120.000	119.929	0.036	120.000	119.966	0.012	120.000	119.978	0.000
160	max	160.250	159.790	0.710	160.100	159.855	0.345	160.063	159.957	0.146	160.040	159.986	0.079	160.040	160.000	0.065
	min	160.000	159.540	0.210	160.000	159.755	0.145	160.000	159.917	0.043	160.000	159.961	0.014	160.000	159.975	0.000

**Table 8.2.44b Limits of Fits**  
(Dimensions in mm)

Basic size		Preferred hole basis transition and interference fits														
		Locational transition			Locational transition			Locational interference			Medium drive			Force		
		Hole H7	Shaft k6	Fit	Hole H7	Shaft n6	Fit	Hole H7	Shaft p6	Fit	Hole H7	Shaft s6	Fit	Hole H7	Shaft u6	Fit
1	max	1.010	1.006	0.010	1.010	1.010	0.006	1.010	1.012	0.004	1.010	1.020	-0.004	1.010	1.024	-0.008
	min	1.000	1.000	-0.006	1.000	1.004	-0.010	1.000	1.006	-0.012	1.000	1.014	-0.020	1.000	1.018	-0.024
1.2	max	1.210	1.206	0.010	1.210	1.210	0.006	1.210	1.212	0.004	1.210	1.220	-0.004	1.210	1.224	-0.008
	min	1.200	1.200	-0.006	1.200	1.204	-0.010	1.200	1.206	-0.012	1.200	1.214	-0.020	1.200	1.218	-0.024
1.6	max	1.610	1.606	0.010	1.610	1.610	0.006	1.610	1.612	0.004	1.610	1.620	-0.004	1.610	1.624	-0.008
	min	1.600	1.600	-0.006	1.600	1.604	-0.010	1.600	1.606	-0.012	1.600	1.614	-0.020	1.600	1.618	-0.024
2	max	2.010	2.006	0.010	2.010	2.010	0.006	2.010	2.012	0.004	2.010	2.020	-0.004	2.010	2.024	-0.008
	min	2.000	2.000	-0.006	2.000	2.004	-0.010	2.000	2.006	-0.012	2.000	2.014	-0.020	2.000	2.018	-0.024
2.5	max	2.510	2.506	0.010	2.510	2.510	0.006	2.510	2.512	0.004	2.510	2.520	-0.004	2.510	2.524	-0.008
	min	2.500	2.500	-0.006	2.500	2.504	-0.010	2.500	2.506	-0.012	2.500	2.514	-0.020	2.500	2.518	-0.024
3	max	3.010	3.006	0.010	3.010	3.010	0.006	3.010	3.012	0.004	3.010	3.020	-0.004	3.010	3.024	-0.008
	min	3.000	3.000	-0.006	3.000	3.004	-0.010	3.000	3.006	-0.012	3.000	3.014	-0.020	3.000	3.018	-0.024
4	max	4.012	4.009	0.011	4.012	4.016	0.004	4.012	4.020	0.000	4.012	4.027	-0.007	4.012	4.031	-0.011
	min	4.000	4.001	-0.009	4.000	4.008	-0.016	4.000	4.012	-0.020	4.000	4.019	-0.027	4.000	4.023	-0.031
5	max	5.012	5.009	0.011	5.012	5.016	0.004	5.012	5.020	0.000	5.012	5.027	-0.007	5.012	5.031	-0.011
	min	5.000	5.001	-0.009	5.000	5.008	-0.016	5.000	5.012	-0.020	5.000	5.019	-0.027	5.000	5.023	-0.031
6	max	6.012	6.009	0.011	6.012	6.016	0.004	6.012	6.020	0.000	6.012	6.027	-0.007	6.012	6.031	-0.011
	min	6.000	6.001	-0.009	6.000	6.008	-0.016	6.000	6.012	-0.020	6.000	6.019	-0.027	6.000	6.023	-0.031
8	max	8.015	8.010	0.014	8.015	8.019	0.005	8.015	8.024	0.000	8.015	8.032	-0.008	8.015	8.037	-0.013
	min	8.000	8.001	-0.010	8.000	8.010	-0.019	8.000	8.015	-0.024	8.000	8.023	-0.032	8.000	8.028	-0.037
10	max	10.015	10.010	0.014	10.015	10.019	0.005	10.015	10.024	0.000	10.015	10.032	-0.008	10.015	10.037	-0.013
	min	10.000	10.001	-0.010	10.000	10.010	-0.019	10.000	10.015	-0.024	10.000	10.023	-0.032	10.000	10.028	-0.037
12	max	12.018	12.012	0.017	12.018	12.023	0.006	12.018	12.029	0.000	12.018	12.039	-0.010	12.018	12.044	-0.015
	min	12.000	12.001	-0.012	12.000	12.012	-0.023	12.000	12.018	-0.029	12.000	12.028	-0.039	12.000	12.033	-0.044
16	max	16.018	16.012	0.017	16.018	16.023	0.006	16.018	16.029	0.000	16.018	16.039	-0.010	16.018	16.044	-0.015
	min	16.000	16.001	-0.012	16.000	16.012	-0.023	16.000	16.018	-0.029	16.000	16.028	-0.039	16.000	16.033	-0.044
20	max	20.021	20.015	0.019	20.021	20.028	0.006	20.021	20.035	-0.001	20.021	20.048	-0.014	20.021	20.054	-0.020
	min	20.000	20.002	-0.015	20.000	20.015	-0.028	20.000	20.022	-0.035	20.000	20.035	-0.048	20.000	20.041	-0.054
25	max	25.021	25.015	0.019	25.021	25.028	0.006	25.021	25.035	-0.001	25.021	25.048	-0.014	25.021	25.061	-0.027
	min	25.000	25.002	-0.015	25.000	25.015	-0.028	25.000	25.022	-0.035	25.000	25.035	-0.048	25.000	25.048	-0.061
30	max	30.021	30.015	0.019	30.021	30.028	0.006	30.021	30.035	-0.001	30.021	30.048	-0.014	30.021	30.061	-0.027
	min	30.000	30.002	-0.015	30.000	30.015	-0.028	30.000	30.022	-0.035	30.000	30.035	-0.048	30.000	30.048	-0.061
40	max	40.025	40.018	0.023	40.025	40.033	0.008	40.025	40.042	-0.001	40.025	40.059	-0.018	40.025	40.076	-0.035
	min	40.000	40.002	-0.018	40.000	40.017	-0.033	40.000	40.026	-0.042	40.000	40.043	-0.059	40.000	40.060	-0.076
50	max	50.025	50.018	0.023	50.025	50.033	0.008	50.025	50.042	-0.001	50.025	50.059	-0.018	50.025	50.086	-0.045
	min	50.000	50.002	-0.018	50.000	50.017	-0.033	50.000	50.026	-0.042	50.000	50.043	-0.059	50.000	50.070	-0.086
60	max	60.030	60.021	0.028	60.030	60.039	0.010	60.030	60.051	-0.002	60.030	60.072	-0.023	60.030	60.106	-0.057
	min	60.000	60.002	-0.021	60.000	60.020	-0.039	60.000	60.032	-0.051	60.000	60.053	-0.072	60.000	60.087	-0.106
80	max	80.030	80.021	0.028	80.030	80.039	0.010	80.030	80.051	-0.002	80.030	80.078	-0.029	80.030	80.121	-0.072
	min	80.000	80.002	-0.021	80.000	80.020	-0.039	80.000	80.032	-0.051	80.000	80.059	-0.078	80.000	80.102	-0.121
100	max	100.035	100.025	0.032	100.035	100.045	0.012	100.035	100.059	-0.002	100.035	100.093	-0.036	100.035	100.146	-0.089
	min	100.000	100.003	-0.025	100.000	100.023	-0.045	100.000	100.037	-0.059	100.000	100.071	-0.093	100.000	100.124	-0.146
120	max	120.035	120.025	0.032	120.035	120.045	0.012	120.035	120.059	-0.002	120.035	120.101	-0.044	120.035	120.166	-0.109
	min	120.000	120.003	-0.025	120.000	120.023	-0.045	120.000	120.037	-0.059	120.000	120.079	-0.101	120.000	120.144	-0.166
160	max	160.040	160.028	0.037	160.040	160.052	0.013	160.040	160.068	-0.003	160.040	160.125	-0.060	160.040	160.215	-0.150
	min	160.000	160.003	-0.028	160.000	160.027	-0.052	160.000	160.043	-0.068	160.000	160.100	-0.125	160.000	160.190	-0.215

SOURCE: ANSI B4.2-1978 (R94), reprinted by permission.

**Table 8.2.45  $K_f$  Values for Plain Press Fits**  
Obtained from fatigue tests in bending)

Shaft material	Shaft diam, in	Collar or hub material	$K_f$	Remarks
0.42% carbon steel	1½	0.42% carbon steel	2.0	No external reaction through collar
0.45% carbon axle steel	2	Ni-Cr-Mo steel (case-hardened)	2.3	No external reaction through collar
0.45% carbon axle steel	2	Ni-Cr-Mo steel (case-hardened)	2.9	External reaction taken through collar
Cr-Ni-Mo steel (310 Brinell)	2	Ni-Cr-Mo steel (case-hardened)	3.9	External reaction taken through collar
2.6% Ni steel (57,000 lb/in <sup>2</sup> fatigue limit)	2	Ni-Cr-Mo steel (case-hardened)	3.3–3.8	External reaction taken through collar
Same, heat-treated to 253 Brinell	2	Ni-Cr-Mo steel (case-hardened)	3.0	External reaction taken through collar

**Marine-engine shafts and diesel-engine crankshafts** should be designed not only for strength but for avoidance of critical speed. (See Applied Mechanics, *Trans. ASME*, **50**, no. 8, for methods of calculating critical speeds of diesel engines.)

**Table 8.2.46 Constants for Use in  $(S_d/S_u)^m + (K S_m/S_u)^p = 1$**

Failure theory	$K$	$P$	$m$
Soderberg	$S_u/S_y$	1	1
Bagci	$S_u/S_y$	4	1
Modified Goodman	1	1	1
Gerber parabolic	1	2	1
Kececioglu	1	2	$m^\dagger$
Quadratic (elliptic)	1	2	2

$$\dagger m = \begin{cases} 0.8914 \text{ UNS G 10180 } H_B = 130 \\ 0.9266 \text{ UNS G 10380 } H_B = 164 \\ 1.0176 \text{ UNS G 41300 } H_B = 207 \\ 0.9685 \text{ UNS G 43400 } H_B = 233 \end{cases}$$

**PULLEYS, SHEAVES, AND FLYWHEELS**

**Arms** of pulleys, sheaves, and flywheels are subjected to stresses due to condition of founding, to details of construction (such as split or solid), and to conditions of service, which are difficult to analyze. For these reasons, no accurate stress relations can be established, and the following formulas must be understood to be only approximately correct. It has been established experimentally by Benjamin (*Am. Mach.*, Sept. 22, 1898) that thin-rim pulleys do not distribute equal loads to the several pulley arms. For these, it will be safe to assume the tangential force on the pulley rim as acting on half of the number of arms. Pulleys with comparatively thick rims, such as engine band wheels, have all the arms taking the load. Furthermore, while the stress action in the arms is similar to that in a beam fixed at both ends, the amount of restraint at the rim depending on the rim's elasticity, it may nevertheless be assumed for purposes of design that cantilever action is predominant. The bending moment at the hub in arms of thin-rim pulleys will be  $M = PL/(½N)$ , where  $M$  = bending moment, in·lb;  $P$  = tangential load on the rim, lb;  $L$  = length of the arm, in; and  $N$  = number of arms. For thick-rim pulleys and flywheels,  $M = PL/N$ .

For arms of elliptical section having a width of two times the thickness, where  $E$  = **width of arm section** at the rim, in, and  $s_t$  = intensity of tensile stress, lb/in<sup>2</sup>

$$E = \sqrt[3]{40PL/(s_t N)} \text{ (thin rim)} = \sqrt[3]{20PL/(s_t N)} \text{ (thick rim)}$$

For single-thickness belts,  $P$  may be taken as  $50B$  lb and for double-thickness belts  $P = 75B$  lb, where  $B$  is the width of pulley face, in. Then  $E = k \times \sqrt[3]{BL/(s_t N)}$ , where  $k$  has the following values: for thin rim, single belt, 13; thin rim, double belt, 15; thick rim, single belt, 10; thick rim, double belt, 12. For cast iron of good quality,  $s_t$  due to bending may be taken at 1,500 to 2,000. The arm section at the rim may be made from ⅓ to ¾ the dimensions at the hub.

For high-speed pulleys and flywheels, it becomes necessary to check the arm for tension due to rim expansion. It will be safe to assume that each arm is in tension due to one-half the centrifugal force of that portion of the rim which it supports. That is,  $T = As_t = Wv^2/(2NgR)$ , lb, where  $T$  = tension in arm, lb;  $N$  = number of arms;  $v$  = speed of rim, ft/s;  $R$  = radius of pulley, ft;  $A$  = area of arm section, in<sup>2</sup>;  $W$  = weight of pulley rim, lb; and  $s_t$  = intensity of tensile stress in arm section, lb/in<sup>2</sup>, whence  $s_t = WRn^2/(6,800NA)$ , where  $n = r/\text{min}$  of pulley.

**Arms of flywheels** having heavy rims may be subjected to severe stress action due to the inertia of the rim at sudden load changes. There being no means of predicting the probable maximum to which the inertia may rise, it will be safe to make the arms equal in strength to ¾ of the shaft strength in torsion. Accordingly, for elliptical arm sections,

$$N \times 0.5E^3 s_t = \frac{3}{4} \times 2s_s d^3 \quad \text{or} \quad E = 1.4d \sqrt[3]{s_s/(s_t N)}$$

For steel shafts with  $s_s = 8,000$  and cast-iron arms with  $s = 1,500$ ,

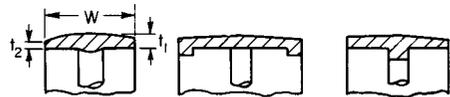
$$E = 2.4d \sqrt[3]{N} = 1.3d \text{ (for 6 arms)} = 1.2d \text{ (for 8 arms)}$$

where  $2E$  = width of elliptical arm section at hub, in (thickness =  $E$ ), and  $d$  = shaft diameter, in.

**Rims of belted pulleys** cast whole may have the following proportions (see Fig. 8.2.86):

$$t_2 = \frac{3}{4}h + 0.005D \quad t_1 = 2t_2 + C \quad W = \frac{1}{8}B \text{ to } \frac{1}{4}B$$

where  $h$  = belt thickness,  $C = \frac{1}{23}W$ , and  $B$  = belt width, all in inches.



**Fig. 8.2.86 Rims for belted pulleys.**

Engine band wheels, flywheels, and pulleys run at **high speeds** are subjected to the following stress actions in the rim:

Considering the rim as a free ring, i.e., without arm restraint, and made of cast iron or steel,  $s_t = v^2/10$  (approx), where  $s_t$  = intensity of tensile stress, lb/in<sup>2</sup>, and  $v$  = rim speed, ft/s. For beam action between the arms of a solid rim,  $M = Pl/12$  (approx), where  $M$  = bending moment in rim, in·lb;  $P$  = centrifugal force of that portion of rim between arms, lb, and  $l$  = length of rim between arms, in; from which  $s_t = WR^2n^2/(450N^2Z)$ , where  $W$  = weight of entire rim, lb;  $R$  = radius of wheel, ft;  $n = r/\text{min}$  of wheel; and  $Z$  = section modulus of rim section, in<sup>3</sup>. In case the rim section is of the forms shown in Fig. 8.2.86, care must be taken that the flanges do not reduce the section modulus from that of the rectangular section. For **split rims** fastened with bolts the stress analysis is as follows:

Let  $w$  = weight of rim portion, lb (with length  $L$ , in) lb;  $w_1$  = weight of lug, lb;  $L_1$  = lever arm of lug, in; and  $s_t$  = intensity of tensile stress lb/in<sup>2</sup> in rim section joining arm. Then  $s_t = 0.00034n^2R(w_1L_1 + wL/2)/Z$ , where  $n = r/\text{min}$  of wheel;  $R$  = wheel radius, ft; and  $Z$  = section modulus of

rim section, in<sup>3</sup>. The above equation gives the value of  $s_r$  for bending when the bolts are loose, which is the worst possible condition that may arise. On this basis of analysis,  $s_r$  should not be greater than 8,000 lb/in<sup>2</sup>. The stress due to bending in addition to the stress due to rim expansion as analyzed previously will be the probable maximum intensity of stress for which the rim should be checked for strength. The flange bolts, because of their position, do not materially relieve the bending action. In case a tie rod leads from the flange to the hub, it will be *safe* to consider it as an additional factor of safety. When the tie rod is kept tight, it very materially strengthens the rim.

A more accurate method for calculating maximum stresses due to centrifugal force in flywheels with arms cast integral with the rim is given by Timoshenko, "Strength of Materials," Pt. II, 1941, p. 98. More exact equations for calculating stresses in the arms of flywheels and pulleys due to a combination of belt pull, centrifugal force, and changes in velocity are given by Heusinger, *Forschung*, 1938, p. 197. In both treatments, shrinkage stresses in the arms due to casting are neglected.

**Large flywheels for high rim speeds** and severe working conditions (as for rolling-mill service) have been made from flat-rolled steel plates with holes bored for the shaft. A group of such plates may be welded together by circumferential welds to form a large flywheel. By this means, the welds do not carry direct centrifugal loads, but serve merely to hold the parts in position. Flywheels up to 15-ft diam for rolling-mill service have been constructed in this way.

**BELT DRIVES**

**Flat-Belt Drives**

The primary drawback of flat belts is their reliance on belt tension to produce frictional grip over the pulleys. Such high tension can shorten bearing life. Also, tracking may be a problem. However, flat belts, being thin, are not subject to centrifugal loads and so work well over small pulleys at high speeds in ranges exceeding 9,000 ft/min. In light service flat belts can make effective clutching drives. Flat-belt drives have efficiencies of about 90 percent, which compares favorably to geared drives. Flat belts are also quiet and can absorb torsional vibration readily.

**Leather belting** has an ultimate tensile strength ranging from 3,000 to 5,000 lb/in<sup>2</sup>. Average values of breaking strength of good oak-tanned belting (determined by Benjamin) are as follows: single (double) in solid leather 900 (1,400); at riveted joint 600 (1,200); at laced joint 350 lb/in of width. Well-made cemented joints have strengths equal to the belt, leather-laced and riveted joints about one-third to two-thirds as strong, and wire-laced joints about 85 to 95 percent as strong.

**Rubber belting** is made from fabric or cord impregnated and bound together by vulcanized rubber compounds. The fabric or cord may be of

cotton or rayon. Nylon cord and steel cord or cable are also available. Advantages are high tensile strength, strength to hold metal fasteners satisfactorily, and resistance to deterioration by moisture. The best rubber fabric construction for most types of service is made from hard or tight-woven fabric with a "skim coat" or thin layer of rubber between plies. The cord type of construction allows the use of smaller pulley diameters than the fabric type, and also develops less stretch in service. It must be used in the endless form, except in cases where the oil-field type of clamp may be used.

Initial tensions in rubber belts run from 15 to 25 lb/ply/in width. A common rule is to cut belts 1 percent less than the minimum tape-line measurement around the pulleys. For heavy loads, a 1½ percent allowance is usually required, although, because of shrinkage, less initial tension is required for wet or damp conditions. Initial tensions of 25 lb/ply/in may overload shafts or bearings. Maximum safe tight-side tensions for rubber belts are as follows:

Duck weight, oz	28	32	32.66	34.66	36
Tension, lb/ply/in width	25	28	30	32	35

Centrifugal forces at high speeds require higher tight-side tensions to carry rated horsepower.

Rubber belting may be gotten in endless form or made endless in the field by means of a vulcanized splice produced by a portable electric vulcanizer. For endless belts the drive should provide take-up of 2 to 4 percent to allow for length variation as received and for stretch in service. The amount of take-up will vary with the type of belt used. For certain drives, it is possible to use endless belts with no provision for take-up, but this involves a heavier belt and a higher initial unit tension than would be the case otherwise. Ultimate tensile strength of rubber belting varies from 280 to 600 lb or more per inch width per ply. The weight varies from 0.02 to 0.03 or more lb/in width/ply. Belts with steel reinforcement are considerably heavier. For horsepower ratings of rubber belts, see Table 8.2.47c.

**Arrangements for Belt Drives** In belt drives, the centerline of the belt advancing on the pulley should lie in a plane passing through the midsection of the pulley at right angles to the shaft. Shafts inclined to each other require connections as shown in Fig. 8.2.87a. In case guide pulleys are needed their positions can be determined as shown in Fig. 8.2.87a, b, and c. In Fig. 8.2.87d the center circles of the two pulleys to be connected are set in correct relative position in two planes,  $a$  being the angle between the planes (= supplement of angle between shafts). If any two points as  $E$  and  $F$  are assumed on the line of intersection  $MN$  of the planes, and tangents  $EG$ ,  $EH$ ,  $FJ$ , and  $FK$  are drawn from them to the circles, the center circles of the guide pulleys must be so arranged that these tangents are also tangents to them, as shown. In other words, the middle planes of the guide pulleys must lie in the planes  $GEH$  and  $JFK$ .

Table 8.2.47a Service Factors S

Application	Squirrel-cage ac motor		Wound rotor ac motor (slip ring)	Single-phase capacitor motor	DC shunt-wound motor	Diesel engine, 4 or more cyl, above 700 r/min
	Normal torque, line start	High torque				
Agitators	1.0-1.2	1.2-1.4	1.2			
Compressors	1.2-1.4		1.4	1.2	1.2	1.2
Belt conveyors (ore, coal, sand)	—	1.4	—	—	1.2	
Screw conveyors	—	1.8	—	—	1.6	
Crushing machinery	—	1.6	1.4	—	—	1.4-1.6
Fans, centrifugal	1.2	—	1.4	—	1.4	1.4
Fans, propeller	1.4	2.0	1.6	—	1.6	1.6
Generators and exciters	1.2	—	—	—	1.2	2.0
Line shafts	1.4	—	1.4	1.4	1.4	1.6
Machine tools	1.0-1.2	—	1.2-1.4	1.0	1.0-1.2	
Pumps, centrifugal	1.2	1.4	1.4	1.2	1.2	
Pumps, reciprocating	1.2-1.4	—	1.4-1.6	—	—	1.8-2.0

**Table 8.2.47b Arc of Contact Factor *K*—Rubber Belts**

Arc of contact, deg	140	160	180	200	220
Factor <i>K</i>	0.82	0.93	1.00	1.06	1.12

When these conditions are met, the belts will run in either direction on the pulleys.

To avoid the necessity of taking up the slack in belts which have become stretched and permanently lengthened, a **belt tightener** such as shown in Fig. 8.2.88 may be employed. It should be placed on the slack side of the belt and nearer the driving pulley than the driven pulley. Pivoted motor drives may also be used to maintain belt tightness with minimum initial tension.

**Length of Belt for a Given Drive** The length of an **open belt** for a given drive is equal to  $L = 2C + 1.57(D + d) + (D - d)^2/(4C)$ , where  $L$  = length of belt, in;  $D$  = diam of large pulley, in;  $d$  = diam of small pulley, in; and  $C$  = distance between pulley centers, in. Center

distance  $C$  is given by  $C = 0.25b + 0.25\sqrt{b^2 - 2(D - d)^2}$ , where  $b = L - 1.57(D + d)$ . When a **crossed belt** is used, the length in  $L = 2C + 1.57(D + d) + (D + d)^2/(4C)$ .

**Step or Cone Pulleys** For belts operating on step pulleys, the pulley diameters must be such that the belt will fit over any pair with equal tightness. With **crossed belts**, it will be apparent from the equation for length of belt that the sum of the pulley diameters need only be constant in order that the belt may fit with equal tightness on each pair of pulleys. With open belts, the length is a function of both the sum and the difference of the pulley diameters; hence no direct solution of the problem is possible, but a graphical approach can be of use.

**A graphical method** devised by Smith (*Trans. ASME*, 10) is shown in Fig. 8.2.89. Let  $A$  and  $B$  be the centers of any pair of pulleys in the set, the diameters of which are known or assumed. Bisect  $AB$  in  $C$ , and draw  $CD$  at right angles to  $AB$ . Take  $CD = 0.314$  times the center distance  $L$ , and draw a circle tangent to the belt line  $EF$ . The belt line of any other pair of pulleys in the set will then be tangent to this circle. If the angle

**Table 8.2.47c Horsepower Ratings of Rubber Belts**  
(hp/in of belt width for 180° wrap)

	Ply	Belt speed, ft/min											
		500	1,000	1,500	2,000	2,500	3,000	4,000	5,000	6,000	7,000	8,000	
32-oz fabric	3	0.7	1.4	2.1	2.7	3.3	3.9	4.9	5.6	6.0			
	4	0.9	1.9	2.8	3.6	4.4	5.2	6.5	7.4	7.9			
	5	1.2	2.3	3.4	4.5	5.5	6.5	8.1	9.2	9.8			
	6	1.4	2.8	4.1	5.4	6.6	7.8	9.6	11.0	11.7			
	7	1.6	3.2	4.7	6.2	7.7	9.0	11.2	12.8	13.6			
	8	1.8	3.6	5.3	7.0	8.7	10.2	12.7	14.6	15.5			
32-oz hard fabric	3	0.7	1.5	2.2	2.9	3.5	4.1	5.1	5.8	6.2	6.1	5.5	
	4	1.0	2.0	3.0	3.9	4.7	5.5	6.8	7.8	8.3	8.1	7.3	
	5	1.3	2.5	3.7	4.9	5.9	6.9	8.5	9.8	10.3	9.1	9.0	
	6	1.5	3.0	4.5	5.9	7.1	8.3	10.2	11.7	12.3	12.1	10.7	
	7	1.7	3.5	5.2	6.9	8.3	9.7	11.9	13.6	14.3	14.1	12.4	
	8	1.9	4.0	5.9	7.9	9.5	11.1	13.6	15.5	16.3	16.0	14.1	
	9	2.1	4.5	6.6	8.9	10.6	12.4	15.3	17.4	18.3	17.9	15.8	
	10	2.3	5.0	7.3	9.8	11.7	13.7	17.0	19.3	20.3	19.8	17.5	
	No. 70 rayon cord	3	1.6	3.1	4.6	6.0	7.3	8.6	10.6	12.0	12.7	12.3	10.7
		4	2.1	4.1	6.1	8.0	9.8	11.5	14.5	16.6	17.8	17.8	16.4
5		2.6	5.1	7.6	10.1	12.3	14.5	18.3	21.1	23.0	23.5	22.2	
6		3.1	6.2	9.2	12.1	14.8	17.5	22.1	25.7	28.1	28.9	27.9	
7		3.6	7.2	10.7	14.1	17.4	20.4	26.0	30.3	33.2	34.5	33.7	
8		4.1	8.2	12.2	16.2	19.9	23.4	29.8	34.8	38.4	40.0	39.4	

**Table 8.2.47d Minimum Pulley Diameters—Rubber Belts, in**

	Ply	Belt speed, ft/min											
		500	1,000	1,500	2,000	2,500	3,000	4,000	5,000	6,000	7,000	8,000	
32-oz fabric	3	4	4	4	4	5	5	5	6	6			
	4	4	5	6	6	7	7	8	9	10			
	5	6	7	9	10	10	11	12	13	14			
	6	9	10	11	13	14	14	16	18	19			
	7	13	14	16	17	18	19	21	22	24			
	8	18	19	21	22	23	24	25	27	29			
32-oz hard fabric	3	3	3	3	4	4	4	4	5	5	6	7	
	4	4	4	5	5	6	6	7	7	8	9	12	
	5	5	6	7	8	8	9	10	11	12	13	16	
	6	6	8	10	11	11	12	13	15	16	18	21	
	7	10	12	14	15	15	16	17	19	20	22	26	
	8	14	16	17	18	19	20	21	23	24	27	31	
	9	18	20	21	22	23	24	25	27	28	31	36	
	10	22	24	25	26	27	28	29	31	33	35	41	
	No. 70 rayon cord	3	5	6	7	7	8	8	9	10	11	12	13
		4	7	8	9	9	10	11	12	12	14	15	17
5		9	10	11	12	13	13	15	16	17	19	21	
6		13	14	15	16	16	17	18	19	21	23	25	
7		16	17	18	19	20	21	22	23	24	26	29	
8		19	20	22	23	23	24	25	26	28	30	33	

$EF$  makes with  $AB$  is greater than  $18^\circ$ , draw a tangent to the circle  $D$ , making an angle of  $18^\circ$  with  $AB$ ; and from a center on  $CD$  distant  $0.298L$  above  $C$ , draw an arc tangent to this  $18^\circ$  line. All belt lines with angles greater than  $18^\circ$  will be tangent to this last drawn arc.

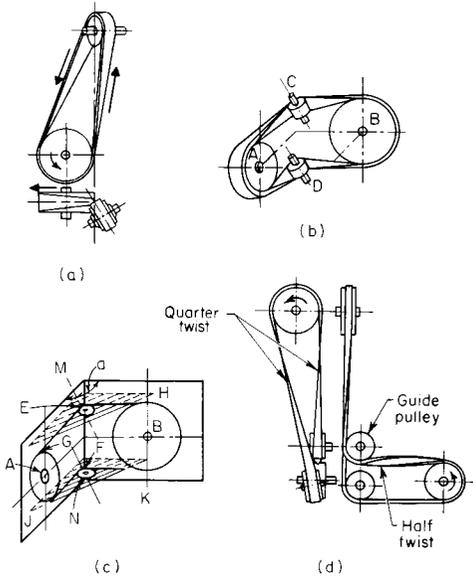


Fig. 8.2.87 Arrangements for flat-belt drives.

A very slight error in a graphical solution drawn to any scale much under full size will introduce an error seriously affecting the equality of belt tensions on the various pairs of pulleys in the set, and where much power is to be transmitted it is advisable to calculate the pulley diameters from the following formulas derived from Burmester's graphical method ("Lehrbuch der Mechanik").

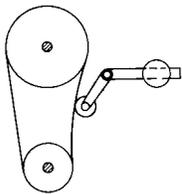


Fig. 8.2.88 Belt tightener.

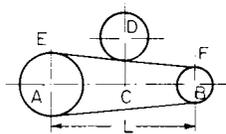


Fig. 8.2.89 Symbols for cone pulley graphical method.

Let  $D_1$  and  $D_2$  be, respectively, the diameters of the smaller and larger pulleys of a pair,  $n = D_2/D_1$ , and  $l$  = distance between shaft centers, all in inches. Also let  $m = 1.5814l - D_0$ , where  $D_0$  = diam of both pulleys for a speed ratio  $n = 1$ . Then  $(D_1 + m)^2 + (nD_1 + m)^2 = 5l^2$ . First settle on values of  $D_0$ ,  $l$ , and  $n$ , and then substitute in the equation and solve for  $D_1$ . The diameter  $D_2$  of the other pulley of the pair will then be  $nD_1$ . The values are correct to the fourth decimal place.

The speeds given by cone pulleys should increase in a **geometric ratio**; i.e., each speed should be multiplied by a constant  $a$  in order to obtain the next higher speed. Let  $n_1$  and  $n_2$  be, respectively, the lowest and highest speeds (r/min) desired and  $k$  the number of speed changes. Then  $a = \sqrt[k-1]{n_2/n_1}$ . In practice,  $a$  ranges from 1.25 up to 1.75 and even 2. The ideal value for  $a$  in machine-tool practice, according to Carl G. Barth, would be 1.189. In the example below, this would mean the use of 18 speeds instead of 8.

EXAMPLE. Let  $n_1 = 16$ ,  $n_2 = 400$ , and  $k = 8$ , to be obtained with four pairs of pulleys and a back gear. From formula,  $a = \sqrt[7]{25} = 1.584$ , whence speeds will be

16,  $(16 \times 1.584 =) 25.34$ ,  $(25.34 \times 1.584 =) 40.14$ , and similarly 63.57, 100.7, 159.5, 252.6, and 400. The first four speeds are with the back gear in; hence the back-gear ratio must be  $100.7 \div 16 = 6.29$ .

**Transmission of Power by Flat Belts** The theory of flat-belt drives takes into account changes in belt tension caused by friction forces between belt and pulley, which, in turn, cause belt elongation or contraction, thus inducing relative movement between belt and pulley. The transmission of power is a complex affair. A lengthy mathematical presentation can be found in Firkbank, "Mechanics of the Flat Belt Drive," ASME Paper 72-PTG-21. A simpler, more conventional analysis used for many years yields highly serviceable designs.

The turning force (tangential) on the rim of a pulley driven by a flat belt is equal to  $T_1 - T_2$ , where  $T_1$  and  $T_2$  are, respectively, the tensions in the driving (tight) side and following (slack) side of the belt. (For the relations of  $T_1$  and  $T_2$  at low peripheral speeds, see Sec. 3.)  $\log (T_1/T_2) = 0.0076fa$  when the effect of centrifugal force is neglected and  $T_1/T_2 = 10^{0.0076fa}$ . Figure 8.2.90 gives values of this function. When the speeds are high, however, the relations of  $T_1$  to  $T_2$  are modified by centrifugal stresses in the belt, in which case  $\log (T_1/T_2) = 0.0076f(1-x)a$ , where  $f$  = coefficient of friction between the belt and pulley surface,  $a$  = angle of wrap, and  $x = 12wv^2/(gt)$  in which  $w$  =

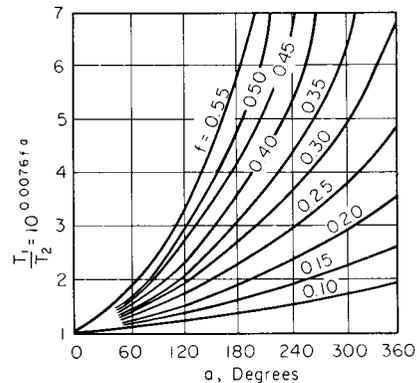


Fig. 8.2.90 Values of  $10^{0.0076fa}$ .

weight of 1 in<sup>3</sup> of belt material, lb;  $v$  = belt speed, ft/s;  $g = 32.2$  ft/s<sup>2</sup>; and  $t$  = allowable working tension, lb/in<sup>2</sup>. Values of  $x$  for leather belting (with  $w = 0.035$  and  $t = 300$ ) are as follows:

$u$	30	40	50	60	70	
$x$	0.039	0.070	0.118	0.157	0.214	
$uv$	80	90	100	110	120	130
$x$	0.279	0.352	0.435	0.526	0.626	0.735

Researches by Barth (*Trans. ASME*, 1909) seem to show that  $f$  is a function of the belt velocity, varying according to the formula  $f = 0.54 - 140/(500 + V)$  for leather belts on iron pulleys, where  $V$  = belt velocity for ft/min. For practical design, however, the following values of  $f$  may be used: for leather belts on cast-iron pulleys,  $f = 0.30$ ; on wooden pulleys,  $f = 0.45$ ; on paper pulleys,  $f = 0.55$ . The treatment of belts with belt dressing, pulleys with cork inserts, and dampness are all factors which greatly modify these values, tending to make them higher.

The **arc of contact** on the smaller of two pulleys connected by an open belt, in degrees, is approximately equal to  $180 - 60(D - d)/l$ , where  $D$  and  $d$  are the larger and smaller pulley diameters and  $l$  the distance between their shaft centers, all in inches. This formula gives an error not exceeding 0.5 percent.

**Selecting a Belt** Selecting an appropriate belt involves calculating horsepower per inch of belt width as follows:

$$\text{hp/in} = (\text{demanded hp} \times S)/(K \times W)$$

where demanded hp = horsepower required by the job at hand;  $S$  = service factor;  $K$  = arc factor;  $W$  = proposed belt width (determined from pulley width). One enters a belt manufacturer's catalog with  $hp/in$ , *belt speed*, and *small pulley diameter*, then selects that belt which has a matching maximum hp/in rating. See Table 8.2.47a, b, c, and d for typical values of  $S$ ,  $K$ , hp/in ratings, and minimum pulley diameters.

#### V-Belt Drives

V-belt drives are widely used in power transmission, despite the fact that they may range in efficiency from about 70 to 96 percent. Such drives consist essentially of endless belts of trapezoidal cross section which ride in V-shaped pulley grooves (see Fig. 8.2.93a). The belts are formed of cord and fabric, impregnated with rubber, the cord material being cotton, rayon, synthetic, or steel. V-belt drives are quiet, able to absorb shock and operate at low bearing pressures. A V belt should ride with the top surface approximately flush with the top of the pulley groove; clearance should be present between the belt base and the base of the groove so that the belt rides on the groove walls. The friction between belt and groove walls is greatly enhanced beyond normal values because sheave groove angles are made somewhat less than belt-section angles, causing the belt to wedge itself into the groove. See Table 8.2.56a for standard groove dimensions of sheaves.

The cross section and lengths of V belts have been standardized by ANSI in both inch and SI (metric) units, while ANSI and SAE have standardized the special category of automobile belts, again in both units. Standard designations are shown in Table 8.2.48, which also includes minimum sheave diameters. V belts are specified by combining a standard designation (from Table 8.2.48) and a belt length; inside length for the inch system, and pitch (effective) length for metric system.

**Table 8.2.48 V-Belt Standard Designations—A Selection**

Type	Inch standard		Metric standard	
	Section	Minimum sheave diameter, in	Section	Minimum sheave diameter, mm
Heavy-duty	A	3.0	13 C	80
	B	5.4	16 C	140
	C	9.0	22 C	214
	D	13.0	32 C	355
	E	21.04		
Automotive	0.25	2.25	6 A	57
	0.315	2.25	8 A	57
	0.380	2.40	10 A	61
	0.440	2.75	11 A	70
	0.500	3.00	13 A	76
	$1/16$	3.00	15 A	76
	$3/4$	3.00	17 A	76
	$7/8$	3.50	20 A	89
	1.0	4.00	23 A	102
	Heavy-duty narrow	3 V	2.65	
5 V		7.1		
8 V		12.3		
Notched narrow	3 VX	2.2		
	5 VX	4.4		
Light-duty	2 L	0.8		
	3 L	1.5		
	4 L	2.5		
	5 L	3.5		
	Synchronous belts	MXL		
XL				
L				
H				
XH				
XXH				

NOTE: The use of smaller sheaves than minimum will tend to shorten belt life.

SOURCE: Compiled from ANSI/RMA IP-20, 21, 22, 23, 24, 25, 26; ANSI/SAE J636C.

Sheaves are specified by their pitch diameters, which are used for velocity ratio calculations in which case inside belt lengths must be converted to pitch lengths for computational purposes. Pitch lengths are calculated by adding a conversion factor to inside length (i.e.,  $L_p = L_s + \Delta$ ). See Table 8.2.49 for conversion factors. Table 8.2.50 lists standard inside inch lengths  $L_s$  and Table 8.2.51 lists standard metric pitch (effective) lengths  $L_p$ .

**Table 8.2.49 Length Conversion Factors  $\Delta$**

Belt section	Size interval	Conversion factor	Belt section	Size interval	Conversion factor
A	26–128	1.3	D	120–210	3.3
B	35–210	1.8	D	$\geq 240$	0.8
B	$\geq 240$	0.3	E	180–210	4.5
C	51–210	2.9	E	$\geq 240$	1.0
C	$\geq 240$	0.9			

SOURCE: Adapted from ANSI/RMA IP-20-1977 (R88) by permission.

**Table 8.2.50 Standard Lengths  $L_s$ , in, and Length Correction Factors  $K_2$ : Conventional Heavy-Duty V Belts**

$L_s$	Cross section				
	A	B	C	D	E
26	0.78				
31	0.82				
35	0.85	0.80			
38	0.87	0.82			
42	0.89	0.84			
46	0.91	0.86			
51	0.93	0.88	0.80		
55	0.95	0.89			
60	0.97	0.91	0.83		
68	1.00	0.94	0.85		
75	1.02	0.96	0.87		
80	1.04				
81		0.98	0.89		
85	1.05	0.99	0.90		
90	1.07	1.00	0.91		
96	1.08		0.92		
97		1.02			
105	1.10	1.03	0.94		
112	1.12	1.05	0.95		
120	1.13	1.06	0.96	0.88	
128	1.15	1.08	0.98	0.89	
144		1.10	1.00	0.91	
158		1.12	1.02	0.93	
173		1.14	1.04	0.94	
180		1.15	1.05	0.95	0.92
195		1.17	1.06	0.96	0.93
210		1.18	1.07	0.98	0.95
240		1.22	1.10	1.00	0.97
270		1.24	1.13	1.02	0.99
300		1.27	1.15	1.04	1.01
330			1.17	1.06	1.03
360			1.18	1.07	1.04
390			1.20	1.09	1.06
420			1.21	1.10	1.07
480				1.13	1.09
540				1.15	1.11
600				1.17	1.13
660				1.18	1.15

SOURCE: ANSI/RMA IP-20-1977 (R88), reproduced by permission.

**Table 8.2.51 Standard Pitch Lengths  $L_p$  (Metric Units) and Length Correction Factors  $K_2$**

13C		16C		22C		32C	
$L_p$	$K_2$	$L_p$	$K_2$	$L_p$	$K_2$	$L_p$	$K_2$
710	0.83	960	0.81	1,400	0.83	3,190	0.89
750	0.84	1,040	0.83	1,500	0.85	3,390	0.90
800	0.86	1,090	0.84	1,630	0.86	3,800	0.92
850	0.88	1,120	0.85	1,830	0.89	4,160	0.94
900	0.89	1,190	0.86	1,900	0.90	4,250	0.94
950	0.90	1,250	0.87	2,000	0.91	4,540	0.95
1,000	0.92	1,320	0.88	2,160	0.92	4,720	0.96
1,075	0.93	1,400	0.90	2,260	0.93	5,100	0.98
1,120	0.94	1,500	0.91	2,390	0.94	5,480	0.99
1,150	0.95	1,600	0.92	2,540	0.96	5,800	1.00
1,230	0.97	1,700	0.94	2,650	0.96	6,180	1.01
1,300	0.98	1,800	0.95	2,800	0.98	6,560	1.02
1,400	1.00	1,900	0.96	3,030	0.99	6,940	1.03
1,500	1.02	1,980	0.97	3,150	1.00	7,330	1.04
1,585	1.03	2,110	0.99	3,350	1.01	8,090	1.06
1,710	1.05	2,240	1.00	3,550	1.02	8,470	1.07
1,790	1.06	2,360	1.01	3,760	1.04	8,850	1.08
1,865	1.07	2,500	1.02	4,120	1.06	9,240	1.09
1,965	1.08	2,620	1.03	4,220	1.06	10,000	1.10
2,120	1.10	2,820	1.05	4,500	1.07	10,760	1.11
2,220	1.11	2,920	1.06	4,680	1.08	11,530	1.13
2,350	1.13	3,130	1.07	5,060	1.10	12,290	1.14
2,500	1.14	3,330	1.09	5,440	1.11		
2,600	1.15	3,530	1.10	5,770	1.13		
2,730	1.17	3,740	1.11	6,150	1.14		
2,910	1.18	4,090	1.13	6,540	1.15		
3,110	1.20	4,200	1.14	6,920	1.16		
3,310	1.21	4,480	1.15	7,300	1.17		
		4,650	1.16	7,680	1.18		
		5,040	1.18	8,060	1.19		
		5,300	1.19	8,440	1.20		
		5,760	1.21	8,820	1.21		
		6,140	1.23	9,200	1.22		
		6,520	1.24				
		6,910	1.25				
		7,290	1.26				
		7,670	1.27				

SOURCE: ANSI/RMA IP-20-1988 revised, reproduced by permission.

For given large and small sheave diameters and center-to-center distance, the needed V-belt length can be computed from

$$L_p = 2C + 1.57(D + d) + \frac{(D - d)^2}{4C}$$

$$C = \frac{K + \sqrt{K^2 - 32(D - d)^2}}{16} \quad K = 4L_p - 6.28(D + d)$$

where  $C$  = center-to-center distance;  $D$  = pitch diameter of large sheave;  $d$  = pitch diameter of small sheave;  $L_p$  = pitch (effective) length.

Arc of contact on the smaller sheave (degrees) is approximately

$$\theta = 180 - \frac{60(D - d)}{C}$$

**Transmission of Power by V Belts** Unfortunately there is no theory or mathematical analysis that is able to explain all experimental results reliably. Empirical formulations based on experimental results, however, do provide very serviceable design procedures, and together with data published in V-belt manufacturers' catalogs provide the engineer with the necessary V-belt selection tools.

For satisfactory performance under most conditions, ANSI provides the following empirical single V-belt power-rating formulation (inch and metric units) for 180° arc of contact and average belt length:

$$H_r = \left( C_1 - \frac{C_2}{d} - C_3(r - d)^2 - C_4 \log rd \right) rd + C_2 r \left( 1 - \frac{1}{K_A} \right)$$

where  $H_r$  = rated horsepower for inch units (rated power kW for metric units);  $C_1, C_2, C_3, C_4$  = constants from Table 8.2.53;  $r$  = r/min of high-speed shaft times  $10^{-3}$ ;  $K_A$  = speed ratio factor from Table 8.2.52;  $d$  = pitch diameter of small sheave, in (mm).

**Selecting a Belt** Selecting an appropriate belt involves calculating horsepower per belt as follows:

$$NH_r = (\text{demanded hp} \times K_s) / (K_1 K_2)$$

where  $H_r$  = hp/belt rating, either from ANSI formulation above, or from manufacturer's catalog (see Table 8.2.55); demanded hp = horsepower required by the job at hand;  $K_s$  = service factor accounting for driver and driven machine characteristics regarding such things as shock, torque level, and torque uniformity (see Table 8.2.54);  $K_1$  = angle of contact correction factor (see Fig. 8.2.91a);  $K_2$  = length correction factor (see Tables 8.2.50 and 8.2.51);  $N$  = number of belts.

See Fig. 8.2.91b for selection of V-belt cross section.

V band belts, effectively joined V belts, serve the function of multiple single V belts (see Fig. 8.2.93b).

Long center distances are not recommended for V belts because excess slack-side vibration shortens belt life. In general  $D \leq C \leq 3(D + d)$ . If longer center distances are needed, then link-type V belts can be used effectively.

Since belt-drive capacity is normally limited by slippage of the smaller sheave, V-belt drives can sometimes be used with a flat, larger pulley rather than with a grooved sheave, with little loss in capacity. For instance the flat surface of the flywheel in a large punch press can serve such a purpose. The practical range of application is when speed ratio is over 3 : 1, and center distance is equal to or slightly less than the diameter of the large pulley.

**Table 8.2.52 Approximate Speed-Ratio Factor  $K_A$  for Use in Power-Rating Formulation**

$D/d$ range	$K_A$	$D/d$ range	$K_A$
1.00–1.01	1.0000	1.15–1.20	1.0586
1.02–1.04	1.0112	1.21–1.27	1.0711
1.05–1.07	1.0226	1.28–1.39	1.0840
1.08–1.10	1.0344	1.40–1.64	1.0972
1.11–1.14	1.0463	Over 1.64	1.1106

SOURCE: Adapted from ANSI/RMA IP-20-1977 (R88), by permission.

**Table 8.2.53 Constants  $C_1, C_2, C_3, C_4$  for Use in Power-Rating Formulation**

Belt section	$C_1$	$C_2$	$C_3$	$C_4$
A	0.8542	1.342	$2.436 \times 10^{-4}$	0.1703
B	1.506	3.520	$4.193 \times 10^{-4}$	0.2931
C	2.786	9.788	$7.460 \times 10^{-4}$	0.5214
D	5.922	34.72	$1.522 \times 10^{-3}$	1.064
E	8.642	66.32	$2.192 \times 10^{-3}$	1.532
Metric				
13C	0.03316	1.088	$1.161 \times 10^{-8}$	$5.238 \times 10^{-3}$
16C	0.05185	2.273	$1.759 \times 10^{-8}$	$7.934 \times 10^{-3}$
22C	0.1002	7.040	$3.326 \times 10^{-8}$	$1.500 \times 10^{-2}$
32C	0.2205	26.62	$7.037 \times 10^{-8}$	$3.174 \times 10^{-2}$

SOURCE: Compiled from ANSI/RMA IP-20-1977 (R88), by permission.

**Table 8.2.54 Approximate Service Factor  $K_s$  for V-Belt Drives**

Power source torque	Load			
	Uniform	Light shock	Medium shock	Heavy shock
Average or normal	1.0–1.2	1.1–1.3	1.2–1.4	1.3–1.5
Nonuniform or heavy	1.1–1.3	1.2–1.4	1.4–1.6	1.5–1.8

SOURCE: Adapted from ANSI/RMA IP-20-1977 (R88), by permission.

Table 8.2.55 Horsepower Ratings of V Belts

Belt section	Speed of faster shaft, r/min	Rated horsepower per belt for small sheave pitch diameter, in							Additional horsepower per belt for speed ratio				
		2.60	3.00	3.40	3.80	4.20	4.60	5.00	1.02–1.04	1.08–1.10	1.15–1.20	1.28–1.39	1.65–over
A	200	0.20	0.27	0.33	0.40	0.46	0.52	0.59	0.00	0.01	0.01	0.02	0.03
	800	0.59	0.82	1.04	1.27	1.49	1.70	1.92	0.01	0.04	0.06	0.08	0.11
	1,400	0.87	1.25	1.61	1.97	2.32	2.67	3.01	0.02	0.06	0.10	0.15	0.19
	2,000	1.09	1.59	2.08	2.56	3.02	3.47	3.91	0.03	0.09	0.15	0.21	0.27
	2,600	1.25	1.87	2.47	3.04	3.59	4.12	4.61	0.04	0.12	0.19	0.27	0.35
	3,200	1.37	2.08	2.76	3.41	4.01	4.57	5.09	0.05	0.14	0.24	0.33	0.43
	3,800	1.43	2.23	2.97	3.65	4.27	4.83	5.32	0.06	0.17	0.28	0.40	0.51
	4,400	1.44	2.29	3.07	3.76	4.36	4.86*	5.26*	0.07	0.20	0.33	0.46	0.59
	5,000	1.39	2.28	3.05	3.71	4.24*	4.48*	4.64*	0.07	0.22	0.37	0.52	0.65
	5,600	1.29	2.17	2.92	3.50*				0.08	0.25	0.42	0.58	0.75
	6,200	1.11	1.98	2.65*					0.09	0.28	0.46	0.64	0.83
	6,800	0.87	1.68*	2.24*					0.10	0.30	0.51	0.71	0.91
	7,400	0.56	1.28*						0.11	0.33	0.55	0.77	0.99
	7,800	0.31*							0.12	0.35	0.58	0.81	1.04
		4.60	5.20	5.80	6.40	7.00	7.60	8.00					
B	200	0.69	0.86	1.02	1.18	1.34	1.50	1.61	0.01	0.02	0.04	0.05	0.07
	600	1.68	2.12	2.56	2.99	3.41	3.83	4.11	0.02	0.07	0.12	0.16	0.21
	1,000	2.47	3.16	3.84	4.50	5.14	5.78	6.20	0.04	0.12	0.19	0.27	0.35
	1,400	3.13	4.03	4.91	5.76	6.59	7.39	7.91	0.05	0.16	0.27	0.38	0.49
	1,800	3.67	4.75	5.79	6.79	7.74	8.64	9.21	0.07	0.21	0.35	0.49	0.63
	2,200	4.08	5.31	6.47	7.55	8.56	9.48	10.05	0.09	0.26	0.43	0.60	0.77
	2,600	4.36	5.69	6.91	8.01	9.01	9.87	10.36	0.10	0.30	0.51	0.71	0.91
	3,000	4.51	5.89	7.11	8.17	9.04	9.73*		0.12	0.35	0.58	0.82	1.05
	3,400	4.51	5.88	7.03	7.95*				0.13	0.40	0.66	0.93	1.19
	3,800	4.34	5.64	6.65*					0.15	0.44	0.74	1.04	1.33
	4,200	4.01	5.17*						0.16	0.49	0.82	1.15	1.47
	4,600	3.48							0.18	0.54	0.90	1.25	1.61
	5,000	2.76*							0.19	0.59	0.97	1.36	1.75
			7.00	8.00	9.00	10.00	11.00	12.00	13.00				
C	100	1.03	1.29	1.55	1.81	2.06	2.31	2.56	0.01	0.03	0.05	0.08	0.10
	400	3.22	4.13	5.04	5.93	6.80	7.67	8.53	0.04	0.13	0.22	0.30	0.39
	800	5.46	7.11	8.73	10.31	11.84	13.34	14.79	0.09	0.26	0.43	0.61	0.78
	1,000	6.37	8.35	10.26	12.11	13.89	15.60	17.24	0.11	0.33	0.54	0.76	0.97
	1,400	7.83	10.32	12.68	14.89	16.94	18.83	20.55	0.15	0.46	0.76	1.06	1.36
	1,800	8.76	11.58	14.13	16.40	18.37	20.01	21.31*	0.20	0.59	0.98	1.37	1.75
	2,000	9.01	11.90	14.44	16.61	18.37	19.70*		0.22	0.65	1.08	1.52	1.95
	2,400	9.04	11.87	14.14					0.26	0.78	1.30	1.82	2.34
	2,800	8.38	10.85*						0.30	0.91	1.52	2.12	2.73
	3,000	7.76							0.33	0.98	1.63	2.28	2.92
	3,200	6.44*							0.36	1.07	1.79	2.50	3.22

\* For footnote see end of table on next page.

In general the V-beltting of skew shafts is discouraged because of the decrease in life of the belts, but where design demands such arrangements, special deep-groove sheaves are used. In such cases center distances should comply with the following:

For 90° turn  $C_{min} = 5.5(D + W)$   
 For 45° turn  $C_{min} = 4.0(D + W)$   
 For 30° turn  $C_{min} = 3.0(D + W)$

where  $W$  = width of group of individual belts. Selected values of  $W$  are shown in Table 8.2.56d.

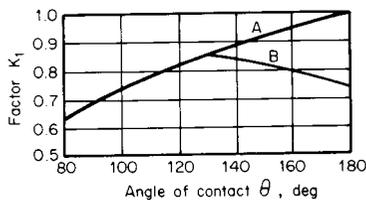


Fig. 8.2.91a Angle-of-contact correction factor, where A = grooved sheave to grooved pulley distance (V to V) and B = grooved sheave to flat-face pulley distance (V to flat).

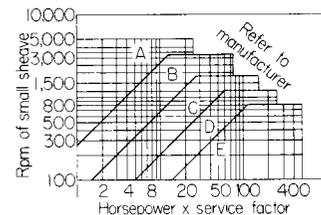


Fig. 8.2.91b V-belt section for required horsepower ratings. Letters A, B, C, D, E refer to belt cross section. (See Table 8.2.54 for service factor.)

Figure 8.2.92 shows a 90° turn arrangement, from which it can be seen that the horizontal shaft should lie some distance  $Z$  higher than the center of the vertical-shaft sheave. Table 8.2.56c lists the values of  $Z$  for various center distances in a 90° turn arrangement.

**Cogged V belts** have cogs molded integrally on the underside of the belt (Fig. 8.2.94a). Sheaves can be up to 25 percent smaller in diameter with cogged belts because of the greater flexibility inherent in the cogged construction. An extension of the cogged belt mating with a sheave or pulley notched at the same pitch as the cogs leads to a drive particularly useful for timing purposes.

Table 8.2.55 Horsepower Ratings of V Belts (Continued)

Belt section	Speed of faster shaft, r/min	Rated horsepower per belt for small sheave pitch diameter, in							Additional horsepower per belt for speed ratio				
		12.00	14.00	16.00	18.00	20.00	22.00	24.00	1.02-1.04	1.08-1.10	1.15-1.20	1.28-1.39	1.65-over
D	50	1.96	2.52	3.08	3.64	4.18	4.73	5.27	0.02	0.06	0.10	0.13	0.17
	200	6.28	8.27	10.24	12.17	14.08	15.97	17.83	0.08	0.23	0.38	0.54	0.69
	400	10.89	14.55	18.12	21.61	25.02	28.35	31.58	0.15	0.46	0.77	1.08	1.38
	600	14.67	19.75	24.64	29.33	33.82	38.10	42.15	0.23	0.69	1.15	1.61	2.07
	800	17.70	23.91	29.75	35.21	40.24	44.83	48.94	0.31	0.92	1.54	2.15	2.77
	1,000	19.93	26.94	33.30	38.96	43.86	47.93	51.12	0.38	1.15	1.92	2.69	3.46
	1,200	21.32	28.71	35.05	40.24	44.18*			0.46	1.39	2.31	3.23	4.15
	1,400	21.76	29.05	34.76*					0.54	1.62	2.69	3.77	4.84
	1,600	21.16	27.81*						0.62	1.85	3.08	4.30	5.53
	1,800	19.41							0.69	2.08	3.46	4.84	6.22
	1,950	17.28*							0.75	2.25	3.75	5.25	6.74
			18.00	21.00	24.00	27.00	30.00	33.00	36.00				
E	50	4.52	5.72	6.91	8.08	9.23	10.38	11.52	0.04	0.11	0.18	0.26	0.33
	100	8.21	10.46	12.68	14.87	17.04	19.19	21.31	0.07	0.22	0.37	0.51	0.66
	200	14.68	18.86	22.97	27.00	30.96	34.86	38.68	0.15	0.44	0.73	1.03	1.32
	300	20.37	26.29	32.05	37.67	43.13	48.43	53.58	0.22	0.66	1.10	1.54	1.98
	400	25.42	32.87	40.05	46.95	53.55	59.84	65.82	0.29	0.88	1.47	2.06	2.64
	500	29.86	38.62	46.92	54.74	62.05	68.81	75.00	0.37	1.10	1.84	2.57	3.30
	600	33.68	43.47	52.55	60.87	68.36	74.97	80.63	0.44	1.32	2.20	3.08	3.96
	700	36.84	47.36	56.83	65.15	72.22	77.93*		0.51	1.54	2.57	3.60	4.62
	800	39.29	50.20	59.61	67.36	73.30*			0.59	1.76	2.94	4.11	5.28
	900	40.97	51.89	60.73					0.66	1.98	3.30	4.63	5.94
	1,000	41.84	52.32	60.04*					0.73	2.21	3.67	5.14	6.60
	1,100	41.81	51.40*						0.81	2.43	4.04	5.65	7.26
	1,200	40.83							0.88	2.65	4.41	6.17	7.93
1,300	38.84*							0.95	2.87	4.77	6.68	8.59	

\* Rim speed above 6,000 ft/min. Special sheaves may be necessary.  
SOURCE: Compiled from ANSI/RMA IP-20-1988 revised, by permission.

Ribbed V belts are really flat belts molded integrally with longitudinal ribbing on the underside (Fig. 8.2.94b). Traction is provided principally by friction between the ribs and sheave grooves rather than by wedging action between the two, as in conventional V-belt operation. The flat

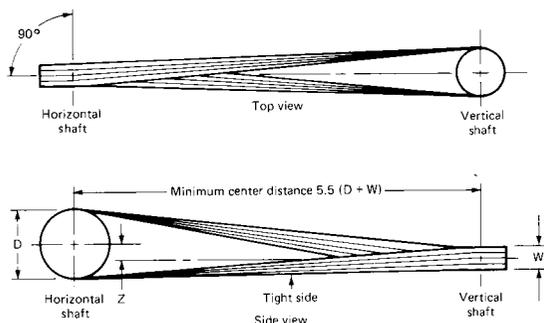


Fig. 8.2.92 Quarter-turn drive for V belts.

upper portion transmits the tensile belt loads. Ribbed belts serve well when substituted for multiple V-belt drives and for all practical purposes eliminate the necessity for belt-matching in multiple V-belt drives.

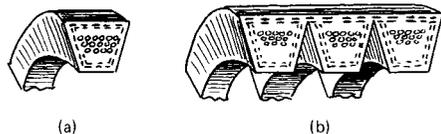


Fig. 8.2.93 V- and V-band belt cross section.

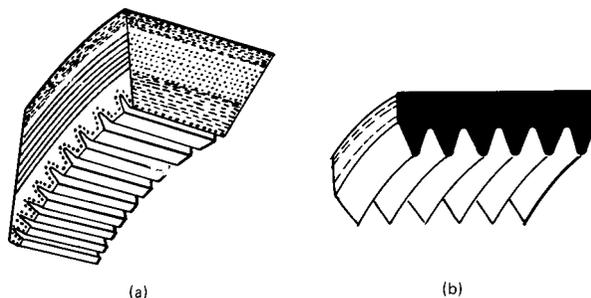


Fig. 8.2.94 Special V belts. (a) Cogged V belt; (b) ribbed V belt.

Adjustable Motor Bases

To maintain proper belt tensions on short center distances, an adjustable motor base is often used. Figure 8.2.95 shows an embodiment of such a base made by the Automatic Motor Base Co., in which adjustment for proper belt tension is made by turning a screw which opens or closes the center distance between pulleys, as required. The carriage portion of the base is spring loaded so that after the initial adjustment for belt tension

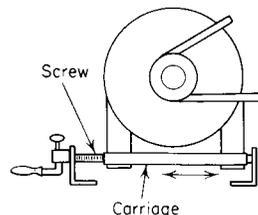
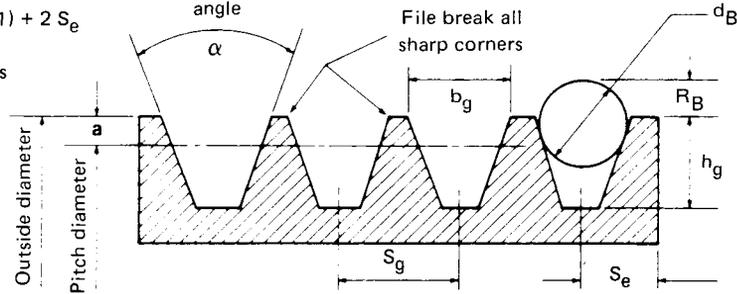


Fig. 8.2.95 Adjustable motor base.

Table 8.2.56a Standard Sheave Groove Dimensions, in

Face width of standard and deep groove sheaves  
 Face width =  $S_g (N_g - 1) + 2 S_e$   
 where:  
 $N_g$  = number of grooves



Cross section	Standard groove dimensions								Drive design factors		
	Outside diameter range	Groove angle $\alpha \pm 0.33^\circ$	$b_g$	$h_g$ min	$R_B$ min	$d_B \pm 0.0005$	$S_g \pm 0.025$	$S_e$	Pitch diameter range	Minimum recommended pitch diameter	$2a$
A	Up through 5.65	34	0.494	0.460	0.148	0.4375	0.625	0.375 + 0.090 - 0.062	Up through 5.40 Over 5.40	3.0	0.250
	Over 5.65	38	0.504 ± 0.005		0.149	( <sup>7</sup> / <sub>16</sub> )					
B	Up through 7.35	34	0.637	0.550	0.189	0.5625	0.750	0.500 + 0.120 - 0.065	Up through 7.00 Over 7.00	5.4	0.350
	Over 7.35	38	0.650 ± 0.006		0.190	( <sup>9</sup> / <sub>16</sub> )					
A/B	Up through 7.35	34	0.612	0.612	0.230	0.5625	0.750	0.500 + 0.120 + 0.065		A = 3.0 B = 5.4	A = 0.620* B = 0.280
	Over 7.35	38	0.625 ± 0.006		0.226	( <sup>9</sup> / <sub>16</sub> )					
C	Up through 8.39	34	0.879	0.750	0.274	0.7812	1.000	0.688 + 0.160 - 0.70	Up through 7.99 Over 7.99 to and incl. 12.00 Over 12.00	9.0	0.400
	Over 8.39 to and incl. 12.40	36	0.877 ± 0.007		0.276	( <sup>25</sup> / <sub>32</sub> )					
	Over 12.40	38	0.895		0.277						
D	Up through 13.59	34	1.256	1.020	0.410	1.1250	1.438	+ 0.220 - 0.80	Up through 12.99 Over 12.99 to and incl. 17.00 Over 17.00	13.0	0.600
	Over 13.59 to and incl. 17.60	36	1.271 ± 0.008		0.410	(1 <sup>1</sup> / <sub>8</sub> )					
	Over 17.60	38	1.283		0.411						
E	Up through 24.80	36	1.527	1.270	0.476	1.3438	1.750	1.125 + 0.280 - 0.090	Up through 24.00 Over 24.00	21.0	0.800
	Over 24.80	38	1.542 ± 0.010		0.477	(1 <sup>11</sup> / <sub>32</sub> )					

\* The  $a$  values shown for the A/B combination sheaves are the geometrically derived values. These values may be different than those shown in manufacturers' catalogs.  
 SOURCE: "Dayco Engineering Guide for V-Belt Drives," Dayco Corp., Dayton, OH, 1981, reprinted by permission.

Table 8.2.56b Classical Deep Groove Sheave Dimensions, in

Cross section	Outside diameter range	Deep groove dimensions							
		Groove angle $\alpha \pm 0.33^\circ$	$b_g$	$h_g$ min	$2a$	$S_g \pm 0.025$	$S_e$		
A	Up through 5.96	34	0.539	0.615	0.560	0.750	0.438	+0.090	
	Over 5.96	38	0.611					-0.062	
B	Up through 7.71	34	0.747	0.730	0.710	0.875	0.562	+0.120	
	Over 7.71	38	0.774					-0.065	
C	Up through 9.00	34	1.066	1.055	1.010	1.250	0.812	+0.160	
	Over 9.00 to and incl. 13.01	36	1.085					+0.007	-0.070
	Over 13.01	38	1.105						
D	Up through 14.42	34	1.513	1.435	1.430	1.750	1.062	+0.220	
	Over 14.42 to and incl. 18.43	36	1.541					$\pm 0.008$	-0.080
	Over 18.43	38	1.569						
E	Up through 25.69	36	1.816	1.715	1.690	2.062	1.312	+0.280	
	Over 25.69	38	1.849					+0.010	-0.090

SOURCE: "Dayco Engineering Guide for V-Belt Drives," Dayco Corp., Dayton, OH, 1981, reprinted by permission.

Table 8.2.56c Z Dimensions for Quarter-Turn Drives, in

Center distance	3V, 5V, 8V Z dimension	A B, C, D Z dimension
20		0.2
30		0.2
40		0.4
50		0.4
60	0.2	0.5
80	0.3	0.5
100	0.4	1.0
120	0.6	1.5
140	0.9	2.0
160	1.2	2.5
180	1.5	3.5
200	1.8	4.0
220	2.2	5.0
240	2.6	6.0

SOURCE: "Dayco Engineering Guide for V-Belt Drives," Dayco Corp., Dayton, OH, 1981, reprinted by permission.

Table 8.2.56d Width  $W$  of Set of Belts Using Deep-Groove Sheaves, in

No. of belts	V-belt cross section						
	3V	5V	8V	A	B	C	D
1	0.4	0.6	1.0	0.5	0.7	0.9	1.3
2	0.9	1.4	2.3	1.3	1.6	2.2	3.1
3	1.4	2.2	3.6	2.0	2.5	3.4	4.8
4	1.9	3.0	4.0	2.8	3.3	4.7	6.6
5	2.4	3.8	6.2	3.5	4.2	5.9	8.3
6	2.9	4.7	7.6	4.3	5.1	7.2	10.1
7	3.4	5.5	8.9	5.0	6.0	8.4	11.8
8	3.9	6.3	10.2	5.8	6.8	9.7	13.6
9	4.4	7.1	11.5	6.5	7.7	10.9	15.3
10	4.9	7.9	12.8	7.3	8.6	12.2	17.1

SOURCE: "Dayco Engineering Guide for V-Belt Drives," Dayco Corp., Dayton, OH, 1981, reprinted by permission.

has been made by the screw, the spring will compensate for a normal amount of stretch in the belts. When there is more stretch than can be accommodated by the spring, the screw is turned to provide the necessary belt tensions. The carriage can be moved while the unit is in operation, and the motor base is provided for vertical as well as horizontal mounting.

## CHAIN DRIVES

### Roller-Chain Drives

The advantages of finished steel roller chains are high efficiency (around 98 to 99 percent), no slippage, no initial tension required, and chains may travel in either direction. The basic construction of roller chains is shown in Fig. 8.2.96 and Table 8.2.57.

The shorter the pitch, the higher the permissible operating speed of roller chains. Horsepower capacity in excess of that provided by a single chain may be had by the use of multiple chains, which are essentially parallel single chains assembled on pins common to all strands. Because of its lightness in relation to tensile strength, the effect of centrifugal pull does not need to be considered; even at the unusual speed of 6,000 ft/min, this pull is only 3 percent of the ultimate tensile strength.

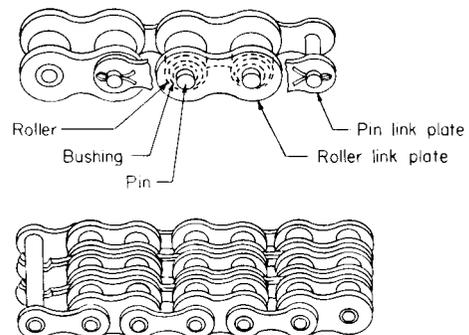


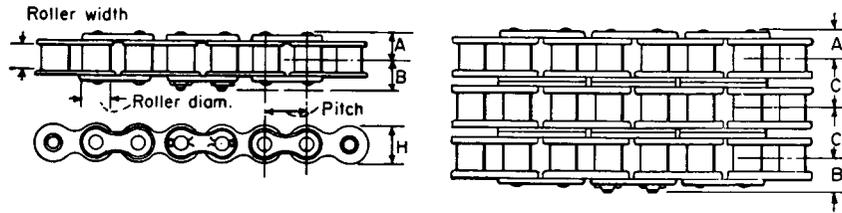
Fig. 8.2.96 Roller chain construction.

Sprocket wheels with fewer than 16 teeth may be used for relatively slow speeds, but 18 to 24 teeth are desirable for high-speed service. Sprockets with fewer than 25 teeth, running at speeds above 500 or 600 r/min, should be heat-treated to give a tough wear-resistant surface testing between 35 and 45 on the Rockwell C hardness scale.

If the speed ratio requires the larger sprocket to have as many as 128 teeth, or more than eight times the number on the smaller sprocket, it is usually better, with few exceptions, to make the desired reduction in two or more steps. The ANSI tooth form ASME B29.1 M-1993 allows roller chain to adjust itself to a larger pitch circle as the pitch of the chain elongates owing to natural wear in the pin-bushing joints. The greater the number of teeth, the sooner the chain will ride out too near the ends of the teeth.

Idler sprockets may be used on either side of the standard roller chain,

Table 8.2.57 Roller-Chain Data and Dimensions, in\*



ANSI chain no.	ISO chain no.	Roller				Roller link plate			Dimension			Tensile strength per strand, lb	Recommended max speed r/min		
		Pitch	Width	Diam	Pin diam	Thickness	Height H	A	B	C	12 teeth		18 teeth	24 teeth	
25	04C-1	¼	⅝	0.130	0.091	0.030	0.230	0.150	0.190	0.252	925	5,000	7,000	7,000	
35	06C-1	⅜	¾	0.200	0.141	0.050	0.344	0.224	0.290	0.399	2,100	2,380	3,780	4,200	
41	085	½	¾	0.306	0.141	0.050	0.383	0.256	0.315	—	2,000	1,750	2,725	2,850	
40	08A-1	½	⅝	0.312	0.156	0.060	0.452	0.313	0.358	0.566	3,700	1,800	2,830	3,000	
50	10A-1	⅝	¾	0.400	0.200	0.080	0.594	0.384	0.462	0.713	6,100	1,300	2,030	2,200	
60	12A-1	¾	½	0.469	0.234	0.094	0.679	0.493	0.567	0.897	8,500	1,025	1,615	1,700	
80	16A-1	1	⅝	0.625	0.312	0.125	0.903	0.643	0.762	1.153	14,500	650	1,015	1,100	
100	20A-1	1¼	¾	0.750	0.375	0.156	1.128	0.780	0.910	1.408	24,000	450	730	850	
120	24A-1	1½	1	0.875	0.437	0.187	1.354	0.977	1.123	1.789	34,000	350	565	650	
140	28A-1	1¾	1	1.000	0.500	0.218	1.647	1.054	1.219	1.924	46,000	260	415	500	
160	32A-1	2	1¼	1.125	0.562	0.250	1.900	1.250	1.433	2.305	58,000	225	360	420	
180		2¼	1½	1.406	0.687	0.281	2.140	1.421	1.770	2.592	76,000	180	290	330	
200	40A-1	2½	1½	1.562	0.781	0.312	2.275	1.533	1.850	2.817	95,000	170	260	300	
240	48A-1	3	1¾	1.875	0.937	0.375	2.850	1.722	2.200	3.458	135,000	120	190	210	

\* For conversion to metric units (mm) multiply table values by 25.4.

Table 8.2.58 Selected Values of Horsepower Ratings of Roller Chains

ANSI no. and pitch, in	Number of teeth in small socket	Small sprocket, r/min										
		50	500	1,200	1,800	2,500	3,000	4,000	5,000	6,000	8,000	10,000
25 ¼	11	0.03	0.23	0.50	0.73	0.98	1.15	1.38	0.99	0.75	0.49	0.35
	15	0.04	0.32	0.70	1.01	1.36	1.61	2.08	1.57	1.20	0.78	0.56
	20	0.06	0.44	0.96	1.38	1.86	2.19	2.84	2.42	1.84	1.201	0.86
	25	0.07	0.56	1.22	1.76	2.37	2.79	3.61	3.38	2.57	1.67	1.20
	30	0.08	0.68	1.49	2.15	2.88	3.40	4.40	4.45	3.38	2.20	1.57
	40	0.12	0.92	2.03	2.93	3.93	4.64	6.00	6.85	5.21	3.38	2.42
35 ⅜	11	0.10	0.77	1.70	2.45	3.30	2.94	1.91	1.37	1.04	0.67	0.48
	15	0.14	1.08	2.38	3.43	4.61	4.68	3.04	2.17	1.65	1.07	0.77
	20	0.19	1.48	3.25	4.68	6.29	7.20	4.68	3.35	2.55	1.65	1.18
	25	0.24	1.88	4.13	5.95	8.00	9.43	6.54	4.68	3.56	2.31	1.65
	30	0.29	2.29	5.03	7.25	9.74	11.5	8.59	6.15	4.68	3.04	2.17
	40	0.39	3.12	6.87	9.89	13.3	15.7	13.2	9.47	7.20	4.68	—
41 ½	11	0.13	1.01	1.71	0.93	(0.58)	0.43	0.28	0.20	0.15	0.10	
	15	0.18	1.41	2.73	1.49	(0.76)	0.69	0.45	0.32	0.24	0.16	
	20	0.24	1.92	4.20	2.29	(1.41)	1.06	0.69	0.49	0.38		
	25	0.31	2.45	5.38	3.20	(1.97)	1.49	0.96	0.69	0.53		
	30	0.38	2.98	6.55	4.20	(2.58)	1.95	1.27	0.91	0.69		
	40	0.51	4.07	8.94	6.47	(3.97)	3.01	1.95	1.40			
40 ½	11	0.23	1.83	4.03	4.66	(3.56)	2.17	1.41	1.01	0.77	0.50	
	15	0.32	2.56	5.64	7.43	(4.56)	3.45	2.24	1.60	1.22	0.79	
	20	0.44	3.50	7.69	11.1	(7.03)	5.31	3.45	2.47	1.88		
	25	0.56	4.45	9.78	14.1	(9.83)	7.43	4.82	3.45	2.63		
	30	0.68	5.42	11.9	17.2	(12.9)	9.76	6.34	4.54	3.45		
	40	0.93	7.39	16.3	23.4	(19.9)	15.0	9.76	6.99			
50 ⅝	11	0.45	3.57	7.85	5.58	(3.43)	2.59	1.68	1.41	1.20	0.92	
	15	0.63	4.99	11.0	8.88	(5.46)	4.13	2.68	2.25	1.92		
	20	0.86	6.80	15.0	13.7	(8.40)	6.35	4.13	3.46	2.95		
	25	1.09	8.66	19.0	19.1	(11.7)	8.88	5.77	4.83			
	30	1.33	10.5	23.2	25.1	(15.4)	11.7	7.58				
	40	1.81	14.4	31.6	38.7	(23.7)	18.0					

Table 8.2.58 Selected Values of Horsepower Ratings of Roller Chains (Continued)

ANSI no. and pitch, in	Number of teeth in small sprocket	Small sprocket, r/min										
		10	50	100	200	500	700	1,000	1,400	2,000	2,700	4,000
60 $\frac{3}{4}$	11	0.18	0.77	1.44	2.69	6.13	8.30	11.4	9.41	5.51	(3.75)	1.95
	15	0.25	1.08	2.01	3.76	8.57	11.6	16.0	15.0	8.77	(6.18)	3.10
	20	0.35	1.47	2.75	5.13	11.7	15.8	21.8	23.1	13.5	(9.20)	
	25	0.44	1.87	3.50	6.52	14.9	20.1	27.8	32.2	18.9	(12.9)	
	30	0.54	2.28	4.26	7.94	18.1	24.5	33.8	42.4	24.8	(16.9)	
	40	0.73	3.11	5.81	10.8	24.7	33.5	46.1	62.5	38.2		
80 1	11	0.42	1.80	3.36	6.28	14.3	19.4	19.6	11.8	6.93	4.42	
	15	0.59	2.52	4.70	8.77	20.0	27.1	31.2	18.9	11.0	7.04	
	20	0.81	3.44	6.41	12.0	27.3	37.0	48.1	29.0	17.0		
	25	1.03	4.37	8.16	15.2	34.7	47.0	64.8	40.6	23.8		
	30	1.25	5.33	9.94	18.5	42.3	57.3	78.9	53.3	31.2		
	40	1.71	7.27	13.6	25.3	57.7	78.1	108	82.1	48.1		
100 $1\frac{1}{4}$	11	0.81	3.45	6.44	12.0	27.4	37.1	23.4	14.2	8.29		
	15	1.13	4.83	9.01	16.8	38.3	51.9	37.3	22.5	13.2		
	20	1.55	6.58	12.3	22.9	52.3	70.8	57.5	34.7	20.3		
	25	1.97	8.38	15.5	29.2	66.6	90.1	80.3	48.5	28.4		
	30	2.40	10.2	19.0	35.5	81.0	110	106	63.7	10.0		
	40	3.27	13.9	26.0	48.5	111	150	163	98.1			
120 $1\frac{1}{2}$	11	1.37	5.83	10.9	20.3	46.3	46.3	27.1	16.4	9.59		
	15	1.91	8.15	15.2	28.4	64.7	73.8	43.2	26.1			
	20	2.61	11.1	20.7	38.7	88.3	114	66.5	40.1			
	25	3.32	14.1	26.4	49.3	112	152	92.9	56.1			
	30	4.05	17.2	32.1	60.0	137	185	122	73.8			
	40	5.52	23.5	43.9	81.8	187	253	188	59.5			
140 $1\frac{3}{4}$	11	2.12	9.02	16.8	31.4	71.6	52.4	30.7	18.5			
	15	2.96	12.6	23.5	43.9	100	83.4	48.9	29.5			
	20	4.04	17.2	32.1	59.9	137	128	75.2	45.4			
	25	5.14	21.9	40.8	76.2	174	180	105	63.5			
	30	6.26	26.7	49.7	92.8	212	236	138				
	40	8.54	36.4	67.9	127	289	363	213				
160 2	11	3.07	13.1	24.4	45.6	96.6	58.3	34.1				
	15	4.30	18.3	34.1	63.7	145	92.8	54.4				
	20	5.86	25.0	46.4	86.9	198	143	83.7				
	25	7.40	31.8	59.3	111	252	200	117				
	30	9.08	38.7	72.2	125	307	263	154				
	40	12.4	52.8	98.5	184	419	404					
180 $2\frac{1}{4}$	11	4.24	18.1	33.7	62.9	106	64.1	37.5				
	15	5.93	25.3	47.1	88.0	169	102	59.7				
	20	8.10	34.5	64.3	120	260	157	92.0				
	25	10.3	43.9	81.8	153	348	220					
	30	12.5	53.4	99.6	186	424	289					
	40	17.1	72.9	136	254	579	398					
200 $2\frac{1}{2}$	11	5.64	24.0	44.8	83.5	115						
	15	7.88	33.5	62.6	117	184						
	20	10.7	45.8	85.4	159	283						
	25	13.7	58.2	109	203	396						
240 3	11	9.08	38.6	72.1	135							
	15	12.7	54.0	101	188							
	20	17.3	73.7	138	257							
	25	22.0	93.8	175	327							

NOTE: The sections separated by heavy lines denote the method of lubrication as follows: type A (left section), manual; type B (middle section), bath or disk; type C (right section), oil stream.  
SOURCE: Supplementary section of ANSI B29.1-1975 (R93), adapted by permission.

to take up slack, to guide the chain around obstructions, to change the direction of rotation of a driven shaft, or to provide more wrap on another sprocket. Idlers should not run faster than the speeds recommended as maximum for other sprockets with the same number of teeth. It is desirable that idlers have at least two teeth in mesh with the chain, and it is advisable, though not necessary, to have an idler contact the idle span of chain.

**Horsepower ratings** are based upon the number of teeth and the rotational speed of the smaller sprocket, either driver or follower. The pin-bushing bearing area, as it affects allowable working load, is the important factor for medium and higher speeds. For extremely slow speeds, the chain selection may be based upon the ultimate tensile strength of the chain. For chain speeds of 25 ft/min and less, the chain pull should be not more than  $\frac{1}{5}$  of the ultimate tensile strength; for 50 ft/min,  $\frac{1}{6}$ ; for

100 ft/min,  $\frac{1}{7}$ ; for 150 ft/min,  $\frac{1}{8}$ ; for 200 ft/min,  $\frac{1}{6}$ ; and for 250 ft/min,  $\frac{1}{10}$  of the ultimate tensile strength.

Ratings for **multiple-strand chains** are proportional to the number of strands. The recommended numbers of strands for multiple chains are 2, 3, 4, 6, 8, 10, 12, 16, 20, and 24, with the maximum overall width in any case limited to 24 in.

The **horsepower ratings** in Table 8.2.58 are modified by the **service factors** in Table 8.2.59. Thus for a drive having a nominal rating of 3 hp, subject to heavy shock, abnormal conditions, 24-h/day operation, the chain rating obtained from Table 8.2.58 should be at least  $3 \times 1.7 = 5.1$  hp.

Table 8.2.59 Service Factors for Roller Chains

Power source	Load		
	Smooth	Moderate shock	Heavy shock
Internal combustion engine with hydraulic drive	1.0	1.2	1.4
Electric motor or turbine	1.0	1.3	1.5
Internal combustion engine with mechanical drive	1.2	1.4	1.7

SOURCE: ANSI B29.1-1975, adapted by permission.

**Chain-Length Calculations** Referring to Fig. 8.2.97,  $L$  = length of chain, in;  $P$  = pitch of chain, in;  $R$  and  $r$  = pitch radii of large and small sprockets, respectively, in;  $D$  = center distance, in;  $A$  = tangent length, in;  $a$  = angle between tangent and centerline;  $N$  and  $n$  = number

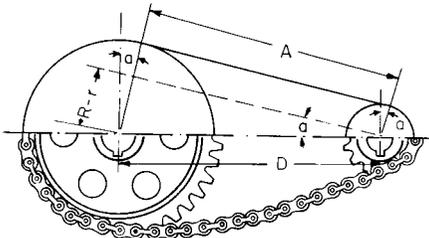


Fig. 8.2.97 Symbols for chain length calculations.

of teeth on larger and smaller sprockets, respectively;  $180 + 2a$  and  $180 - 2a$  are angles of contact on larger and smaller sprockets, respectively, deg.

$$a = \sin^{-1}[(R - r)/D] \quad A = D \cos a$$

$$L = NP(180 + 2a)/360 + nP(180 - 2a)/360 + 2D \cos a$$

If  $L_p$  = length of chain in pitches and  $D_p$  = center distance in pitches,

$$L_p = (N + n)/2 + a(N - n)/180 + 2D_p \cos a$$

Avoiding the use of trigonometric tables,

$$L_p = 2C + (N + n)/2 + K(N - n)^2/C$$

where  $C$  is the center distance in pitches and  $K$  is a variable depending upon the value of  $(N - n)/C$ . Values of  $K$  are as follows:

$(N - n)/C$	0.1	1.0	2.0	3.0
$K$	0.02533	0.02538	0.02555	0.02584
$(N - n)/C$	4.0	5.0	6.0	
$K$	0.02631	0.02704	0.02828	

Formulas for chain length on multisprocket drives are cumbersome except when all sprockets are the same size and on the same side of the chain. For this condition, the chain length in pitches is equal to the sum of the consecutive center distances in pitches plus the number of teeth on one sprocket.

Actual chain lengths should be in even numbers of pitches. When necessary, an odd number of pitches may be secured by the use of an offset link, but such links should be avoided if possible. An offset link is one pitch; half roller link at one end and half pin link at the other end. If center distances are to be nonadjustable, they should be selected to give an initial snug fit for an even number of pitches of chain. For the average application, a center distance equal to  $40 \pm 10$  pitches of chain represents good practice.

There should be at least  $120^\circ$  of wrap in the arc of contact on a power sprocket. For ratios of 3 : 1 or less, the wrap will be  $120^\circ$  or more for any center distance or number of teeth. To secure a wrap of  $120^\circ$  or more, for ratios greater than 3 : 1, the center distance must be not less than the difference between the pitch diameters of the two sprockets.

**Sprocket Diameters**  $N$  = number of teeth;  $P$  = pitch of chain, in;  $D$  = diameter of roller, in. The pitch of a standard roller chain is measured from the center of a pin to the center of an adjacent pin.

$$\text{Pitch diam} = P/\sin \frac{180}{N}$$

$$\text{Bottom diam} = \text{pitch diam} - D$$

$$\text{Outside diam} = P \left( 0.6 + \cot \frac{180}{N} \right)$$

$$\text{Caliper diam} = \left( \text{pitch diam} \times \cos \frac{90}{N} \right) - D$$

The exact bottom diameter cannot be measured for an odd number of teeth, but it can be checked by measuring the distance (caliper diameter) between bottoms of the two tooth spaces nearest opposite to each other. Bottom and caliper diameters must not be oversize—all tolerances must be negative. ANSI negative tolerance =  $0.003 + 0.001P\sqrt{N}$  in.

**Design of Sprocket Teeth for Roller Chains** The section profile for the teeth of roller chain sprockets, recommended by ANSI, has the proportions shown in Fig. 8.2.98. Let  $P$  = chain pitch;  $W$  = chain width (length of roller);  $n$  = number of strands of multiple chain;  $M$  = overall width of tooth profile section;  $H$  = nominal thickness of the link plate, all in inches. Referring to Fig. 8.2.98,  $T = 0.93W - 0.006$ , for

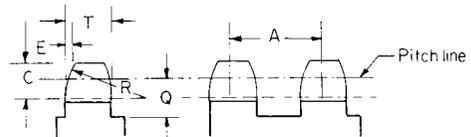


Fig. 8.2.98 Sprocket tooth sections.

single-strand chain; =  $0.90W - 0.006$ , for double- and triple-strand chains; =  $0.88W - 0.006$ , for quadruple- or quintuple-strand chains; and =  $0.86W - 0.006$ , for sextuple-strand chain and over.  $C = 0.5P$ .  $E = \frac{1}{8}P$ .  $R(\text{min}) = 1.063P$ .  $Q = 0.5P$ .  $A = W + 4.15H + 0.003$ .  $M = A(n - 1) + T$ .

**Inverted-tooth (silent) chain drives** have a typical tooth form shown in Fig. 8.2.99. Such chains should be operated in an oil-retaining casing with provisions for lubrication. The use of offset links and chains with an uneven number of pitches should be avoided.

**Horsepower ratings** per inch of silent chain width, given in ANSI B29.2-1957 (R1971), for various chain pitches and speeds, are shown in

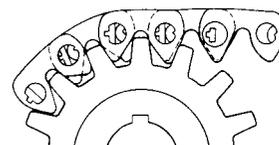


Fig. 8.2.99 Inverted tooth (silent-chain) drive.

Table 8.2.60 Horsepower Rating per Inch of Chain Width, Silent-Chain Drive (Small Pitch)

Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min												
		500	600	700	800	900	1,200	1,800	2,000	3,500	5,000	7,000	9,000	
$\frac{3}{16}$	21	0.41	0.48	0.55	0.62	0.68	0.87	1.22	1.33	2.03	2.58	3.12	3.35	
	25	0.49	0.58	0.66	0.74	0.82	1.05	1.47	1.60	2.45	3.13	3.80	4.10	
	29	0.57	0.67	0.76	0.86	0.95	1.21	1.70	1.85	2.83	3.61	4.40	4.72	
	33	0.64	0.75	0.86	0.97	1.07	1.37	1.90	2.08	3.17	4.02	4.85	—	
	37	0.71	0.84	0.96	1.08	1.19	1.52	2.11	2.30	3.48	4.39	5.24	—	
	45	0.86	1.02	1.15	1.30	1.43	1.83	2.53	2.75	4.15	5.21	—	—	
Type*		I					II			III				
Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min												
		100	500	1,000	1,200	1,500	1,800	2,000	2,500	3,000	3,500	4,000	5,000	6,000
$\frac{3}{8}$	21	0.58	2.8	5.1	6.0	7.3	8.3	9.0	10	11	12	12	12	10
	25	0.69	3.3	6.1	7.3	8.8	10	11	13	14	15	5	15	14
	29	0.80	3.8	7.3	8.5	10	12	13	15	16	18	19	19	18
	33	0.90	4.4	8.3	9.8	12	14	15	18	19	21	21	21	20
	37	1.0	4.9	9.1	11	14	15	16	20	21	24	24	24	—
	45	1.3	6.0	11	13	16	19	20	24	26	28	29	—	—
Type*		I			II				III					
Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min												
		100	500	700	1,000	1,200	1,800	2,000	2,500	3,000	3,500	4,000		
$\frac{1}{2}$	21	1.0	5.0	6.3	8.8	10	14	14	15	16	16	—		
	25	1.2	5.0	7.5	10	13	16	18	20	21	21	20		
	29	1.4	6.3	8.8	13	14	19	21	24	25	25	25		
	33	1.6	7.5	10	14	16	23	24	28	29	30	29		
	37	1.9	8.8	11	16	19	25	26	30	33	33	—		
	45	2.5	10	14	19	23	30	30	36	39	—	—		
Type*		I		II				III						
Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min												
		100	500	700	1,000	1,200	1,800	2,000	2,500	3,000	3,500			
$\frac{5}{8}$	21	1.6	7.5	10	13	15	19	20	20	20	—			
	25	1.9	8.8	11	16	19	24	25	26	26	24			
	29	2.1	10	14	19	21	28	30	31	31	29			
	33	2.5	11	16	21	25	33	34	36	36	34			
	37	2.8	13	18	24	28	36	39	43	41	—			
	45	3.4	16	21	29	34	44	46	—	—	—			
Type*		I		II			III							
Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min												
		100	500	700	1,000	1,200	1,500	1,800	2,000	2,500				
$\frac{3}{4}$	21	2.3	10	14	18	20	23	24	25	24				
	25	2.8	13	16	21	25	29	31	31	30				
	29	3.1	15	20	26	30	34	36	38	38				
	33	3.6	16	23	30	34	39	43	44	44				
	37	4.0	19	25	34	39	44	48	49	49				
	45	4.9	23	30	40	46	53	56	58	—				
Type*		I		II			III							

\* Type I: manual, brush, or oil cup. Type II: bath or disk. Type III: circulating pump.

NOTE: For best results, smaller sprocket should have at least 21 teeth.

SOURCE: Adapted from ANSI B29.2M-1982.

Tables 8.2.60 and 8.2.61. These ratings are based on ideal drive conditions with relatively little shock or load variation, an average life of 20,000 h being assumed. In utilizing the horsepower ratings of the tables, the nominal horsepower of the drive should be multiplied by a service factor depending on the application. Maximum, or worst-case scenario, service factors are listed in Table 8.2.62.

For details on lubrication, sprocket dimensions, etc., see ANSI B29.2M-1957(R71). In utilizing Tables 8.2.60 and 8.2.61 (for a complete set of values, see ANSI B29.2M-1982) the required chain width is obtained by dividing the design horsepower by the horsepower ratings given. For calculating silent-chain lengths, the procedure for roller-chain drives may be used.

Table 8.2.61 Horsepower Rating per Inch of Chain Width, Silent-Chain Drive (Large Pitch)

Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min										
		100	200	300	400	500	700	1,000	1,200	1,500	1,800	2,000
1	21	3.8	7.5	11	15	18	23	29	31	33	33	—
	25	5.0	8.8	14	18	21	28	35	39	41	41	41
	29	5.0	11	16	20	25	33	11	46	50	51	50
	33	6.3	13	18	24	29	38	49	54	59	59	58
	37	6.8	14	20	26	33	43	54	60	65	66	—
	45	8.8	16	25	31	39	51	65	71	76	—	—
Type*		I			II			III				
Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min										
		100	200	300	400	500	600	700	800	1,000	1,200	1,500
1 1/4	21	6.3	11	18	23	26	30	33	36	40	41	—
	25	7.5	14	20	26	31	36	40	44	50	53	53
	29	8.6	16	24	31	38	43	48	53	59	63	64
	33	9.9	19	28	35	43	49	55	60	69	73	74
	37	11	21	30	40	48	55	63	68	76	81	—
	45	13	26	38	49	59	68	75	81	91	—	—
Type*		I			II			III				
Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min										
		100	200	300	400	500	600	700	800	900	1,000	1,200
1 1/2	21	8.8	16	24	30	36	40	44	46	49	49	—
	25	10	20	29	38	44	50	55	59	61	65	64
	29	13	24	34	44	51	59	65	70	74	75	76
	33	14	28	39	50	59	68	75	80	85	88	89
	37	16	30	44	59	66	76	84	90	96	99	—
	45	19	38	54	68	81	93	101	108	113	—	—
Type*		I			II			III				
Pitch, in	No. of teeth in small sprocket	Small sprocket, r/min										
		100	200	300	400	500	600	700	800	900	1,000	1,200
2	21	16	29	40	50	53	63	65	—	—	—	
	25	18	35	49	61	70	78	83	85	85	85	
	29	21	41	58	73	84	93	99	103	103	103	
	33	25	46	66	83	96	106	114	118	118	118	
	37	28	53	75	92	110	124	128	131	—	—	
	45	34	64	90	113	131	144	151	—	—	—	
Type*		I			II			III				

\* Type I: manual, brush, or oil cup. Type II: bath or disk. Type III: circulating pump.

NOTE: For best results, small sprocket should have at least 21 teeth.

SOURCE: Adapted from ANSI B29.2-1982.

Table 8.2.62 Service Factors\* for Silent-Chain Drives

Application	Fluid-coupled engine or electric motor		Engine with straight mechanical drive		Torque converter drives		Application	Fluid-coupled engine or electric motor		Engine with straight mechanical drive		Torque converter drives	
	10 h	24 h	10 h	24 h	10 h	24 h		10 h	24 h	10 h	24 h	10 h	24 h
Agitators	1.1	1.4	1.3	1.6	1.5	1.8	Line shafts	1.6	1.9	1.8	2.1	2.0	2.3
Brick and clay machinery	1.4	1.7	1.6	1.9	1.8	2.1	Machine tools	1.4	1.7	—	—	—	—
Centrifuges	1.4	1.7	1.6	1.9	1.8	2.1	Mills—ball, hardinge, roller, etc.	1.6	1.9	1.8	2.1	—	—
Compressors	1.6	1.9	1.8	2.1	2.0	2.3	Mixer—concrete, liquid	1.6	1.9	1.8	2.1	2.0	2.3
Conveyors	1.6	1.9	1.8	2.1	2.0	2.3	Oil field machinery	1.6	1.9	1.8	2.1	2.0	2.3
Cranes and hoists	1.4	1.7	1.6	1.9	1.8	2.1	Oil refinery equipment	1.5	1.8	1.7	2.0	1.9	2.2
Crushing machinery	1.6	1.9	1.8	2.1	2.0	2.3	Paper machinery	1.5	1.8	1.7	2.0	1.9	2.2
Dredges	1.6	1.9	1.8	2.1	2.0	2.3	Printing machinery	1.5	2.0	1.4	1.7	1.6	1.9
Elevators	1.4	1.7	1.6	1.9	1.8	2.1	Pumps	1.6	1.9	1.8	2.1	2.0	2.3
Fans and blowers	1.5	1.8	1.7	2.0	1.9	2.2	Rubber plant machinery	1.5	1.8	1.7	2.0	1.9	2.2
Mills—flour, feed, cereal	1.4	1.7	1.6	1.9	1.8	2.1	Rubber mill equipment	1.6	1.9	1.8	2.1	2.0	2.3
Generator and exciters	1.2	1.5	1.4	1.7	1.6	1.9	Screens	1.5	1.8	1.7	2.0	1.9	2.2
Laundry machinery	1.2	1.5	1.4	1.7	1.6	1.9	Steel plants	1.3	1.6	1.5	1.8	1.7	2.0
							Textile machinery	1.1	1.4	—	—	—	—

\* The values shown are for the maximum worst-case scenario for each application. The table was assembled from ANSI B29.2M-1982, by permission. Because the listings above are maximum

## ROTARY AND RECIPROCATING ELEMENTS

## Slider Crank Mechanism

**Kinematics** The slider crank mechanism is widely used in automobile engines, punch presses, feeding mechanisms, etc. For such mechanisms, displacements, velocities, and accelerations of the parts are important design parameters. The basic mechanism is shown in Fig. 8.2.100. The slider displacement  $x$  is given by

$$x = r \cos \theta + l \sqrt{1 - [(r/l) \sin \theta]^2}$$

where  $r$  = crank length,  $l$  = connecting-rod length,  $\theta$  = crank angle measured from top dead center position. Slider velocity is given by

$$\dot{x} = V = -r\omega \left( \sin \theta + \frac{r \sin 2\theta}{2l \cos \beta} \right)$$

where  $\omega$  = instantaneous angular velocity of the crank at position  $\theta$  and

$$\cos \beta = \sqrt{1 - (r/l)^2 \sin^2 \theta}$$

Slider acceleration is given by

$$a = \ddot{x} = -r\alpha \left( \sin \theta + \frac{r \sin 2\theta}{l \cos \beta} \right) - r\omega^2 \left( \cos \theta + \frac{r \cos 2\theta}{l \cos \beta} + \frac{r \sin^2 2\theta}{l^3 4 \cos^3 \beta} \right)$$

where  $\alpha$  = instantaneous angular acceleration of the crank at position  $\theta$ .

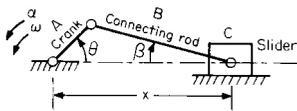


Fig. 8.2.100 Basic slider-crank mechanism.

**Forces** Neglecting gravity effects, the forces in a mechanism arise from those produced by input and output forces or torques (herewith called static components). Such forces may be produced by driving motors, shaft loads, expanding cylinder gases, etc. The net forces on the various links cause accelerations of the mechanism's masses, and can be thought of as dynamic components. The static components must be borne by the various links, thus giving rise to internal stresses in those parts. The supporting bearings and slide surfaces also feel the effects of these components, as do the support frames. Stresses are also induced by the dynamic components in the links, and such components cause shaking forces and shaking moments in the support frame.

By building onto the basic mechanism appropriately designed counterweights, the support frame and its bearings can be relieved of a significant portion of the dynamic component effects, sometimes called inertia effects. The augmented mechanism is then considered to be "balanced." The static components cannot be relieved by any means, so that the support frame and its bearings must be designed to carry safely the static component forces and not be overstressed. Figure 8.2.101 illustrates a common, simple form of approximate balancing. Sizing of the counterweight is somewhat complicated because the total

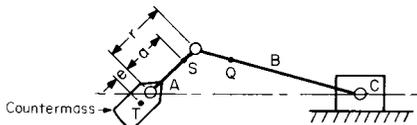


Fig. 8.2.101 "Balanced" slider-crank mechanism where  $T$  = center of mass,  $S$  = center of mass of crank  $A$ , and  $Q$  = center of mass of connecting rod  $B$ .

inertia effect is the vector sum of the separate link inertias, which change in magnitude and direction at each position of the crank. The counterweight  $D$  is sized to contain sufficient mass to completely balance the crank plus an additional mass (effective mass) to "balance" the other links (connecting rod and slider). In simple form, the satisfying

condition is approximately

$$M_D e \omega^2 = M_A a \omega^2 + M_{\text{eff}} e \omega^2$$

where  $a$  = distance from crankshaft to center of mass of crank (note that most often  $a$  is approximately equal to the crank radius  $r$ );  $e$  = distance from crankshaft to center of mass of counterweight;  $M_{\text{eff}}$  = additional mass of counterweight to "balance" connecting rod and slider.

From one-half to two-thirds the slider mass (e.g.,  $\frac{1}{2}M_c \leq M_{\text{eff}} \leq \frac{2}{3}M_c$ ) is usually added to the counterweight for a single-cylinder engine. For critical field work, single and multiple slider crank mechanisms are dynamically balanced by experimental means.

**Forces and Torques** Figure 8.2.102a shows an exploded view of the slider crank mechanism and the various forces and torques on the links (neglecting gravity and weight effects). Inertial effects are shown as broken-line vectors and are manipulated in the same manner as the actual or real forces. The inertial effects are also known as **D'Alembert forces**. The meanings are as follows:  $F_1 = -M_D e \omega^2$ , parallel to crank  $A$ ;  $F_2 = -M_a e a$ , perpendicular to crank  $A$ ;  $F_3 = (z/l)F_9$ ;  $F_4$  = as found in Fig. 8.2.102b;  $F_5 = F_2 - F_7$ ;  $F_6 = F_1 - F_8$ ;  $F_7 = -M_A r \omega^2$ , parallel to crank  $A$ ;  $F_8 = -M_A r \alpha$ , perpendicular to crank  $A$ ;  $F_9 = -M_B \times$  absolute acceleration of  $Q$ , where acceleration of point  $Q$  can be found

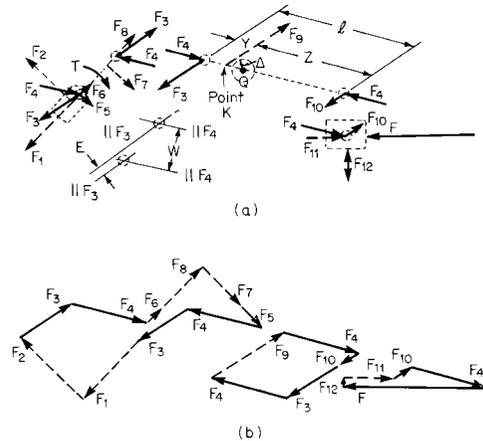


Fig. 8.2.102 (a) Forces and torques; (b) force polygons.

graphically by constructing an acceleration polygon of the mechanism (see for example Shigley, "Kinematic Analysis of Mechanisms," McGraw-Hill);  $F_{10} = (y/l)F_9$ ;  $F_{11} = -M_c \times$  absolute acceleration of slider  $\ddot{x}$ ;  $F_{12}$  = normal wall force (neglecting friction);  $F$  = external force on slider's face, where the vector sum  $F + F_4 + F_{10} + F_{11} + F_{12} = 0$ ;  $T$  = external crankshaft torque, where the algebraic sum  $T + F_3 E + F_4 W + F_2 e + F_7 a = 0$  (note that signs must reflect direction of torque);  $T_t = F_3 E + F_{11} W$  = transmitted torque;  $K$  = the effective location of force  $F_9$ , from the center of mass of connecting rod  $B$  (see Fig. 8.2.102a) is given by  $\Delta = J_{B(\text{cm})} \times$  angular acceleration of link  $B/M_B \times$  absolute acceleration of point  $Q$ . Figure 8.2.102b shows the force polygons of the separate links of the mechanism.

## Flywheel

One can surmise that both  $F$  and  $T$  may be functions of crank angle  $\theta$ . Even if one or the other were deliberately kept constant, the remaining one would still be a function of  $\theta$ . If a steady-speed crank is desired ( $\omega = \text{constant}$  and  $\alpha = \text{zero}$ ), then external crankshaft torque  $T$  must be constantly adjusted to equal transmitted torque  $T_t$ . In such a situation a motor at the crankshaft would suffer fatigue effects. In a combustion engine the crankshaft would deliver a fluctuating torque to its load.

Inserting a flywheel at the crankshaft allows the peak and valley excursions of  $\omega$  to be considerably reduced because of the flywheel's ability to absorb energy over periods when  $T > T_t$  and to deliver back

into the system such excess energy when  $T < T_r$ . Figure 8.2.103a illustrates the above concepts, also showing that over one cycle of a repeated event the excess (+) energies and the deficient (-) energies are equal. The greatest crank speed change tends to occur across a single large positive loop, as illustrated in Fig. 8.2.103a.

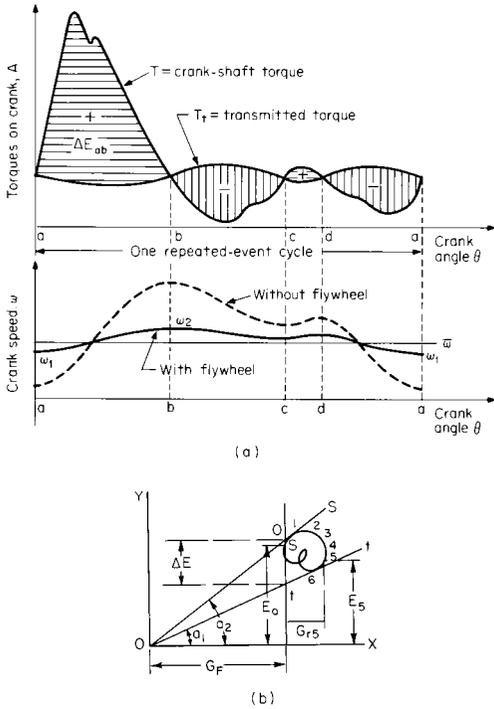


Fig. 8.2.103 Sizing the flywheel. (a) Variation of torque and crank speed vs. crank angle, showing  $\Delta E_{ab}$ ; (b) graphics for Wittenbauer's analysis.

**Sizing the Flywheel** For the single largest energy change we can write

$$\Delta E_{ab} \int_a^b (T - T_r) d\theta = \frac{J_0}{2} (\omega_2^2 - \omega_1^2) = \frac{J_0}{2} (\omega_2 - \omega_1)(\omega_2 + \omega_1)$$

where  $J_0$  = flywheel moment of inertia plus effective mechanism moment of inertia.

Define  $\bar{\omega} \doteq (\omega_2 + \omega_1)/2$  and  $C_s$  = coefficient of speed fluctuation =  $(\omega_2 - \omega_1)/\bar{\omega}$ . Hence  $\Delta E_{ab} = J_0 C_s \bar{\omega}^2$ . Acceptable values of  $C_s$  are:

Pumps	0.03–0.05
Machine tools	0.025–0.03
Looms	0.025
Paper mills	0.025
Spinning mills	0.015
Crusher	0.02
Electric generators, ac	0.003
Electric generators, dc	0.002

Evaluating  $\Delta E_{ab}$  involves finding the integral

$$\int_a^b (T - T_r) d\theta$$

which can be done graphically or by a numerical technique such as Simpson's rule.

**Wittenbauer's Analysis for Flywheel Performance** This method does not involve more computation work than the one described above, but it is more accurate where the reciprocating parts are comparatively heavy. Wittenbauer's method avoids the inaccuracy resulting from the evaluation of the inertia forces on the reciprocating parts on the basis of the uniform nominal speed of rotation for the engine.

Let the crankpin velocity be represented by  $v_r$  and the velocity of any moving masses ( $m_1, m_2, m_3$ , etc.) at any instant of phase be represented, respectively, by  $v_1, v_2, v_3$ , etc. The kinetic energy of the entire engine system of moving masses may then be expressed as

$$E = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2 + m_3 v_3^2 + \dots) = \frac{1}{2} M_r v_r^2$$

or, the single reduced mass  $M_r$  at the crankpin which possesses the equivalent kinetic energy is

$$M_r = [m_1(v_1/v_r)^2 + m_2(v_2/v_r)^2 + m_3(v_3/v_r)^2 + \dots]$$

In an engine mechanism, sufficiently accurate values of  $M_r$  can be obtained if the weight of the connecting rod is divided between the crankpin and the wrist pin so as to retain the center of gravity of the rod in its true position; usually 0.55 to 0.65 of the weight of the connecting rod should be placed on the crankpin, and 0.45 to 0.35 of the weight on the wrist pin.  $M_r$  is a variable in engine mechanisms on account of the reciprocating parts and should be found for a number of crank positions. It should include all moving masses except the flywheel.

The total energy  $E$  used in accelerating reciprocating parts from the beginning of the forward stroke up to any crank position can be obtained by finding from the indicator cards the total work done in the cylinder (on both sides of the piston) up to that time and subtracting from it the work done in overcoming the resisting torque, which may usually be assumed constant. The mean energy of the moving masses is  $E_0 = \frac{1}{2} M_r v_r^2$ .

In Fig. 8.2.103b, the reduced weights of the moving masses  $G_F + G_{r5}$  are plotted on the X axis corresponding to different crank positions.  $G_F = gM_F$  is the reduced flywheel weight and  $G_{r5} = gM_{r5}$  is the sum of the other reduced weights. Against each of these abscissas is plotted the energy  $E$  available for acceleration measured from the beginning of the forward stroke. The curve O123456 is the locus of these plotted points.

The diagram possesses the following property: Any straight line drawn from the origin  $O$  to any point in the curve is a measure of the velocity of the moving masses; tangents bounding the diagram measure the limits of velocity between which the crankpin will operate. The maximum linear velocity of the crankpin in feet per second is  $v_2 = \sqrt{2g} \tan \alpha_2$ , and the minimum velocity is  $v_1 = \sqrt{2g} \tan \alpha_1$ . Any desired change in  $v_1$  and  $v_2$  may be accomplished by changing the value of  $G_F$ , which means a change in the flywheel weight or a change in the flywheel weight reduced to the crankpin. As  $G_F$  is very large compared with  $G_r$  and the point  $O$  cannot be located on the diagram unless a very large drawing is made, the tangents are best formed by direct calculation:

$$\tan \alpha_2 = \frac{v_r^2}{2g} (1 + k) \quad \tan \alpha_1 = \frac{v_r^2}{2g} (1 - k)$$

where  $k$  is the coefficient of velocity fluctuation. The two tangents  $ss$  and  $tt$  to the curve O123456, thus drawn, cut a distance  $\Delta E$  and on the ordinate  $E_0$ . The reduced flywheel weight is then found to be

$$G_F(\Delta E)g/(v_r^2 k)$$

**SPRINGS**

It is assumed in the following formulas that the springs are in no case stressed beyond the elastic limit (i.e., that they are perfectly elastic) and that they are subject to Hooke's law.

**Notation**

- $P$  = safe load, lb
- $f$  = deflection for a given load  $P$ , in
- $l$  = length of spring, in
- $V$  = volume of spring, in<sup>3</sup>
- $S_s$  = safe stress (due to bending), lb/in<sup>2</sup>
- $S_v$  = safe shearing stress, lb/in<sup>2</sup>
- $U$  = resilience, in · lb

For sheet metal and wire gages, ferrous and nonferrous, see Table 8.2.76 and metal suppliers' catalogs.

The work in inch-pounds performed in deflecting a spring from 0 to  $f$  (spring duty) is  $U = Pf/2 = s_s^2V/(CE)$ . This is based upon the assumption that the deflection is proportional to the load, and  $C$  is a constant dependent upon the shape of the springs.

The time of vibration  $T$  (in seconds) of a spring (weight not considered) is equal to that of a simple circular pendulum whose length  $l_0$  equals the deflection  $f$  (in feet that is produced in the spring by the load  $P$ .  $T = \pi\sqrt{l_0/g}$ , where  $g$  = acceleration of gravity, ft/s<sup>2</sup>.

**Springs Subjected to Bending**

1. **Rectangular plate spring** (Fig. 8.2.104).

$$P = bh^2S_s/(6l) \quad I = bh^3/12 \quad U = Pf/2 = VS_s^2/(18E)$$

$$f = Pl^3/(3EI) = 4Pl^3/(bh^3E) = 2l^2S_s/(3hE)$$

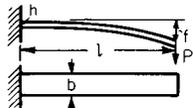


Fig. 8.2.104 Rectangular plate spring.

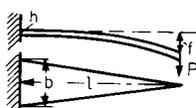


Fig. 8.2.105 Triangular plate spring.

2. **Triangular plate spring** (Fig. 8.2.105). The elastic curve is a circular arc.

$$P = bh^2S_s/(6l) \quad I = bh^3/12 \quad U = Pf/2 = S_s^2V/(6E)$$

$$f = Pl^3/(2EI) = 6Pl^3/(bh^3E) = l^2S_s/(hE)$$

3. **Rectangular plate spring with end tapered in the form of a cubic parabola** (Fig. 8.2.106). The elastic curve is a circular arc;  $P$ ,  $l$ , and  $f$  same as for triangular plate spring (Fig. 8.2.105);  $U = Pf/2 = S_s^2V/(9E)$ .

The strength and deflection of single-leaf flat springs of various forms

are given (Bruce, *Am. Mach.*, July 19, 1900) by the formulas  $h = al^2/f$  and  $b = cPl/h^2$ . The volume of the spring is given by  $V = vlbh$ . The values of constants  $a$  and  $c$  and the resilience in inch-pounds per cubic inch are given in Table 8.2.63, in terms of the safe stress  $S_s$ . Values of  $v$  are given also.

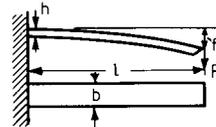


Fig. 8.2.106 Rectangular plate spring: tapered end.

4. **Compound (leaf or laminated) springs**. If several springs of rectangular section are combined, the resulting compound spring should (1) form a beam of uniform strength that (2) does not open between the joints while bending (i.e., elastic curve must be a circular arc). Only the type immediately following meets both requirements, the others meeting only the second requirement.

5. **Laminated triangular plate spring** (Fig. 8.2.107). If the triangular plate spring shown at I is cut into an even number ( $= 2n$ ) of strips of equal width (in this case eight strips of width  $b/2$ ), and these strips are combined, a laminated spring will be formed whose carrying capacity will equal that of the original uncut spring; or  $P = nbh^2S_s/(6l)$ ;  $n = 6Pl/(bh^2S_s)$ .

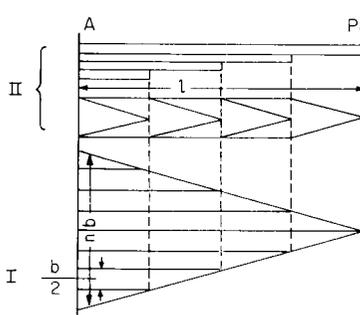


Fig. 8.2.107 Laminated triangular plate spring.

Table 8.2.63 Strength and Deflection of Single-Leaf Flat Springs

Load applied at end of spring; $c = 6/S_s$				Load applied at center of spring; $c = 6/4S_s$			
Plans and elevations of springs	$a$	$U$	$v$	Plans and elevations of springs	$a$	$U$	$v$
	$\frac{S_s}{E}$	$\frac{S_s^2}{6E}$	$\frac{1}{2}$		$\frac{S_s}{4E}$	$\frac{S_s^2}{6E}$	$\frac{1}{2}$
	$\frac{4S_s}{3E}$	$\frac{S_s^2}{6E}$	$\frac{2}{3}$		$\frac{0.87S_s}{4E}$	$\frac{0.70S_s^2}{6E}$	$\frac{5}{8}$
	$\frac{2S_s}{3E}$	$\frac{0.33S_s^2}{6E}$	1		$\frac{S_s}{3E}$	$\frac{S_s^2}{6E}$	$\frac{2}{3}$
	$\frac{0.87S_s}{E}$	$\frac{0.70S_s^2}{6E}$	$\frac{5}{8}$		$\frac{1.09S_s}{4E}$	$\frac{0.725S_s^2}{6E}$	$\frac{3}{4}$
	$\frac{1.09S_s}{E}$	$\frac{0.725S_s^2}{6E}$	$\frac{3}{4}$		$\frac{S_s}{6E}$	$\frac{0.33S_s^2}{6E}$	1

6. **Laminated rectangular plate spring with lead ends tapered in the form of a cubical parabola** (Fig. 8.2.108); see case 3.

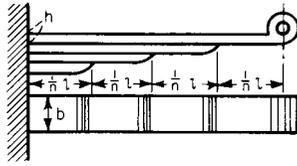


Fig. 8.2.108 Laminated rectangular plate spring with leaf end tapered.

7. **Laminated trapezoidal plate spring with leaf ends tapered** (Fig. 8.2.109). The ends of the leaves are trapezoidal and are tapered according to the formula

$$z = \frac{h}{\sqrt[3]{1 + (b_1/b)(a/x - 1)}}$$

8. **Semielliptic springs** (for locomotives, trucks, etc.). Referring to Fig. 8.2.110, the load  $2P$  ( $lb$ ) acting on the spring center band produces a tensional stress  $P/\cos a$  in each of the inclined shackle links. This is resolved into the vertical force  $P$  and the horizontal force  $P \tan a$ , which together produce a bending moment  $M = P(l + p \tan a)$ . The equations

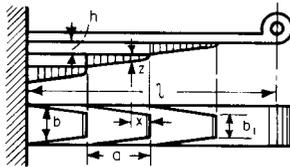


Fig. 8.2.109 Laminated trapezoidal plate spring with leaf ends tapered.

given in (1), (2), and (3) apply to curved as well as straight springs. The bearing force  $= 2P = (2nbh^2/6)[S_s/(l + p \tan a)]$ , and the deflection  $= [6l^2/(nbh^3)]P(l + p \tan a)/E = l^2S_2/(hE)$ .

In addition to the bending moment, the leaves are subjected to the tension force  $P \tan a$  and the transverse force  $P$ , which produce in the upper leaf an additional stress  $S = P \tan a/(bh)$ , as well as a transverse shearing stress.

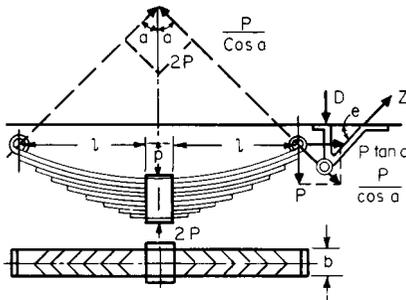


Fig. 8.2.110 Semielliptic springs.

In determining the number of leaves  $n$  in a given spring, allowance should be made for an excess load on the spring caused by the vibration. This is usually done by decreasing the allowable stress about 15 percent.

The foregoing does not take account of initial stresses caused by the band. For more detailed information, see Wahl, "Mechanical Springs," Penton.

9. **Elliptic springs.** Safe load  $P = nbh^2S_s/(6l)$ , where  $l = 1/2$  distance

between bolt eyes (less  $1/2$  length of center band, where used); deflection  $f = 4l^2S_sK/(hE)$ , where

$$K = \frac{1}{(1-r)^3} \left[ \frac{1-r^2}{2} - 2r(1-r) - r^2 \ln r \right]$$

$r$  being the number of full-length leaves  $\div$  total number ( $n$ ) of leaves in the spring. All dimensions in inches. For semielliptic springs, the deflection is only half as great. Safe load  $= nbh^2S_s/(3l)$ . (Peddle, *Am. Mach.*, Apr. 17, 1913.)

**Coiled Springs** In these, the load is applied as a couple  $Pr$  which turns the spring while winding or holds it in place when wound up. If the spindle is not to be subjected to bending moment,  $P$  must be replaced by two equal and opposite forces ( $P/2$ ) acting at the circumference of a circle of radius  $r$ . The formulas are the same in both cases. The springs are assumed to be fixed at one end and free at the other. The bending moment acting on the section of least resistance is always  $Pr$ . The length of the straightened spring  $= l$ . See Benjamin and French, Experiments on Helical Springs, *Trans. ASME*, 23, p. 298.

For heavy closely coiled helical springs the usual formulas are inaccurate and result in stresses greatly in excess of those assumed. See Wahl, Stresses in Heavy Closely-Coiled Helical Springs, *Trans. ASME*, 1929. In springs 10 to 12 and 15 to 18, the quantity  $k$  is unity for lighter springs and has the stated values (supplied by Wahl) for heavy closely coiled springs.

10. **Spiral coiled springs of rectangular cross section** (Fig. 8.2.111).

$$P = bh^2S_s/(6rk) \quad I = bh^3/12 \quad U = Pf/2 = S_s^2V/(8Ek^2) \\ f = ra = Plr^2/(EI) = 12Plr^2/(Ebh^3) = 2rIS_s/(hEk)$$

For heavy closely coiled springs,  $k = (3c - 1)/(3c - 3)$ , where  $c = 2R/h$  and  $R$  is the minimum radius of curvature at the center of the spiral.

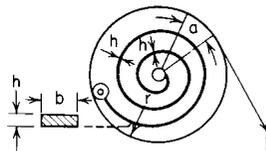


Fig. 8.2.111 Spiral coiled spring: rectangular cross section.

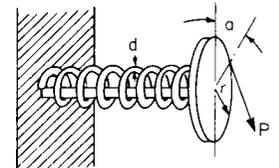


Fig. 8.2.112 Cylindrical helical spring: circular cross section.

11. **Cylindrical helical spring of circular cross section** (Fig. 8.2.112).

$$P = \pi d^3S_s/(32rk) \quad I = \pi d^4/64 \quad U = Pf/2 = S_s^2V/(8Ek^2) \\ f = ra = Plr^2/(EI) = 64Plr^2/(\pi Ed^4) = 2rIS_s/(dEk)$$

For heavy closely coiled springs,  $k = (4c - 1)/(4c - 4)$ , where  $c = 2R/d$ .

12. **Cylindrical helical spring of rectangular cross section** (Fig. 8.2.113).

$$P = bh^2S_s/(6rk) \quad I = bh^3/12 \quad U = Pf/2 = S_s^2V/(8Ek^2) \\ f = ra = Plr^2/(EI) = 12Plr^2/(Ebh^3) = 2rIS_s/(hEk)$$

For heavy closely coiled springs,  $k = (3c - 1)/(3c - 3)$ , where  $c = 2r/h$ .

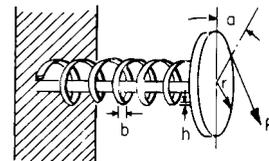


Fig. 8.2.113 Cylindrical helical spring: rectangular cross section.

**Springs Subjected to Torsion**

The statements made concerning coiled springs subjected to bending apply also to springs 13 and 14.

13. **Straight bar spring of circular cross section** (Fig. 8.2.114).

$$P = \pi d^3 S_v / (16r) = 0.1963 d^3 S_v / r \quad U = P f / 2 = S_v^2 V / (4G)$$

$$f = ra = 32 r^2 P / (\pi d^4 G) = 2 r l S_v / (dG)$$

14. **Straight bar spring of rectangular cross section** (Fig. 8.2.115).

$$P = 2 b^2 h S_v / (9r) \quad K = b/h$$

$$U = P f / 2 = 4 S_v^2 V (K^2 + 1) / (45G) \quad \text{max when } K = 1$$

$$f = ra = 3.6 r^2 P (b^2 + h^2) / (b^3 h^3 G) = 0.8 r l S_v (b^2 + h^2) / (b h^2 G)$$

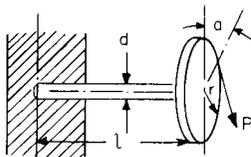


Fig. 8.2.114 Straight bar spring: circular cross section.

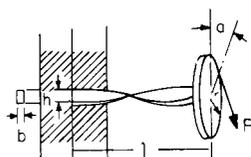


Fig. 8.2.115 Straight bar spring: rectangular cross section.

**Springs Loaded Axially in Either Tension or Compression**

NOTE. For springs 15 to 18,  $r$  = mean radius of coil;  $n$  = number of coils.

15. **Cylindrical helical spring of circular cross section** (Fig. 8.2.116).

$$P = \pi d^3 S_v / (16rk) = 0.1963 d^3 S_v / (rk)$$

$$U = P f / 2 = S_v^2 V / (4Gk^2)$$

$$f = 64 n r^3 P / (d^4 G) = 4 \pi m r^2 S_v / (dGk)$$

For heavy closely coiled springs,  $k = (4c - 1)/(4c - 4) + 0.615/c$ , where  $c = 2r/d$ .



Fig. 8.2.116 Cylindrical helical spring: circular cross section.

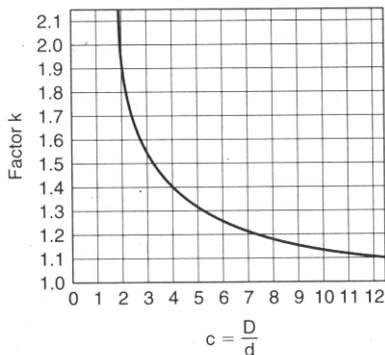


Fig. 8.2.117 Wahl correction factor.

16. **Cylindrical helical spring of rectangular cross section** (Fig. 8.2.118).

$$P = 2 b^2 h S_v / (9rk) \quad K = b/h$$

$$U = P f / 2 = 4 S_v^2 V (K^2 + 1) / (45Gk^2) \quad \text{max when } K = 1$$

$$f = 7.2 \pi m r^3 P (b^2 + h^2) / (b^3 h^3 G) = 1.6 \pi m r^2 S_v (b^2 + h^2) / (b h^2 Gk)$$

For heavy closely coiled springs,  $k = (4c - 1)/(4c - 4) + 0.615/c$ , where  $c = 2r/b$ .

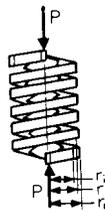


Fig. 8.2.118 Cylindrical helical spring: rectangular cross section.

17. **Conical helical spring of circular cross section** (Fig. 8.2.119a).

$l$  = length of developed spring  
 $d$  = diameter of wire  
 $r$  = maximum mean radius of coil

$$P = \pi d^3 S_v / (16rk) = 0.1963 d^3 S_v / (rk)$$

$$U = P f / 2 = S_v^2 V / (8Gk^2)$$

$$f = 16 r^2 P / (\pi d^4 G) = 16 n r^3 P / (d^4 G)$$

$$= r l S_v / (dGk) = \pi m r^2 S_v / (dGk)$$

For heavy closely coiled springs,  $k = (4c - 1)/(4c - 4) + 0.615/c$ , where  $c = 2r/d$ .

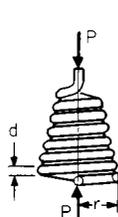


Fig. 8.2.119a Conical helical spring: circular cross section.

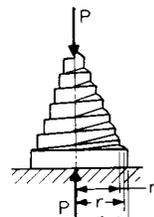


Fig. 8.2.119b Conical helical spring: rectangular cross section.

18. **Conical helical spring of rectangular cross section** (Fig. 8.2.119b).

$b$  = small dimension of section  
 $d$  = large dimension of section  
 $r$  = maximum mean radius of coil

$$K = b/h (\leq 1) \quad P = 2 b^2 h S_v / (9rk)$$

$$U = P f / 2 = 2 S_v^2 V (K^2 + 1) / (45Gk^2) \quad \text{max when } K = 1$$

$$f = 1.8 r^2 P (b^2 + h^2) / (b^3 h^3 G) = 1.8 \pi m r^3 P (b^2 + h^2) / (b^3 h^3 G)$$

$$= 0.4 r l S_v (b^2 + h^2) / (b h^2 Gk) = 0.4 \pi m r^2 S_v (b^2 + h^2) / (b h^2 Gk)$$

For heavy closed coiled springs,  $k = (4c - 1)/(4c - 4) + 0.615/c$ , where  $c = 2r/r_o - r_i$ .

19. **Truncated conical springs** (17 and 18). The formulas under 17 and 18 apply for truncated springs. In calculating deflection  $f$ , however, it is necessary to substitute  $r_1^2 + r_2^2$  for  $r^2$ , and  $\pi m(r_1 + r_2)$  for  $\pi m r$ ,  $r_1$  and  $r_2$  being, respectively, the greatest and least mean radii of the coils.

NOTE. The preceding formulas for various forms of coiled springs are sufficiently accurate when the cross-section dimensions are small in comparison with the radius of the coil, and for small pitch. Springs 15 to 19 are for either tension or compression but formulas for springs 17 and 18 are good for compression only until the largest coil flattens out; then  $r$  becomes a variable, depending on the load.

**Design of Helical Springs**

When sizing a new spring, one must consider the spring's available working space and the loads and deflections the spring must experience. Refinements dictated by temperature, corrosion, reliability, cost, etc. may also enter design considerations. The two basic formulas of load

and deflection (see item 15, Fig. 8.2.116) contain eight variables ( $f, P, d, S, r, k, n, G$ ) which prevent one from being able to use a one-step solution. For instance, if  $f$  and  $P$  are known and  $S$  and  $G$  are chosen, there still remain  $d, r, k$ , and  $n$  to be found.

A variety of solution approaches are available, including: (1) slide-rule-like devices available from spring manufacturers, (2) nomographic methods (Chironis, "Spring Design and Application," McGraw-Hill; Tsai, Speedy Design of Helical Compression Springs by Nomography Method, *J. of Eng. for Industry*, Feb. 1975), (3) table methods (Carlson, "Spring Designer's Handbook," Marcel Dekker), (4) formula method (ibid.), and (5) computer programs and subroutines.

### Design by Tables

**Safe working loads and deflections of cylindrical helical springs** of round steel wire in tension or compression are given in Table 8.2.64. The table is based on the formulas given for spring 15.  $d$  = diameter of steel wire, in;  $D$  = pitch diameter (center to center of wire), in;  $P$  = safe working load for given unit stress, lb;  $f$  = deflection of 1 coil for safe working load, in.

The table is based on the values of unit stress indicated, and  $G = 12,500,000$ . For any other value of unit stress, divide the tabular value by the unit stress used in the table and multiply by the unit stress to be used in the design. For any other value of  $G$ , multiply the value of  $f$  in the table by 12,500,000 and divide by the value of  $G$  chosen. For **square steel wire**, multiply values of  $P$  by 1.06, and values of  $f$  by 0.75. For **round brass wire**, take  $S_s = 10,000$  to 20,000, and multiply values of  $f$  by 2 (Howe).

EXAMPLES OF USE OF TABLE 8.2.64. 1. Required the safe load ( $P$ ) for a spring of  $\frac{3}{8}$ -in round steel with a pitch diameter ( $D$ ) of  $3\frac{1}{2}$  in. In the line headed  $D$ , under  $3\frac{1}{2}$ , is given the value of  $P$ , or 678 lb. This is for a unit stress of 115,000 lb/in<sup>2</sup>. The load  $P$  for any other unit stress may be found by dividing the 678 by 115,000 and multiplying by the unit stress to be used in the design. To determine the number of coils this spring would need to compress (say) 6 in under a load of (say) 678 lb, take the value of  $f$  under 678, or 0.938, which is the deflection of one coil under the given load. Therefore,  $6/0.938 = 6.4$ , say 7, equals the number of coils required. The spring will therefore be  $2\frac{5}{8}$  in long when closed ( $7 \times \frac{3}{8}$ ), counting the working coils only, and must be  $8\frac{3}{8}$  in long when unloaded. Whether there is an extra coil at one end which does not deflect will depend upon the details of the particular design. The deflection in the above example is for a unit stress of 115,000 lb/in<sup>2</sup>. The rule is, divide the deflection by 115,000 and multiply by the unit stress to be used in the design.

2. A  $\frac{3}{16}$ -in steel spring of  $3\frac{1}{2}$ -in OD has its coils in close contact. How much can it be extended without exceeding the limit of safety? The maximum safe load for this spring is found to be 1,074 lb, and the deflection of one coil under this load is 0.810 in. This is for a unit stress of 115,000 lb/in<sup>2</sup>. Therefore, 0.810 is the greatest admissible opening between any two coils. In this way, it is possible to ascertain whether or not a spring is overloaded, without knowledge of the load carried.

### Design by Formula

A design formula can be constructed by equating calculated stress  $s_s$  (from load formula, Fig. 8.2.117) to an allowable working stress in torsion:

$$s_s = \sigma_{\max} = \frac{16rP}{\pi d^3} \left( \frac{4c-1}{4c-4} + \frac{0.615}{c} \right) = \frac{16rPk}{\pi d^3} = \frac{S_v}{K_{sf}}$$

where  $P$  = axial load on spring, lb;  $r = D/2$  = mean radius of coil, in;  $D$  = mean diameter of coil, in (outside diameter minus wire diameter);  $d$  = wire diameter, in;  $\sigma_{\max}$  = torsional stress, lb/in<sup>2</sup>;  $K_{sf}$  = safety factor and  $c = D/d$ . Note that the expression in parentheses  $[(4c-1)/(4c-4) + 0.615/c]$  is the **Wahl correction factor**  $k$ , which accounts for the added stresses in the coils due to curvature and direct shear. See Fig. 8.2.117. Values of  $S_v$ , yield point in shear from standard tests, are strongly dependent on  $d$ , hence the availability of  $S_v$  in the literature is limited. However, an empirical relationship between  $S_{v,T}$  and  $d$  is available (see Shigley, "Mechanical Engineering Design," McGraw-Hill, 4th ed., p. 452). Using also the approximate relations  $S_v = 0.75S_u$

and  $S_v = 0.577S_y$ , results in the following relationship:

$$S_v = \frac{0.43A}{d^m}$$

where  $A$  and  $m$  are constants (see Table 8.2.65).

Substituting  $S_v$  and rearranging yield the following useful formula:

$$d^{3-m} = \frac{K_{sf} 16rPk}{\pi 0.43A}$$

EXAMPLE. A slow-speed follower is kept in contact with its cam by means of a helical compression spring, in which the minimum spring force desired is 20 lb to assure firm contact, while maximum spring force is not to exceed 60 lb to prevent excessive surface wear on the cam. The follower rod is  $\frac{1}{4}$  inch in diameter, and the rod enclosure where the spring is located is  $\frac{7}{8}$  inch in diameter. Maximum displacement is 1.5 in.

Choose  $r = 0.5 \times 0.75 = 0.375$ . From Table 8.2.65 choose  $m = 0.167$  and  $A = 169,000$ . Choose  $K_{sf} = 2$ . Assume  $k = 1$  to start.

$$d^{3-0.167} = \frac{(2)(0.375)(60)(16)}{(\pi)(0.43)(169,000)} = 0.00315$$

$$d = (0.00315)^{0.35298} = 0.131 \text{ in}$$

$$\text{Spring constant} = \frac{\Delta P}{\Delta f} = \frac{40}{1.5} = 26.667$$

and from the deflection formula, the number of active turns

$$n = \frac{(0.131)^4(11,500,000)}{(26.667)(64)(0.375)^3} = 37.6$$

For squared and ground ends add two dead coils, so that

$$n_{\text{total}} = 40$$

$$H = \text{solid height} = (40)(0.131) = 5.24 \text{ in}$$

$$f_0 = \text{displacement from zero to maximum load}$$

$$= 60/26.667 = 2.249$$

$$L_0 = \text{approximate free length} = 5.24 + 2.249 = 7.49 \text{ in}$$

NOTE. Some clearance should be added between coils so that at maximum load the spring is not closed to its solid height. Also, the nearest commercial stock size should be selected, and recalculations made on this stock size for  $S_v$ , remembering to include  $k$  at this juncture. If recalculated  $S_v$  is satisfactory as compared to published values, the design is retained, otherwise enough iterations are performed to arrive at a satisfactory result. Figure 8.2.120 shows a plot of  $S_{v,T}$  versus  $d$ . To convert  $S_{v,T}$  to  $S_v$ , multiply  $S_{v,T}$  by 0.43.

$$c = 2(0.375/0.131) = 5.73$$

$$k = \frac{4(5.73) - 1}{4(5.73) - 4} + \frac{0.615}{5.73} = 1.257$$

$$S_v = \frac{(K_{sf})(16)(0.375)(1.257)(60)}{\pi(0.131)^3} = (K_{sf})(64,072)$$

Now  $S_{v,T} = 235,000$  lb/in<sup>2</sup> (from Fig. 8.2.120) and  $S_v$  (tabulated) =  $(0.43)(235,000) = 101,696$  lb/in<sup>2</sup> so that  $K_{sf} = 101,696/64,072 = 1.59$ , a satisfactory value.

NOTE. The original choice of a generous  $K_{sf} = 2$  was made to hedge against the large statistical variations implied in the empirical formula  $S_v = 0.43A/d^m$ .

The basis for design of springs in parallel or in series is shown in Fig. 8.2.121.

**Belleville Springs** Often called dished, or conical spring, washers, Belleville springs occupy a very small space. They are stressed in a very complex manner, and provide unusual spring rate curves (Fig. 8.2.122a). These springs are nonlinear, but for some proportions, they behave with approximately linear characteristics in a limited range. Likewise, for some proportions they can be used through a spectrum of spring rates, from positive to flat and then through a negative region. The snap-through action, shown at point A in Fig. 8.2.122b, can be useful in particular applications requiring reversal of spring rates. These

**Table 8.2.64 Safe Working Loads  $P$  and Deflections  $f$  of Cylindrical Helical Steel Springs of Circular Cross Section**

(For closely coiled springs, divide given load and deflection values by the curvature factor  $k$ .)

Allowable unit stress, lb/in <sup>2</sup>	Diam, in	Pitch diameter $D$ , in																							
		$D$	$\frac{5}{32}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	1 $\frac{1}{8}$	1 $\frac{1}{4}$	1 $\frac{3}{8}$	1 $\frac{1}{2}$	1 $\frac{5}{8}$	1 $\frac{3}{4}$	1 $\frac{7}{8}$	2	2 $\frac{1}{4}$	2 $\frac{1}{2}$		
150,000	0.035	$P$	16.2	13.4	10.0	8.10	6.66	5.75	4.96	4.05	3.39														
		$f$	.026	.037	.067	.105	.149	.200	.276	.420	.608														
	0.041	$P$	26.2	21.6	16.2	13.0	10.8	9.27	8.10	6.52	5.35	4.57													
		$f$	.023	.032	.057	.089	.128	.175	.229	.362	.512	.697													
	0.047	$P$	39.1	32.6	24.5	19.6	16.4	13.9	12.3	9.80	8.10	6.92	6.14												
		$f$	.019	.028	.049	.078	.112	.153	.200	.311	.449	.610	.800												
	0.054	$P$	59.4	49.6	37.2	29.7	24.6	21.2	18.5	14.7	12.4	10.5	9.25	8.23											
		$f$	.016	.024	.043	.067	.098	.133	.174	.273	.390	.532	.695	.878											
	0.062	$P$		74.9	56.1	44.9	37.3	32.0	28.0	22.4	18.6	16.1	13.9	12.5	11.2										
		$f$		.021	.037	.058	.084	.115	.151	.235	.340	.460	.605	.760	.947										
	0.063	$P$		78.2	58.7	46.9	39.2	33.9	29.4	23.5	19.6	16.8	14.7	13.2	11.9	10.7									
		$f$		.020	.037	.057	.083	.113	.148	.233	.335	.445	.591	.748	.930	1.12									
0.072	$P$		117.	80.7	70.0	58.7	50.2	43.6	35.2	29.0	25.0	21.9	19.5	17.5	16.0										
	$f$		.018	.032	.050	.077	.100	.130	.203	.294	.405	.521	.652	.802	.986										
0.080	$P$			121	96.6	80.5	69.1	60.4	48.2	48.2	34.6	30.1	26.7	24.2	22.1	20.2									
	$f$			.029	.045	.065	.090	.117	.183	.262	.359	.470	.593	.735	.886	1.105									
140,000	0.092	$P$		171	136	113	97.6	85.5	68.9	57.3	48.8	42.6	37.8	34.5	31.3	28.6									
		$f$		.023	.037	.053	.072	.098	.148	.214	.291	.388	.481	.596	.720	.854									
	0.093	$P$			178	142	118	99.5	89.0	71.2	59.1	50.9	44.3	39.6	35.7	32.3	29.6	27.3							
		$f$			.023	.036	.052	.071	.093	.146	.211	.286	.376	.473	.585	.707	.841	.986							
	0.105	$P$				204	170	147	127	102	85.4	73.0	63.4	56.6	51.1	46.3	42.6	38.9							
		$f$				.032	.047	.064	.083	.122	.188	.256	.336	.425	.512	.632	.755	.880							
	0.120	$P$				303	253	217	190	152	126	108	95.2	84.2	76.2	69.2	63.5	58.5	54.3						
		$f$				.028	.041	.055	.073	.114	.174	.223	.296	.368	.449	.551	.657	.768	.893						
	0.125	$P$					286	245	214	171	143	121	107	95.5	85.2	78.0	71.5	65.8	60.8	57.2					
		$f$					.039	.053	.069	.109	.169	.213	.278	.353	.437	.528	.626	.731	.855	.981					
	0.135	$P$						359	309	270	217	171	154	135	120	108	98.7	90.2	82.7	72.2	71.8	67.5			
		$f$						.036	.049	.064	.106	.145	.198	.260	.327	.399	.486	.581	.680	.791	.908	1.04			
0.148	$P$							408	356	285	237	207	178	158	142	130	118	109	102	95.0	89.0				
	$f$							.045	.059	.092	.132	.180	.236	.293	.370	.448	.530	.620	.723	.828	.945				

**Table 8.2.64 Safe Working Loads  $P$  and Deflections  $f$  of Cylindrical Helical Steel Springs of Circular Cross Section (Continued)**

Allowable unit stress, lb/in <sup>2</sup>	Diam, in	Pitch diameter $D$ , in																								
		$D$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	1 $\frac{1}{8}$	1 $\frac{1}{4}$	1 $\frac{3}{8}$	1 $\frac{1}{2}$	1 $\frac{5}{8}$	1 $\frac{3}{4}$	1 $\frac{7}{8}$	2	2 $\frac{1}{4}$	2 $\frac{1}{2}$	2 $\frac{3}{4}$	3	3 $\frac{1}{2}$	4	4 $\frac{1}{2}$	5		
140,000	0.156	$P$	480	418	330	270	234	208	185	167	152	139	128	119	111	104	92.7									
		$f$	0.42	.056	.087	.125	.171	.223	.282	.350	.422	.509	.588	.685	.785	.896	1.12									
	0.162	$P$		468	376	311	276	234	207	187	170	156	143	134	125	117	103									
		$f$		.054	.085	.121	.165	.216	.273	.338	.409	.488	.566	.663	.757	.863	1.09									
	0.177	$P$		608	487	406	347	305	270	243	223	205	187	174	163	152	135	122								
		$f$		.049	.077	.115	.151	.198	.251	.311	.375	.447	.522	.606	.695	.793	1.00	1.24								
125,000	0.187	$P$	642	522	426	367	320	284	256	233	213	197	183	170	160	142	128									
		$f$	.041	.065	.093	.127	.166	.210	.260	.314	.373	.437	.487	.510	.584	.665	.832	1.04								
	0.192	$P$	696	556	465	396	348	309	278	254	233	214	199	186	174	154	139	126								
		$f$	.040	.063	.091	.124	.160	.205	.252	.308	.366	.428	.499	.571	.652	.825	1.02	1.23								
	0.207	$P$		694	579	495	432	385	346	315	288	266	247	232	216	192	173	158	144							
		$f$		.059	.085	.115	.151	.191	.236	.286	.342	.396	.462	.570	.607	.757	.943	1.11	1.36							
	0.218	$P$		812	678	580	509	452	408	360	339	310	291	270	255	225	204	185	169							
		$f$		.055	.080	.109	.142	.180	.223	.269	.321	.374	.437	.488	.570	.710	.891	1.08	1.28							
	0.225	$P$		895	746	640	560	498	447	407	372	345	320	299	280	248	224	203	187							
		$f$		.054	.078	.106	.138	.175	.213	.262	.312	.372	.425	.486	.565	.691	.866	1.05	1.24							
	0.244	$P$		1120	950	811	711	632	570	517	475	438	406	381	356	316	284	259	237							
		$f$		.049	.071	.098	.138	.161	.200	.240	.287	.336	.391	.449	.537	.646	.800	.965	1.14							
	0.250	$P$			1027	880	760	685	617	560	513	476	440	410	385	342	308	281	266	222						
		$f$			.070	.095	.131	.157	.191	.236	.281	.328	.385	.439	.524	.624	.780	.946	1.12	1.53						

**Table 8.2.64 Safe Working Loads  $P$  and Deflections  $f$  of Cylindrical Helical Steel Springs of Circular Cross Section (Continued)**

Allowable unit stress, lb/in <sup>2</sup>	Diam, in	Pitch diameter $D$ , in																								
		$D$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	1 $\frac{1}{8}$	1 $\frac{1}{4}$	1 $\frac{3}{8}$	1 $\frac{1}{2}$	1 $\frac{5}{8}$	1 $\frac{3}{4}$	1 $\frac{7}{8}$	2	2 $\frac{1}{4}$	2 $\frac{1}{2}$	2 $\frac{3}{4}$	3	3 $\frac{1}{2}$	4	4 $\frac{1}{2}$	5		
0.263	$P$					1195	1125	895	795	717	652	598	551	501	478	448	400	359	326	298	256					
	$f$					.066	.089	.118	.149	.183	.224	.266	.312	.363	.416	.475	.592	.740	.896	1.06	1.44					
0.281	$P$					1450	1240	1087	969	863	794	724	665	620	580	543	482	437	395	362	310					
	$f$					.062	.085	.111	.140	.172	.209	.250	.292	.340	.390	.443	.562	.692	.840	1.02	1.36					
0.283	$P$					1264	1110	985	886	805	740	682	634	592	564	492	439	402	370	317						
	$f$					.084	.111	.139	.169	.199	.246	.289	.338	.386	.440	.559	.690	.883	.990	1.35						
115,000	$P$					1575	1376	1220	1100	1000	915	845	775	733	687	610	550	500	460	392	343					
	$f$					.070	.092	.116	.144	.174	.207	.242	.283	.322	.368	.467	.577	.697	.829	1.12	1.47					
0.331	$P$					1636	1455	1316	1187	1090	1000	932	870	818	725	653	594	545	468	410						
	$f$					.088	.109	.135	.163	.194	.227	.264	.343	.346	.437	.541	.654	.770	1.05	1.30						
0.341	$P$					1820	1620	1452	1325	1214	1120	1040	970	910	808	728	661	608	520	454						
	$f$					.082	.105	.127	.156	.186	.218	.256	.293	.330	.413	.522	.625	.745	1.02	1.32						
0.362	$P$					2140	1910	1714	1560	1430	1318	1220	1147	1070	950	858	778	714	612	535						
	$f$					.079	.100	.123	.149	.177	.207	.243	.273	.317	.400	.495	.598	.713	.965	.126						
0.375	$P$					2110	1940	1780	1580	1458	1354	1265	1185	1058	950	860	790	678	592	528						
	$f$					.079	.117	.144	.172	.201	.234	.268	.308	.382	.478	.579	.688	.938	1.22	1.54						
0.393	$P$					2430	2180	1984	1820	1680	1560	1458	1365	1212	1092	990	910	780	682	670						
	$f$					.092	.114	.137	.164	.195	.223	.256	.292	.369	.457	.550	.657	.890	1.16	1.47						
0.406	$P$					2400	2170	2000	1840	1710	1600	1500	1330	1200	1090	1000	855	750	666							
	$f$					.108	.134	.159	.168	.217	.248	.284	.353	.444	.525	.640	.867	1.13	1.43							
0.430	$P$					2875	2610	2400	2210	2050	1918	1798	1598	1440	1308	1200	1028	900	800							
	$f$					.104	.126	.150	.175	.204	.234	.267	.338	.418	.503	.600	.815	1.06	1.35							
0.437	$P$					3000	2730	2500	2310	2140	2000	1800	1665	1500	1365	1250	1074	940	835	750						
	$f$					.100	.124	.148	.173	.201	.231	.264	.327	.412	.490	.593	.810	1.05	1.33	1.64						

**Table 8.2.64 Safe Working Loads *P* and Deflections *f* of Cylindrical Helical Steel Springs of Circular Cross Section (Continued)**

Allowable unit stress, lb/in <sup>2</sup>	Diam, in	Pitch diameter <i>D</i> , in																	
		<i>D</i>	1¼	1½	1⅝	1¾	1⅞	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6	
110,000	0.460	<i>P</i>		3065	2800	2580	2400	2230	2100	1865	1680	1530	1400	1200	1058	952	840		
		<i>f</i>		.112	.134	.157	.183	.209	.239	.303	.374	.447	.536	.729	.956	1.21	1.49		
	0.468	<i>P</i>		3265	2940	2725	2530	2375	2210	1970	1770	1610	1472	1265	1110	935	885		
		<i>f</i>		.111	.132	.154	.182	.206	.235	.295	.368	.444	.530	.720	.943	1.19	1.47		
	0.490	<i>P</i>		3675	3270	3115	2890	2710	2535	2245	2025	1840	1690	1445	1268	1125	1015	920	
		<i>f</i>		.106	.126	.148	.172	.196	.225	.284	.351	.424	.506	.688	.900	1.13	1.40	1.70	
	0.500	<i>P</i>			3610	3320	3090	2890	2710	2410	2160	1970	1810	1550	1352	1205	1082	985	
		<i>f</i>			.123	.144	.168	.192	.220	.274	.347	.415	.495	.672	.880	1.11	1.37	1.65	
	0.562	<i>P</i>				4700	4390	4090	3830	3420	3080	2790	2565	2190	1913	1710	1535	1395	1280
		<i>f</i>				.128	.149	.175	.195	.248	.306	.372	.440	.596	.782	.990	1.22	1.47	1.75
0.625	<i>P</i>					6100	5600	5260	4660	4210	3825	3505	3000	2630	2340	2110	1913	1750	
	<i>f</i>					.134	.154	.176	.218	.275	.328	.397	.538	.705	.875	1.05	1.33	1.58	
100,000	0.687	<i>P</i>						6325	5660	5090	4630	4250	3625	3195	2825	2560	2330	2125	
		<i>f</i>						.145	.183	.228	.274	.3278	.443	.580	.733	.908	1.00	1.30	
	0.750	<i>P</i>							7400	6640	6030	5540	4745	4150	3690	3325	3025	2770	
		<i>f</i>							.178	.218	.252	.299	.402	.532	.671	.832	1.00	1.19	
	0.812	<i>P</i>								8420	7660	7000	6000	5260	4675	4200	3825	3500	
		<i>f</i>								.192	.232	.276	.376	.490	.620	.766	.880	1.10	
	0.875	<i>P</i>								10830	9550	8700	7500	6560	5740	5250	4770	4730	
		<i>f</i>								.179	.218	.257	.348	.456	.577	.712	.860	1.02	
90,000	0.937	<i>P</i>									10600	9700	8400	7160	6470	5810	5290	4850	
		<i>f</i>										.179	.217	.290	.383	.480	.591	.715	.855
	1.000	<i>P</i>											11780	10100	8800	7850	7050	6330	5870
		<i>f</i>											.206	.276	.360	.454	.561	.680	.803
	1.125	<i>P</i>												14400	12600	11230	10100	9200	8400
<i>f</i>													.244	.320	.405	.496	.600	.718	
1.250	<i>P</i>													24700	18200	15300	13250	12540	11500
	<i>f</i>													.260	.287	.364	.442	.545	.648
80,000	1.375	<i>P</i>													20400	18100	16150	14850	13600
		<i>f</i>													.280	.294	.364	.440	.522

Table 8.2.65 Constants for Use in  $S_v = 0.43A/d^m$

Material	Size range, in	Size range, mm	Exponent $m$	Constant A	
				kips	MPa
Music wire*	0.004–0.250	0.10–6.55	0.146	196	2,170
Oil-tempered wire†	0.020–0.500	0.50–12	0.186	149	1,880
Hard-drawn wire‡	0.028–0.500	0.70–12	0.192	136	1,750
Chrome vanadium§	0.032–0.437	0.80–12	0.167	169	2,000
Chrome silicon¶	0.063–0.375	1.6–10	0.112	202	2,000

\* Surface is smooth, free from defects, and has a bright, lustrous finish.

† Has a slight heat-treating scale which must be removed before plating.

‡ Surface is smooth and bright, with no visible marks.

§ Aircraft-quality tempered wire; can also be obtained annealed.

¶ Tempered to Rockwell C49 but may also be obtained untempered.

SOURCE: Adapted from "Mechanical Engineering Design," Shigley, McGraw-Hill, 1983 by permission.

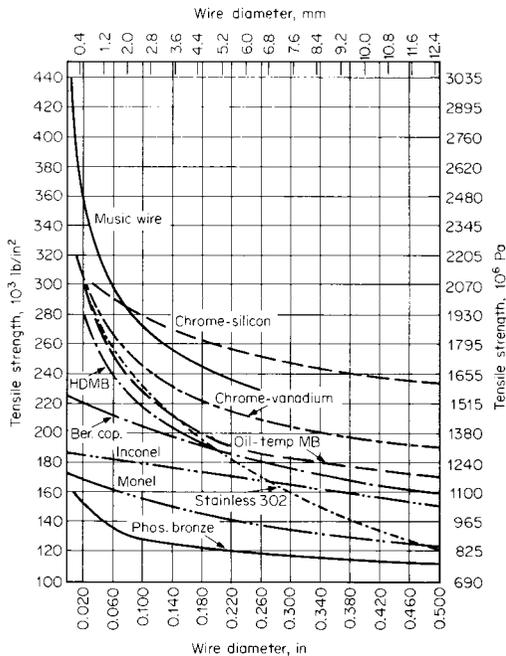
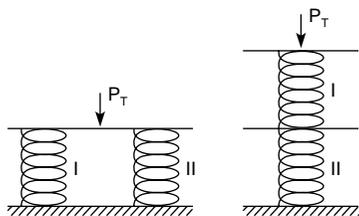


Fig. 8.2.120 Minimum tensile strength for the most popular spring materials, spring-quality wire. (Reproduced from Carlson, "Spring Designer's Handbook," Marcel Dekker, by permission.)



**Parallel**

$$P_T = P_I + P_{II}$$

$$\Delta_T = \Delta_I = \Delta_{II}$$

$$K_T = K_I + K_{II}$$

where  $P$  = load, lb

$\Delta$  = deflection, in.

$K$  = spring rate, lb/in.

**Series**

$$P_T = P_I = P_{II}$$

$$\Delta_T = \Delta_I + \Delta_{II}$$

$$K_T = \frac{K_I K_{II}}{K_I + K_{II}}$$

Fig. 8.2.121 Springs in parallel and in series.

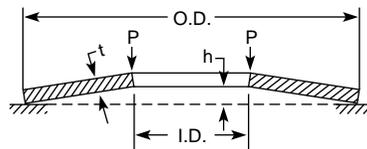


Fig. 8.2.122a Sectional view of Belleville spring.

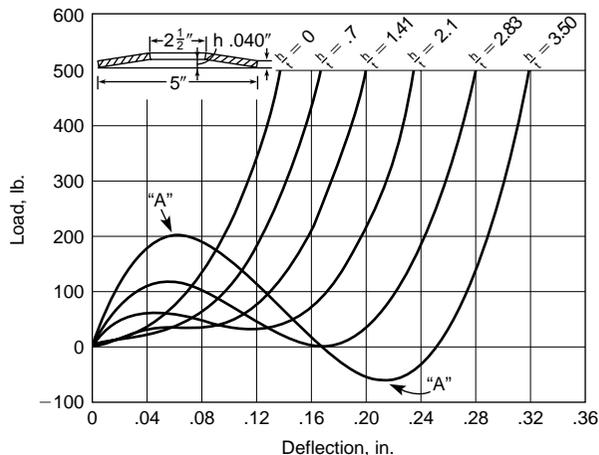


Fig. 8.2.122b Load deflection curves for a family of Belleville springs. (Associated Spring Corp.)

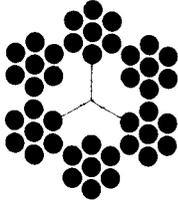
springs are used for very high and special spring rates. They are extremely sensitive to slight variations in their geometry. A wide range is available commercially.

**WIRE ROPE**

When power source and load are located at extreme distances from one another, or loads are very large, the use of wire rope is suggested. Design and use decisions pertaining to wire ropes rest with the user, but manufacturers generally will help users toward appropriate choices. The following material, based on the Committee of Wire Rope Producers, "Wire Rope User's Manual," 2d ed., 1981, may be used as an initial guide in selecting a rope.

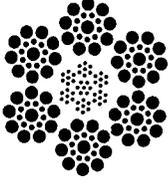
Wire rope is composed of (1) wires to form a strand, (2) strands wound helically around a core, and (3) a core. Classification of wire ropes is made by giving the number of strands, number of minor strands in a major strand (if any), and nominal number of wires per strand. For example  $6 \times 7$  rope means 6 strands with a nominal 7 wires per strand (in this case no minor strands, hence no middle number). A nominal value simply represents a range. A nominal value of 7 can mean anywhere from 3 to 14, of which no more than 9 are outside wires. A full

6 x 7 classification

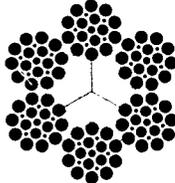


6 x 7  
FC

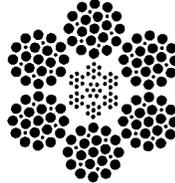
6 x 19 classification



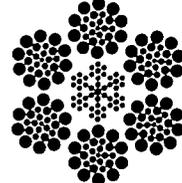
6 x 19 Seale  
IWRC



6 x 12 filler wire  
FC

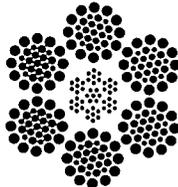


6 x 25 filler wire  
IWRC

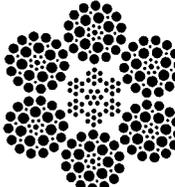


6 x 26 Warrington Seale  
IWRC

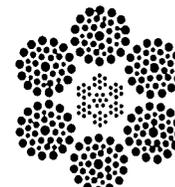
6 x 37 classification



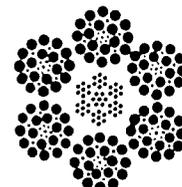
6 x 31 Warrington Seale  
IWRC



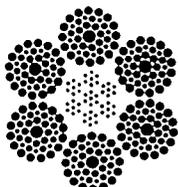
6 x 36 Seale  
filler wire  
IWRC



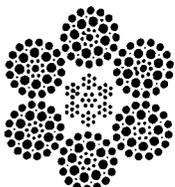
6 x 36 Warrington Seale  
IWRC



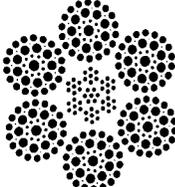
6 x 31 filler wire  
IWRC



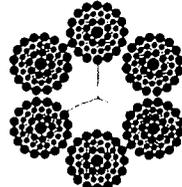
6 x 41 Warrington Seale  
IWRC



6 x 41 Seale  
filler wire  
IWRC

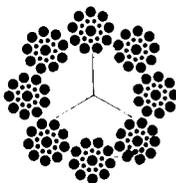


6 x 46 Seale filler wire  
IWRC

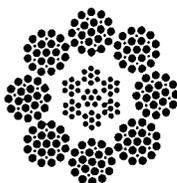


6 x 49 filler  
wire Seale  
FC

8 x 19 classification



8 x 19 Seale  
FC



8 x 25 filler wire  
IWRC

Fig. 8.2.123 Cross sections of some commonly used wire rope construction. (Reproduced from "Wire Rope User's Manual," AISI, by permission.)

rope description will also include length, size (diameter), whether wire is preformed or not prior to winding, direction of lay (right or left, indicating the direction in which strands are laid around the core), grade of rope (which reflects wire strength), and core. The most widely used classifications are:  $6 \times 7$ ,  $6 \times 19$ ,  $6 \times 37$ ,  $6 \times 61$ ,  $6 \times 91$ ,  $6 \times 127$ ,  $8 \times 19$ ,  $18 \times 7$ ,  $19 \times 7$ . Some special constructions are:  $3 \times 7$  (guardrail rope);  $3 \times 19$  (slusher),  $6 \times 12$  (running rope);  $6 \times 24$  and  $6 \times 30$  (hawsers);  $6 \times 42$  and  $6 \times 6 \times 7$  (tiller rope);  $6 \times 3 \times 19$  (spring lay);  $5 \times 19$  and  $6 \times 19$  (marlin clad);  $6 \times 25B$ ,  $6 \times 27H$ , and  $6 \times 30G$  (flattened strand). The diameter of a rope is the circle which just contains the rope. The right-regular lay (in which the wire is twisted in one direction to form the strands and the strands are twisted in the opposite direction to form the rope) is most common. Regular-lay ropes do not kink or untwist and handle easily. Lang-lay ropes (in which wires and strands are twisted in the same direction) are more resistant to abrasive wear and fatigue failure.

Cross sections of some commonly used wire rope are shown in Fig. 8.2.123. Figure 8.2.124 shows rotation-resistant ropes, and Fig. 8.2.125 shows some special-purpose constructions.

The core provides support for the strands under normal bending and loading. Core materials include fibers (hard vegetable or synthetic) or steel (either a strand or an independent wire rope). Most common core designations are: fiber core (FC), independent wire-rope core (IWRC), and wire-strand core (WSC). Lubricated fiber cores can provide lubrication to the wire, but add no real strength and cannot be used in high temperature environments. Wire-strand or wire-rope cores add from 7 to 10 percent to strength, but under nonstationary usage tend to wear from interface friction with the outside strands. Great flexibility can be achieved when wire rope is used as strands. Such construction is very pliable and friction resistant. Some manufacturers will provide plastic coatings (nylon, Teflon, vinyl, etc.) upon request. Such coatings help provide resistance to abrasion, corrosion, and loss of lubricant. *Crushing* refers to rope damage caused by excessive pressures against drum or sheave, improper groove size, and multiple layers on drum or sheave. Consult wire rope manufacturers in doubtful situations.

Wire-rope materials and their strengths are reflected as grades. These are: traction steel (TS), mild plow steel (MPS), plow steel (PS), improved plow steel (IPS), and extra improved plow (EIP). The plow steel strength curve forms the basis for calculating the strength of all steel rope wires. American manufacturers use color coding on their ropes to identify particular grades.

The grades most commonly available and tabulated are IPS and EIP. Two specialized categories, where selection requires extraordinary attention, are elevator and rotation-resistant ropes.

Elevator rope can be obtained in four principal grades: iron, traction steel, high-strength steel, and extra-high-strength steel.

Bronze rope has limited use; iron rope is used mostly for older existing equipment.

**Selection of Wire Rope**

Appraisal of the following is the key to choosing the rope best suited to the job: resistance to breaking, resistance to bending fatigue, resistance to vibrational fatigue, resistance to abrasion, resistance to crushing, and reserve strength. Along with these must be an appropriate choice of safety factor, which in turn requires careful consideration of all loads, acceleration-deceleration, shocks, rope speed, rope attachments, sheave

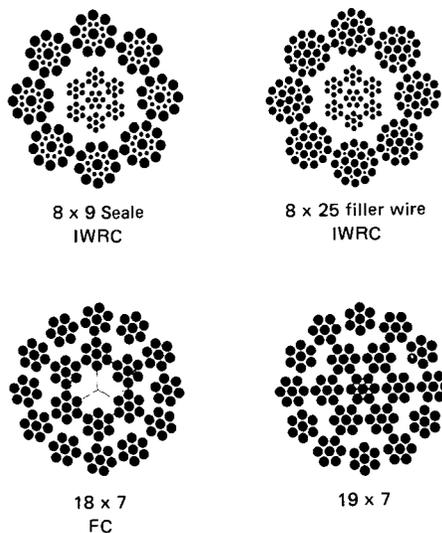


Fig. 8.2.124 Cross section of some rotation-resistant wire ropes. (Reproduced from "Wire Rope User's Manual," AISI, by permission.)

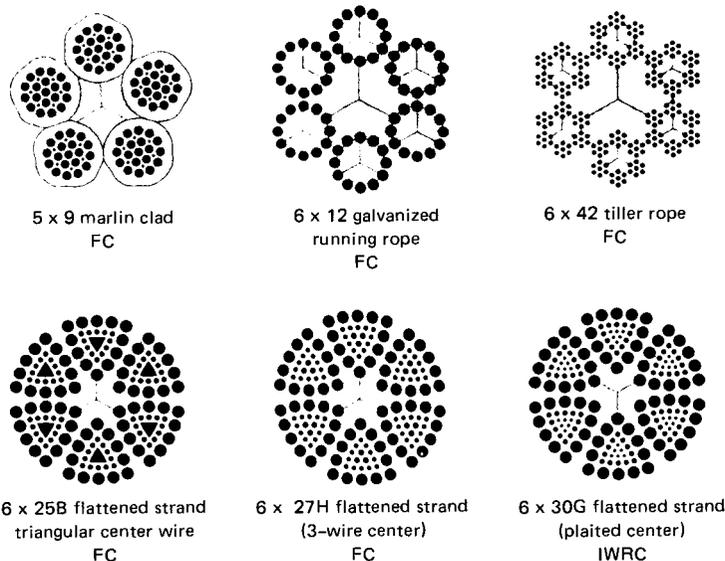


Fig. 8.2.125 Some special constructions. (Reproduced from "Wire Rope User's Manual," AISI, by permission.)

Table 8.2.66 Selected Values of Nominal Strengths of Wire Rope

Classification	Fiber core						IWRC					
	Nominal diameter		Approximate mass		Nominal strength IPS		Approximate mass		Nominal strength			
									IPS		EIP	
	in	mm	lb/ft	kg/m	tons	t	lb/ft	kg/m	tons	t	tons	t
6 × 7 Bright (uncoated)	¼	6.4	0.09	0.14	2.64	2.4	0.10	0.15	2.84	2.58		
	⅜	9.5	0.21	0.31	5.86	5.32	0.23	0.34	6.30	5.72		
	½	13	0.38	0.57	10.3	9.35	0.42	0.63	11.1	10.1		
	⅝	16	0.59	0.88	15.9	14.4	0.65	0.97	17.1	15.5		
	¾	22	1.15	1.71	30.7	27.9	1.27	1.89	33.0	29.9		
	1⅜	29	1.90	2.83	49.8	45.2	2.09	3.11	53.5	48.5		
	1½	35	2.82	4.23	73.1	66.3	3.12	4.64	78.6	71.3		
6 × 19 Bright (uncoated)	¼	6.4	0.11	0.16	2.74	2.49	0.12	0.17	2.94	2.67	3.40	3.08
	⅜	9.5	0.24	0.35	6.10	5.53	0.26	0.39	6.56	5.95	7.55	6.85
	½	13	0.42	0.63	10.7	9.71	0.46	0.68	11.5	10.4	13.3	12.1
	⅝	16	0.66	0.98	16.7	15.1	0.72	1.07	17.7	16.2	20.6	18.7
	¾	22	1.29	1.92	32.2	29.2	1.42	2.11	34.6	31.4	39.8	36.1
	1⅜	29	2.13	3.17	52.6	47.7	2.34	3.48	56.5	51.3	65.0	59.0
	1½	35	3.18	4.73	77.7	70.5	3.5	5.21	83.5	75.7	96.0	87.1
	1¾	42	4.44	6.61	107	97.1	4.88	7.26	115	104	132	120
	1⅞	48	5.91	8.8	141	128	6.5	9.67	152	138	174	158
	2⅜	54	7.59	11.3	179	162	8.35	12.4	192	174	221	200
	2½	60	9.48	14.1	222	201	10.4	15.5	239	217	274	249
	2⅞	67	11.6	17.3	268	243	12.8	19.0	288	261	331	300
6 × 37 Bright (uncoated)	¼	6.4	0.11	0.16	2.74	2.49	0.12	0.17	2.94	2.67	3.4	3.08
	⅜	9.5	0.24	0.35	6.10	5.53	0.26	0.39	6.56	5.95	7.55	6.85
	½	13	0.42	0.63	10.7	9.71	0.46	0.68	11.5	10.4	13.3	12.1
	⅝	16	0.66	0.98	16.7	15.1	0.72	1.07	17.9	16.2	20.6	18.7
	¾	22	1.29	1.92	32.2	29.2	1.42	2.11	34.6	31.4	39.5	36.1
	1⅜	29	2.13	3.17	52.6	47.7	2.34	3.48	56.5	51.3	65.0	59.0
	1½	35	3.18	4.73	77.7	70.5	3.50	5.21	83.5	75.7	96.0	87.1
	1¾	42	4.44	6.61	107	97.1	4.88	7.26	115	104	132	120
	1⅞	48	5.91	8.8	141	128	6.5	9.67	152	138	174	158
	2⅜	54	7.59	11.3	179	162	8.35	12.4	192	174	221	200
	2½	60	9.48	14.1	222	201	10.4	15.5	239	217	274	249
	2⅞	67	11.6	17.3	268	243	12.8	19.0	288	261	331	300
	3⅜	74	13.9	20.7	317	287	15.3	22.8	341	309	392	356
80	16.4	24.4	371	336	18.0	26.8	399	362	458	415		
6 × 61 Bright (uncoated)	1⅜	29	2.13	3.17	50.1	45.4	2.34	3.48	53.9	48.9	61.9	56.2
	1½	42	4.44	6.61	103	93.4	4.88	7.62	111	101	127	115
	2	52	6.77	10.1	154	140	7.39	11.0	165	150	190	172
	2½	67	11.6	17.3	260	236	12.8	18.3	279	253	321	291
	3	77	15.1	22.5	335	304	16.6	24.7	360	327	414	376
	4	103	26.9	40.0	577	523	29.6	44.1	620	562	713	647
6 × 91 Bright (uncoated)	5	128	42.0	62.5	872	791	46.2	68.8	937	850	1,078	978
	2	51	6.77	10.1	146	132	7.39	11.0	157	142	181	164
	3	77	15.1	22.5	318	288	16.6	24.7	342	310	393	357
	4						29.6	44.1	589	534	677	614
	5						46.2	68.7	891	808	1,024	929
6						65.0	96.7	1,240	1,125	1,426	1,294	
6 × 25B	½	13	0.45	0.67	11.8	10.8	0.47	0.70	12.6	11.4	14	12.7
6 × 27H	⅝	14.5	0.57	0.85	14.9	13.5	0.60	0.89	16.0	14.5	17.6	16
6 × 30G	¾	19	1.01	1.50	26.2	23.8	1.06	1.58	28.1	25.5	31	28.1
Flattened strand bright (uncoated)	1	26	1.80	2.68	46.0	41.7	1.89	2.83	49.4	44.8	54.4	49.4
	1¼	32	2.81	4.18	71.0	64.4	2.95	4.39	76.3	60.2	84	76.2
	1½	38	4.05	6.03	101	91.6	4.25	6.32	108	98	119	108
	1¾	45	5.51	8.20	136	123	5.78	8.60	146	132	161	146
	2	52	7.20	10.70	176	160	7.56	11.3	189	171	207	188
8 × 19 Bright (uncoated)	¼	6.4	0.10	0.15	2.35	2.13	0.47	0.70	10.1	9.16	11.6	10.5
	⅜	9.5	0.22	0.33	5.24	4.75	0.73	1.09	15.7	14.2	18.1	16.4
	½	13	0.39	0.58	9.23	8.37	1.44	2.14	30.5	27.7	35.0	31.8
	⅝	16	0.61	0.91	14.3	13.0	2.39	3.56	49.8	45.2	57.3	51.7
	1	26	1.57	2.34	36.0	32.7	4.24	6.31	87.3	79.2	100.0	90.7
	1½	38	3.53	5.25	79.4	72.0						
18 × 7 Rotation resistant, bright (uncoated)	½	13	0.43	0.64	9.85	8.94	0.45	0.67	9.85	8.94	10.8	9.8
	¾	19	0.97	1.44	21.8	19.8	1.02	1.52	21.8	19.8	24.0	21.8
	1	26	1.73	2.57	38.3	34.7	1.82	2.71	38.3	34.7	42.2	38.3
	1¼	32	2.70	4.02	59.2	53.7	2.84	4.23	59.2	53.7	65.1	59.1
	1½	38	3.89	5.79	84.4	76.6	4.08	6.07	84.4	76.6	92.8	84.2

SOURCE: "Wire Rope User's Manual," AISI, adapted by permission.

arrangements as well as their number and size, corrosive and/or abrasive environment, length of rope, etc. An approximate selection formula can be written as:

$$DSL = \frac{(NS)K_b}{K_{sf}}$$

where DSL (demanded static load) = known or dead load plus additional loads caused by sudden starts or stops, shocks, bearing friction, etc., tons; NS (nominal strength) = published test strengths, tons (see Table 8.2.66);  $K_b$  = a factor to account for the reduction in nominal strength due to bending when a rope passes over a curved surface such as a stationary sheave or pin (see Fig. 8.2.126);  $K_{sf}$  = safety factor. (For average operation use  $K_{sf} = 5$ . If there is danger to human life or other critical situations, use  $8 \leq K_{sf} \leq 12$ . For instance, for elevators moving at 50 ft/min,  $K_{sf} = 8$ , while for those moving at 1,500 ft/min,  $K_{sf} = 12$ .)

Having made a tentative selection of a rope based on the demanded static load, one considers next the wear life of the rope. A loaded rope

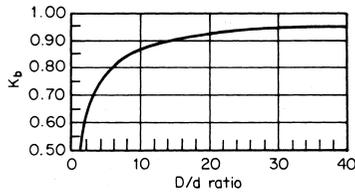


Fig. 8.2.126 Values of  $K_{bend}$  vs.  $D/d$  ratios ( $D$  = sheave diameter,  $d$  = rope diameter), based on standard test data for  $6 \times 9$  and  $6 \times 17$  class ropes. (Compiled from "Wire Rope User's Manual," AISI, by permission.)

bent over a sheave stretches elastically and so rubs against the sheave, causing wear of both members. Drum or sheave size is of paramount importance at this point.

**Sizing of Drums or Sheaves**

Diameters of drums or sheaves in wire rope applications are controlled by two main considerations: (1) the radial pressure between rope and groove and (2) degree of curvature imposed on the rope by the drum or sheave size.

Radial pressures can be calculated from  $p = 2T/(Dd)$ , where  $p$  = unit radial pressure, lb/in<sup>2</sup>;  $T$  = rope load, lb;  $D$  = tread diameter of drum or sheave, in;  $d$  = nominal diameter of rope, in. Table 8.2.67 lists suggested allowable radial bearing pressures of ropes on various sheave materials.

All wire ropes operating over drums or sheaves are subjected to cyclical stresses, causing shortened rope life because of fatigue. Fatigue resistance or relative service life is a function of the ratio  $D/d$ . Adverse

effects also arise out of relative motion between strands during passage around the drum or sheave. Additional adverse effects can be traced to poor match between rope and groove size, and to lack of rope lubrication. Table 8.2.68 lists suggested and minimum sheave and drum ratios for various rope construction. Table 8.2.69 lists relative bending life factors; Figure 8.2.127 shows a plot of relative rope service life versus  $D/d$ . Table 8.2.70 lists minimum drum (sheave) groove dimensions. Periodic groove inspection is recommended, and worn or corrugated grooves should be remachined or the drum replaced, depending on severity of damage.

**Seizing and Cutting Wire Rope** Before a wire rope is cut, seizings (bindings) must be applied on either side of the cut to prevent rope distortion and flattening or loosened strands. Normally, for preformed ropes, one seizing on each side of the cut is sufficient, but for ropes that

Table 8.2.68 Sheave and Drum Ratios

Construction*	Suggested	Minimum
$6 \times 7$	72	42
$19 \times 7$ or $18 \times 7$ Rotation-resistant	51	34
$6 \times 19$ S	51	34
$6 \times 25$ B flattened strand	45	30
$6 \times 27$ H flattened strand	45	30
$6 \times 30$ G flattened strand	45	30
$6 \times 21$ FW	45	30
$6 \times 26$ WS	45	30
$6 \times 25$ FW	39	26
$6 \times 31$ WS	39	26
$6 \times 37$ SFW	39	26
$6 \times 36$ WS	35	23
$6 \times 43$ FWS	35	23
$6 \times 41$ WS	32	21
$6 \times 41$ SFW	32	21
$6 \times 49$ SWS	32	21
$6 \times 46$ SFW	28	18
$6 \times 46$ WS	28	18
$8 \times 19$ S	41	27
$8 \times 25$ FW	32	21
$6 \times 42$ Tiller	21	14

\* WS—Warrington Seale; FWS—Filler Wire Seale; SFW—Seale Filler Wire; SWS—Seale Warrington Seale; S—Seale; FW—Filler Wire.

†  $D$  = tread diameter of sheave;  $d$  = nominal diameter of rope. To find any tread diameter from this table, the diameter for the rope construction to be used is multiplied by its nominal diameter  $d$ . For example, the minimum sheave tread diameter for a  $1/2$ -in  $6 \times 21$  FW rope would be  $1/2$  (nominal diameter)  $\times$  30 (minimum ratio), or 15 in.

NOTE: These values are for reasonable service. Other values are permitted by various standards such as ANSI, API, PCSA, HMI, CMAA, etc. Similar values affect rope life.

SOURCE: "Wire Rope User's Manual," AISI, reproduced by permission.

Table 8.2.67 Suggested Allowable Radial Bearing Pressures of Ropes on Various Sheave Materials

Material	Regular lay rope, lb/in <sup>2</sup>				Lang lay rope, lb/in <sup>2</sup>			Flattened strand lang lay, lb/in <sup>2</sup>	Remarks
	$6 \times 7$	$6 \times 19$	$6 \times 37$	$8 \times 19$	$6 \times 7$	$6 \times 19$	$6 \times 37$		
Wood	150	250	300	350	165	275	330	400	On end grain of beech, hickory, gum.
Cast iron	300	480	585	680	350	550	660	800	Based on minimum Brinell hardness of 125.
Carbon-steel casting	550	900	1,075	1,260	600	1,000	1,180	1,450	30–40 carbon. Based on minimum Brinell hardness of 160.
Chilled cast iron	650	1,100	1,325	1,550	715	1,210	1,450	1,780	Not advised unless surface is uniform in hardness.
Manganese steel	1,470	2,400	3,000	3,500	1,650	2,750	3,300	4,000	Grooves must be ground and sheaves balanced for high-speed service.

SOURCE: "Wire Rope User's Manual," AISI, reproduced by permission.

**Table 8.2.69 Relative Bending Life Factors**

Rope construction	Factor	Rope construction	Factor
6 × 7	0.61	6 × 36 WS	1.16
19 × 7 or 18 × 7	0.67	6 × 43 FWS	1.16
Rotation-resistant	0.81	6 × 41 WS	1.30
6 × 19 S	0.90	6 × 41 SFW	1.30
6 × 25 B flattened strand	0.90	6 × 49 SWS	1.30
6 × 27 H flattened strand	0.90	6 × 43 FW (2 op)	1.41
6 × 30 G flattened strand	0.89	6 × 46 SFW	1.41
6 × 21 FW	0.89	6 × 46 WS	1.41
6 × 26 WS	1.00	8 × 19 S	1.00
6 × 25 FW	1.00	8 × 25 FW	1.25
6 × 31 WS	1.00	6 × 42 Tiller	2.00
6 × 37 SFW			

SOURCE: "Wire Rope User's Manual," AISI, reproduced by permission.

**Table 8.2.70 Minimum Sheave- and Drum-Groove Dimensions\***

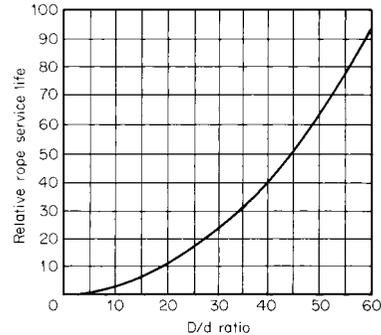
Nominal rope diameter		Groove radius			
		New		Worn	
in	mm	in	mm	in	mm
¼	6.4	0.135	3.43	.129	3.28
⅜	8.0	0.167	4.24	.160	4.06
½	9.5	0.201	5.11	.190	4.83
⅝	11	0.234	5.94	.220	5.59
¾	13	0.271	6.88	.256	6.50
7/16	14.5	0.303	7.70	.288	7.32
⅝	16	0.334	8.48	.320	8.13
¾	19	0.401	10.19	.380	9.65
7/8	22	0.468	11.89	.440	11.18
1	26	0.543	13.79	.513	13.03
1 1/8	29	0.605	15.37	.577	14.66
1 1/4	32	0.669	16.99	.639	16.23
1 3/8	35	0.736	18.69	.699	17.75
1 1/2	38	0.803	20.40	.759	19.28
1 5/8	42	0.876	22.25	.833	21.16
1 3/4	45	0.939	23.85	.897	22.78
1 7/8	48	1.003	25.48	.959	24.36
2	52	1.085	27.56	1.025	26.04
2 1/8	54	1.137	28.88	1.079	27.41
2 1/4	58	1.210	30.73	1.153	29.29
2 3/8	60	1.271	32.28	1.199	30.45
2 1/2	64	1.338	33.99	1.279	32.49
2 5/8	67	1.404	35.66	1.339	34.01
2 3/4	71	1.481	37.62	1.409	35.79
2 7/8	74	1.544	39.22	1.473	37.41
3	77	1.607	40.82	1.538	39.07
3 1/8	80	1.664	42.27	1.598	40.59
3 1/4	83	1.731	43.97	1.658	42.11
3 3/8	87	1.807	45.90	1.730	43.94
3 1/2	90	1.869	47.47	1.794	45.57
3 3/4	96	1.997	50.72	1.918	48.72
4	103	2.139	54.33	2.050	52.07
4 1/4	109	2.264	57.51	2.178	55.32
4 1/2	115	2.396	60.86	2.298	58.37
4 3/4	122	2.534	64.36	2.434	61.82
5	128	2.663	67.64	2.557	64.95
5 1/4	135	2.804	71.22	2.691	68.35
5 1/2	141	2.929	74.40	2.817	71.55
5 3/4	148	3.074	78.08	2.947	74.85
6	154	3.198	81.23	3.075	78.11

\* Values given are applicable to grooves in sheaves and drums; they are not generally suitable for pitch design since this may involve other factors. Further, the dimensions do not apply to traction-type elevators; in this circumstance, drum- and sheave-groove tolerances should conform to the elevator manufacturer's specifications. Modern drum design embraces extensive considerations beyond the scope of this publication. It should also be noted that drum grooves are now produced with a number of oversize dimensions and pitches applicable to certain service requirements.

SOURCE: "Wire Rope User's Manual," AISI, reproduced by permission.

are not preformed a minimum of two seizings on each side is recommended, and these should be spaced six rope diameters apart (see Fig. 8.2.128). Seizings should be made of soft or annealed wire or strand, and the width of the seizing should never be less than the diameter of the rope being seized. Table 8.2.71 lists suggested seizing wire diameters.

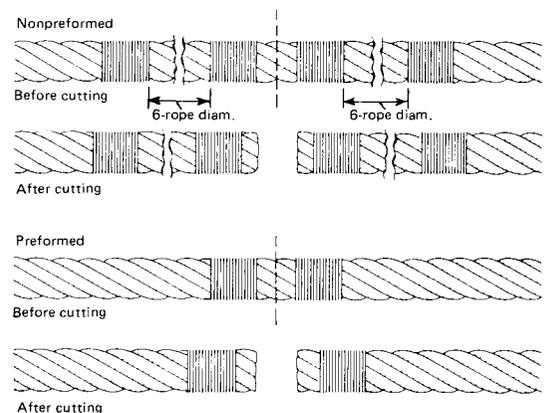
**Wire Rope Fittings or Terminations** End terminations allow forces to be transferred from rope to machine, or load to rope, etc. Figure 8.2.129 illustrates the most commonly used end fittings or terminations. Not all terminations will develop full strength. In fact, if all of the rope elements are not held securely, the individual strands will sustain unequal loads causing unequal wear among them, thus shortening the effective rope service life. Socketing allows an end fitting which reduces the chances of unequal strand loading.



**Fig. 8.2.127** Service life curves for various  $D/d$  ratios. Note that this curve takes into account only bending and tensile stresses. (Reproduced from "Wire Rope User's Manual," AISI, by permission.)

Wire rope manufacturers have developed a recommended procedure for socketing. A tight wire serving band is placed where the socket base will be, and the wires are unlaidd, straightened, and "broomed" out. Fiber core is cut close to the serving band and removed, wires are cleaned with a solvent such as SC-methyl chloroform, and brushed to remove dirt and grease. If additional cleaning is done with muriatic acid this must be followed by a neutralizing rinse (if possible, ultrasonic cleaning is preferred). The wires are dipped in flux, the socket is positioned, zinc (spelter) is poured and allowed to set, the serving band is removed, and the rope lubricated.

A somewhat similar procedure is used in thermoset resin socketing. Socketed terminations generally are able to develop 100 percent of nominal strength.



**Fig. 8.2.128** Seizings. (Reproduced from "Wire Rope User's Manual," AISI, by permission.)

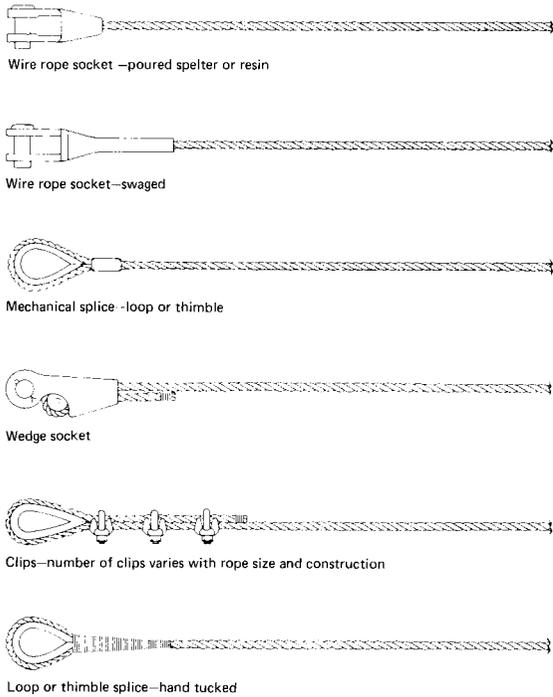


Fig. 8.2.129 End fittings, or terminations, showing the six most commonly used. (Reproduced from "Wire Rope User's Manual," AISI, by permission.)

Table 8.2.71 Seizing\*

Rope diameter		Suggested seizing wire diameter†	
in	mm	in	mm
1/8 - 5/16	3.5 - 8.0	0.032	0.813
3/8 - 9/16	9.4 - 14.5	0.048	1.21
5/8 - 1 1/16	16.0 - 24.0	0.063	1.60
1 - 1 1/16	26.0 - 33.0	0.080	2.03
1 3/8 - 1 11/16	35.0 - 43.0	0.104	2.64
1 3/4 and larger	45.0 and larger	0.124	3.15

\* Length of the seizing should not be less than the rope diameter.

† The diameter of seizing wire for elevator ropes is usually somewhat smaller than that shown in this table. Consult the wire rope manufacturer for specific size recommendations. Soft annealed seizing strand may also be used.

SOURCE: "Wire Rope User's Manual," AISI, reproduced by permission.

FIBER LINES

The breaking strength of various fiber lines is given in Table 8.2.72.

Knots, Hitches, and Bends

No two parts of a knot which would move in the same direction if the rope were to slip should lie alongside of and touching each other. The knots shown in Fig. 8.2.130 are known by the following names:

A, bight of a rope; B, simple or overhand knot; C, figure 8 knot; D, double knot; E, boat knot; F, bowline, first step; G, bowline, second step; H, bowline, completed; I, square or reef knot; J, sheet bend or weaver's knot; K, sheet bend with a toggle; L, carrick bend;

Table 8.2.72 Breaking Strength of Fiber Lines, Lb

Size, in		Manila	Composite	Sisal	Sisal mixed	Sisal hemp	Agave or jute	Nylon	Dacron	Polyethylene	Polypropylene (monofilament)	Esterlon (polyester)
Diam	Cir.											
3/16	5/8	450	—	360	340	310	270	1,000	850	700	800	720
1/4	3/4	600	—	480	450	420	360	1,500	1,380	1,200	1,200	1,150
5/16	1	1,000	—	800	750	700	600	2,500	2,150	1,750	2,100	1,750
3/8	1 1/8	1,350	—	1,080	1,010	950	810	3,500	3,000	2,500	3,100	2,450
7/16	1 1/4	1,750	—	1,400	1,310	1,230	1,050	4,800	4,500	3,400	3,700	3,400
1/2	1 1/2	2,650	—	2,120	1,990	1,850	1,590	6,200	5,500	4,100	4,200	4,400
5/16	1 3/4	3,450	—	2,760	2,590	2,410	2,070	8,300	7,300	4,600	5,100	5,700
5/8	2	4,400	—	3,520	3,300	3,080	2,640	10,500	9,500	5,200	5,800	7,300
3/4	2 1/4	5,400	—	4,320	4,050	3,780	3,240	14,000	12,500	7,400	8,200	9,500
13/16	2 1/2	6,500	—	5,200	4,880	4,550	3,900	17,000	15,000	8,900	9,800	11,500
7/8	2 3/4	7,700	—	—	—	—	—	20,000	17,500	10,400	11,500	13,500
1	3	9,000	—	7,200	6,750	6,300	5,400	24,000	20,000	12,600	14,000	16,500
1 1/16	3 1/4	10,500	—	8,400	7,870	7,350	6,300	28,000	22,500	14,500	16,100	19,000
1 1/8	3 1/2	12,000	—	9,600	9,000	8,400	7,200	32,000	25,000	16,500	18,300	21,500
1 1/4	3 3/4	13,500	—	10,800	10,120	9,450	8,100	36,500	28,500	18,600	21,000	24,300
1 1/16	4	15,000	—	12,000	11,250	10,500	9,000	42,000	32,000	21,200	24,000	28,000
1 1/2	4 1/2	18,500	16,600	14,800	13,900	12,950	11,100	51,000	41,000	26,700	30,000	34,500
1 5/8	5	22,500	20,300	18,000	16,900	15,800	13,500	62,000	50,000	32,700	36,500	41,500
1 3/4	5 1/2	26,500	23,800	21,200	19,900	18,500	15,900	77,500	61,000	39,500	44,000	51,000
2	6	31,000	27,900	24,800	23,200	21,700	18,600	90,000	72,000	47,700	53,000	61,000
2 1/8	6 1/2	36,000	—	—	—	—	—	105,000	81,000	55,800	62,000	70,200
2 1/4	7	41,000	36,900	32,800	30,800	28,700	—	125,000	96,000	63,000	70,000	81,000
2 1/2	7 1/2	46,500	—	—	—	—	—	138,000	110,000	72,500	80,500	92,000
2 5/8	8	52,000	46,800	41,600	39,000	36,400	—	154,000	125,000	81,000	90,000	103,000
2 7/8	8 1/2	58,000	—	—	—	—	—	173,000	140,000	92,000	100,000	116,000
3	9	64,000	57,500	51,200	48,000	44,800	—	195,000	155,000	103,000	116,000	130,000
3 1/4	10	77,000	69,300	61,600	57,800	53,900	—	238,000	190,000	123,000	137,000	160,000
3 1/2	11	91,000	—	—	—	—	—	288,000	230,000	146,000	162,000	195,000
4	12	105,000	94,500	84,000	78,800	73,500	—	342,000	275,000	171,000	190,000	230,000

Breaking strength is the maximum load the line will hold at the time of breaking. The working load of a line is one-fourth to one-fifth of the breaking strength. SOURCE: Adapted, by permission of the U.S. Naval Institute, Annapolis, MD, and Wall Rope Works, Inc., New York, NY.

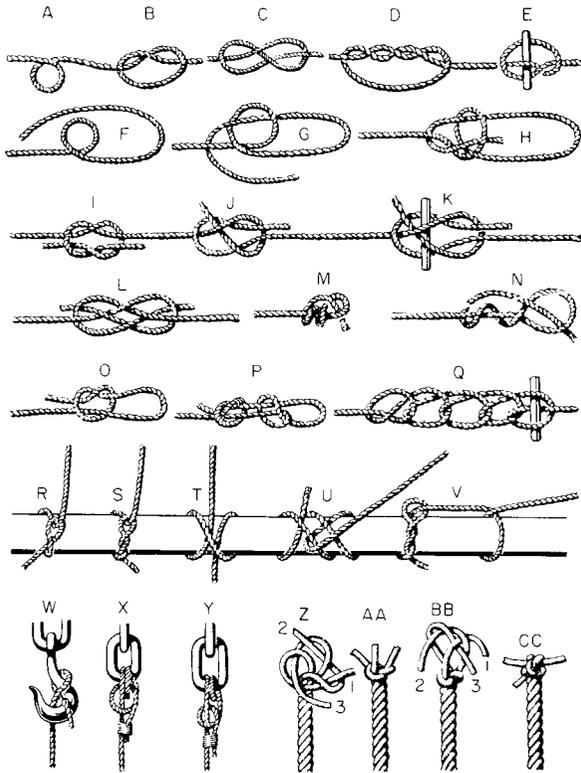


Fig. 8.2.130 Knots, hitches, and bends.

*M*, "stevedore" knot completed; *N*, "stevedore" knot commenced; *O*, slip knot; *P*, Flemish loop; *Q*, chain knot with toggle; *R*, half hitch; *S*, timber hitch; *T*, clove hitch; *U*, rolling hitch; *V*, timber hitch and half hitch; *W*, blackwall hitch; *X*, fisherman's bend; *Y*, round

turn and half hitch; *Z*, wall knot commenced; *AA*, wall knot completed; *BB*, wall-knot crown commenced; *CC*, wall-knot crown completed.

The bowline *H*, one of the most useful knots, will not slip, and after being strained is easily untied. Knots *H*, *K*, and *M* are easily untied after being under strain. The knot *M* is useful when the rope passes through an eye and is held by the knot, as it will not slip, and is easily untied after being strained. The wall knot is made as follows: Form a bight with strand 1 and pass the strand 2 around the end of it, and the strand 3 around the end of 2, and then through the bight of 1, as shown at *Z* in the figure. Haul the ends taut when the appearance is as shown in *AA*. The end of the strand 1 is now laid over the center of the knot, strand 2 laid over 1, and 3 over 2, when the end of 3 is passed through the bight of 1, as shown at *BB*. Haul all the strands taut, as shown at *CC*. The "stevedore" knot (*M, N*) is used to hold the end of a rope from passing through a hole. When the rope is strained, the knot draws up tight, but it can be easily untied when the strain is removed. If a knot or hitch of any kind is tied in a rope, its failure under stress is sure to occur at that place. The shorter the bend in the standing rope, the weaker is the knot. The approximate strength of knots compared with the full strength of (dry) rope (= 100), based on Miller's experiments (*Mach.*, 1900, p. 198), is as follows: eye splice over iron thimble, 90; short splice in rope, 80; *S* and *Y*, 65; *H*, *O*, and *T*, 60; *I* and *J*, 50; *B* and *P*, 45.

#### NAILS AND SPIKES

**Nails** are either **wire nails** of circular cross section and constant diameter or **cut nails** of rectangular cross section with taper from head to point. The larger sizes are called **spikes**. The length of the nail is expressed in the "penny" system, the equivalents in inches being given in Tables 8.2.73 to 8.2.75. The letter *d* is the accepted symbol for penny. A keg of nails weighs 100 lb. **Heavy hinge nails** or **track nails** with countersunk heads have chisel points unless diamond points are specified. **Plasterboard nails** are smooth with circumferential grooves and have diamond points. **Spikes** are made either with flat heads and diamond points or with oval heads and chisel points.

Table 8.2.73 Wire Nails for Special Purposes  
(Steel wire gage)

Length, in	Barrel nails		Barbed roofing nails		Barbed dowel nails	
	Gage	No. per lb	Gage	No. per lb	Gage	No. per lb
5/8	15½	1,570	—	—	8	394
¾	15½	1,315	13	729	8	306
7/8	14½	854	12	478	8	250
1	14½	750	12	416	8	212
1¼	14½	607	12	368	8	183
1½	14	539	11	250	8	16
1¾	13	386	11	228	8	145
1½	13	355	10	167	8	131
1¾	—	—	10	143		
2	—	—	9	104		

Length, in	Clout nails		Slatting nails		Fine nails		
	Gage	No. per lb	Gage	No. per lb	Length, in	Gage	No. per lb
¾	15	999					
7/8	14	733					
1	14	648	12	425	1	16½	1,280
1¼	14	580	10½	229			
1½	13	398	—	—	1	17	1,492
1¾	13	365					
1½	13	336	10½	190	1½	15	757
1¾	—	—	10	144	1½	16	984
2	—	—	9	104			

**Table 8.2.74 Wire Nails and Spikes**  
(Steel wire gage)

Size of nail	Length, in	Casing nails		Finishing nails		Clinch nails		Shingle nails	
		Gage	No. per lb	Gage	No. per lb	Gage	No. per lb	Gage	No. per lb
2d	1	15½	940	16½	1,473	14	723	13	434
3d	1¼	14½	588	15½	880	13	432	12	271
4d	1½	14	453	15	634	12	273	12	233
5d	1¾	14	389	15	535	12	234	12	203
6d	2	12½	223	13	288	11	158		
7d	2¼	12½	200	13	254	11	140		
8d	2½	11½	136	12½	196	10	101		
9d	2¾	11½	124	12½	178	10	91.4		
10d	3	10½	90	11½	124	9	70		
12d	3¼	10½	83	11½	113	9	64.1		
16d	3½	10	69	11	93	8	50		
20d	4	9	51	10	65	7	36.4		
30d	4½	9	45						
40d	5	8	37						

Size of nail	Length, in	Boat nails				Hinge nails				Flooring nails	
		Heavy		Light		Heavy		Light		Gage	No. per lb
		Diam, in	No. per lb	Diam, in	No. per lb	Diam, in	No. per lb	Diam, in	No. per lb		
4d	1½	¼	47	⅜ <sub>16</sub>	82	¼	53	⅜ <sub>16</sub>	90		
6d	2	¼	36	⅜ <sub>16</sub>	62	¼	39	⅜ <sub>16</sub>	66	11	168
8d	2½	¼	29	⅜ <sub>16</sub>	50	¼	31	⅜ <sub>16</sub>	53	10	105
10d	3	⅜	11	¼	24	⅜	12	¼	25	9	72
12d	3¼	⅜	10.4	¼	22	⅜	11	¼	23	8	56
16d	3½	⅜	9.6	¼	20	⅜	10	¼	22	7	44
20d	4	⅜	8	¼	18	⅜	8	¼	19	6	32

Size of nail	Length, in	Common wire nails and brads		Barbed car nails				Spikes			Approx no. per lb
		Gage	No. per lb	Heavy		Light		Size	Length, in	Gage	
				Gage	No. per lb	Gage	No. per lb				
2d	1	15	847	—	—	—	—	10d	3	6	43
3d	1¼	14	548	—	—	—	—	12d	3¼	6	39
4d	1½	12½	294	10	179	12	284	16d	3½	5	31
5d	1¾	12½	254	9	124	10	152	20d	4	4	23
6d	2	11½	167	9	108	10	132	30d	4½	3	18
7d	2¼	11½	150	8	80	9	95	40d	5	2	14
8d	2½	10¼	101	8	72	9	88	50d	5½	1	11
9d	2¾	10¼	92	7	55	8	65	50d	6	1	10
10d	3	9	66	7	50	8	59	—	7	⅜ in.	7
12d	3¼	9	61	6	39	7	46	—	8	⅜	4.1
16d	3½	8	47	6	36	7	43	—	9	⅜	3.7
20d	4	6	30	5	27	6	32	—	10	⅜	3.3
30d	4½	5	23	5	24	6	28	—	12	⅜	2.7
40d	5	4	18	4	18	5	22				
50d	5½	3	14	3	14	4	17				
60d	6	2	11	3	13	4	15				

**Table 8.2.75 Cut Steel Nails and Spikes**

(Sizes, lengths, and approximate number per lb)

Size	Length, in	Common	Clinch	Finishing	Casing and box	Fencing	Spikes	Barrel	Slating	Tobacco	Brads	Shingle
2d	1	740	400	1,100	—	—	—	450	340			
3d	1¼	460	260	880	—	—	—	280	280			
4d	1½	280	180	530	420	—	—	190	220			
5d	1¾	210	125	350	300	100	—	—	180	130		
6d	2	160	100	300	210	80	—	—	—	97	120	
7d	2¼	120	80	210	180	60	—	—	—	85	94	
8d	2½	88	68	168	130	52	—	—	—	68	74	90
9d	2¾	73	52	130	107	38	—	—	—	58	62	72
10d	3	60	48	104	88	26	—	—	—	48	50	60
12d	3¼	46	40	96	70	20	—	—	—	—	40	
16d	3½	33	34	86	52	18	17	—	—	—	27	
20d	4	23	24	76	38	16	14					
25d	4¼	20	—	—	—	—	—					
30d	4½	16½	—	—	30	—	11					
40d	5	12	—	—	26	—	9					
50d	5½	10	—	—	20	—	7½					
60d	6	8	—	—	16	—	6					
—	6½	—	—	—	—	—	5½					
—	7	—	—	—	—	—	5					

## WIRE AND SHEET-METAL GAGES

In the metal industries, the word *gage* has been used in various systems, or scales, for expressing the thickness or weight per unit area of thin plates, sheet, and strip, or the diameters of rods and wire. Specific diameters, thicknesses, or weights per square foot have been or are denoted in gage systems by certain numerals followed by the word *gage*, for example, no. 12 gage, or simply 12 gage. Gage numbers for flat rolled products have been used only in connection with thin materi-

als (Table 8.2.76). Heavier and thicker, flat rolled materials are usually designated by thickness in English or metric units.

There is considerable danger of confusion in the use of gage number in both foreign and domestic trade, which can be avoided by specifying thickness or diameter in inches or millimeters.

## DRILL SIZES

See Table 8.2.77.

**Table 8.2.76 Comparison of Standard Gages\***  
Thickness of diameter, in

Gage no.	BWG; Stubs Iron Wire	AWG; B&S	U.S. Steel Wire; Am. Steel & Wire; Washburn & Moen; Steel Wire	Galv. sheet steel	Manufacturers' standard
0000000	—	—	0.4900	—	—
000000	—	0.580000	0.4615	—	—
00000	—	0.516500	0.4305	—	—
0000	0.454	0.460000	0.3938	—	—
000	0.425	0.409642	0.3625	—	—
00	0.380	0.364796	0.3310	—	—
0	0.340	0.324861	0.3065	—	—
1	0.300	0.289297	0.2830	—	—
2	0.284	0.257627	0.2625	—	—
3	0.259	0.229423	0.2437	—	0.2391
4	0.238	0.204307	0.2253	—	0.2242
5	0.220	0.181940	0.2070	—	0.2092
6	0.203	0.162023	0.1920	—	0.1943
7	0.180	0.144285	0.1770	—	0.1793
8	0.165	0.128490	0.1620	0.1681	0.1644
9	0.148	0.114423	0.1483	0.1532	0.1495
10	0.134	0.101897	0.1350	0.1382	0.1345
11	0.120	0.090742	0.1205	0.1233	0.1196
12	0.109	0.080808	0.1055	0.1084	0.1046
13	0.095	0.071962	0.0915	0.0934	0.0897
14	0.083	0.064084	0.0800	0.0785	0.0747
15	0.072	0.057068	0.0720	0.0710	0.0673
16	0.065	0.050821	0.0625	0.0635	0.0598
17	0.058	0.045257	0.0540	0.0575	0.0538
18	0.049	0.040303	0.0475	0.0516	0.0478
19	0.042	0.035890	0.0410	0.0456	0.0418
20	0.035	0.031961	0.0348	0.0396	0.0359
21	0.032	0.028462	0.03175	0.0366	0.0329
22	0.028	0.025346	0.0286	0.0336	0.0299
23	0.025	0.022572	0.0258	0.0306	0.0269
24	0.022	0.020101	0.0230	0.0276	0.0239
25	0.020	0.017900	0.0204	0.0247	0.0209
26	0.018	0.015941	0.0181	0.0217	0.0179
27	0.016	0.014195	0.0173	0.0202	0.0164
28	0.014	0.012641	0.0162	0.0187	0.0149
29	0.013	0.011257	0.0150	0.0172	0.0135
30	0.012	0.010025	0.0140	0.0157	0.0120
31	0.010	0.008928	0.0132	0.0142	0.0105
32	0.009	0.007950	0.0128	0.0134	0.0097
33	0.008	0.007080	0.0118	—	0.0090
34	0.007	0.006305	0.0104	—	0.0082
35	0.005	0.005615	0.0095	—	0.0075
36	0.004	0.005000	0.0090	—	0.0067
37	—	0.004453	0.0085	—	0.0064
38	—	0.003965	0.0080	—	0.0060
39	—	0.003531	0.0075	—	—
40	—	0.003144	0.0070	—	—

\* Principal uses—BWG: strips, bands, hoops, and wire; AWG or B&S: nonferrous sheets, rod, and wire; U.S. Steel Wire: steel wire except music wire; manufacturers' standard: uncoated steel sheets.

**Table 8.2.77 Diameters of Small Drills**  
 Number, letter, metric, and fractional drills in order of size (rounded to 4 decimal places)

No.	Ltr	mm	in	Diam, in	No.	Ltr	mm	in	Diam, in	No.	Ltr	mm	in	Diam, in
		0.10		0.0039	48				0.0760			4.40		0.1732
		0.15		0.0059			1.95		0.0767	16				0.1770
		0.20		0.0079				$\frac{5}{64}$	0.0781			4.50		0.1772
		0.25		0.0098	47				0.0785	15				0.1800
		0.30		0.0118			2.00		0.0787			4.60		0.1811
80				0.0135			2.05		0.0807	14				0.1820
		0.35		0.0137	46				0.0810	13				0.1850
79			$\frac{1}{64}$	0.0145	45				0.0820			4.70		0.1850
				0.0156			2.10		0.0827			4.75		0.1850
		0.40		0.0157			2.15		0.0846				$\frac{3}{16}$	0.1875
78				0.0160	44				0.0860			4.80		0.1890
		0.45		0.0177			2.20		0.0866	12				0.1890
77				0.0180			2.25		0.0885	11				0.1910
		0.50		0.0197	43				0.0890			4.90		0.1920
76				0.0200			2.30		0.0906	10				0.1935
75				0.0210			2.35		0.0925	9				0.1960
		0.55		0.0216	42				0.0935			5.00		0.1968
74				0.0225				$\frac{3}{32}$	0.0937	8				0.1990
		0.60		0.0236			2.40		0.0945			5.10		0.2008
73				0.0240	41				0.0960					0.2010
72				0.0250			2.45		0.0964				$\frac{13}{64}$	0.2031
		0.65		0.0255	40				0.0980	6				0.2040
71				0.0260			2.50		0.0984			5.20		0.2047
		0.70		0.0275	39				0.0995	5				0.2055
70				0.0280	38				0.1015			5.25		0.2066
69				0.0292			2.60		0.1024			5.30		0.2087
		0.75		0.0295	37				0.1040	4				0.2090
68				0.0310			2.70		0.1063			5.40		0.2126
		0.80	$\frac{1}{32}$	0.0312	36				0.1065	3				0.2130
				0.0314			2.75		0.1082			5.50		0.2165
67				0.0320				$\frac{7}{64}$	0.1093				$\frac{7}{32}$	0.2187
66				0.0330	35				0.1100			5.60		0.2205
		0.85		0.0334			2.80		0.1102	2				0.2210
65				0.0350	34				0.1110			5.70		0.2244
		0.90		0.0354	33				0.1130			5.75		0.2263
64				0.0360			2.90		0.1142	1				0.2280
63				0.0370	32				0.1160			5.80		0.2283
		0.95		0.0374			3.00		0.1181			5.90		0.2323
62				0.0380	31				0.1200		A			0.2340
61				0.0390			3.10		0.1220				$\frac{15}{64}$	0.2340
		1.00		0.0393				$\frac{1}{8}$	0.1250			6.00		0.2362
60				0.0400			3.20		0.1260		B			0.2380
59				0.0410			3.25		0.1279			6.10		0.2402
		1.05		0.0413	30				0.1285		C			0.2420
58				0.0420			3.30		0.1299			6.20		0.2441
57				0.0430			3.40		0.1339		D			0.2460
		1.10		0.0433	29				0.1360			6.25		0.2460
		1.15		0.0452			3.50		0.1378			6.30		0.2480
56				0.0465	28				0.1405		E		$\frac{1}{4}$	0.2500
			$\frac{3}{64}$	0.0468				$\frac{9}{64}$	0.1406			6.40		0.2519
		1.20		0.0472			3.60		0.1417			6.50		0.2559
		1.25		0.0492	27				0.1440		F			0.2570
		1.30		0.0512			3.70		0.1457			6.60		0.2598
55				0.0520	26				0.1470		G			0.2610
		1.35		0.0531			3.75		0.1476			6.70		0.2637
54				0.0550	25				0.1495				$\frac{17}{64}$	0.2656
		1.40		0.0551			3.80		0.1496			6.75		0.2657
		1.45		0.0570	24				0.1520		H			0.2660
		1.50		0.0590			3.90		0.1535			6.80		0.2677
53				0.0595	23				0.1540			6.90		0.2717
		1.55		0.0610				$\frac{5}{32}$	0.1562		I			0.2720
			$\frac{1}{16}$	0.0625	22				0.1570			7.00		0.2756
		1.60		0.0630			4.00		0.1575		J			0.2770
				0.0635	21				0.1590			7.10		0.2795
52				0.0649	20				0.1610		K			0.2810
		1.70		0.0669			4.10		0.1614				$\frac{9}{32}$	0.2812
				0.0670			4.2		0.1654			7.20		0.2835
51				0.0688	19				0.1660			7.25		0.2854
		1.75		0.0700			4.25		0.1673			7.30		0.2874
50				0.0709			4.30		0.1693		L			0.2900
		1.80		0.0728	18				0.1695			7.40		0.2913
		1.85		0.0730				$\frac{11}{64}$	0.1718		M			0.2950
49				0.0748	17				0.1730			7.50		0.2953

Table 8.2.77 Diameters of Small Drills (Continued)

No.	Ltr	mm	in	Diam, in	No.	Ltr	mm	in	Diam, in	No.	Ltr	mm	in	Diam, in
			<sup>19</sup> / <sub>64</sub>	0.2968			9.70		0.3819			17.00		0.6693
		7.60		0.2992			9.75		0.3838				<sup>43</sup> / <sub>64</sub>	0.6718
	N			0.3020			9.80		0.3858				<sup>11</sup> / <sub>16</sub>	0.6875
		7.70		0.3031		W			0.3860			17.50		0.6890
		7.75		0.3051			9.90		0.3898				<sup>45</sup> / <sub>64</sub>	0.7031
		7.80		0.3071				<sup>25</sup> / <sub>64</sub>	0.3906			18.00		0.7087
		7.90		0.3110			10.00		0.3937				<sup>23</sup> / <sub>32</sub>	0.7187
			<sup>5</sup> / <sub>16</sub>	0.3125		X			0.3970			18.50		0.7283
		8.00		0.3150		Y			0.4040				<sup>47</sup> / <sub>64</sub>	0.7374
	O			0.3160				<sup>13</sup> / <sub>32</sub>	0.4062			19.00		0.7480
		8.10		0.3189		Z			0.4130				<sup>3</sup> / <sub>4</sub>	0.7500
		8.20		0.3228			10.50		0.4134				<sup>49</sup> / <sub>64</sub>	0.7656
	P			0.3230				<sup>27</sup> / <sub>64</sub>	0.4218			19.50		0.7677
		8.25		0.3248			11.00		0.4331				<sup>25</sup> / <sub>32</sub>	0.7812
		8.30		0.3268				<sup>7</sup> / <sub>16</sub>	0.4375			20.00		0.7874
			<sup>21</sup> / <sub>64</sub>	0.3281			11.50		0.4528				<sup>51</sup> / <sub>64</sub>	0.7968
		8.40		0.3307				<sup>29</sup> / <sub>64</sub>	0.4531			20.50		0.8070
	Q			0.3320				<sup>15</sup> / <sub>32</sub>	0.4687				<sup>13</sup> / <sub>16</sub>	0.8125
		8.50		0.3346			12.00		0.4724			21.00		0.8267
		8.60		0.3386				<sup>31</sup> / <sub>64</sub>	0.4843				<sup>53</sup> / <sub>64</sub>	0.8281
	R			0.3390			12.50		0.4921				<sup>27</sup> / <sub>32</sub>	0.8437
		8.70		0.3425				<sup>1</sup> / <sub>2</sub>	0.5000			21.50		0.8464
			<sup>11</sup> / <sub>32</sub>	0.3437			13.00		0.5118				<sup>55</sup> / <sub>64</sub>	0.8593
		8.75		0.3444				<sup>33</sup> / <sub>64</sub>	0.5156			22.00		0.8661
		8.80		0.3464				<sup>17</sup> / <sub>32</sub>	0.5312				<sup>7</sup> / <sub>8</sub>	0.8750
	S			0.3480			13.50		0.5315			22.50		0.8858
		8.90		0.3504				<sup>35</sup> / <sub>64</sub>	0.5468				<sup>57</sup> / <sub>64</sub>	0.8906
		9.00		0.3543			14.00		0.5512			23.00		0.9055
	T			0.3580				<sup>9</sup> / <sub>16</sub>	0.5625				<sup>29</sup> / <sub>32</sub>	0.9062
		9.10		0.3583			14.50		0.5708				<sup>59</sup> / <sub>64</sub>	0.9218
			<sup>23</sup> / <sub>64</sub>	0.3593				<sup>37</sup> / <sub>64</sub>	0.5781			23.50		0.9251
		9.20		0.3622			15.00		0.5905				<sup>15</sup> / <sub>16</sub>	0.9375
		9.25		0.3641				<sup>19</sup> / <sub>32</sub>	0.5937			24.00		0.9448
		9.30		0.3661				<sup>39</sup> / <sub>64</sub>	0.6093				<sup>61</sup> / <sub>64</sub>	0.9531
	U			0.3680			15.50		0.6102			24.50		0.9646
		9.40		0.3701				<sup>5</sup> / <sub>8</sub>	0.6250				<sup>31</sup> / <sub>32</sub>	0.9687
		9.50		0.3740			16.00		0.6299			25.00		0.9842
	V		<sup>3</sup> / <sub>8</sub>	0.3750				<sup>41</sup> / <sub>64</sub>	0.6406				<sup>63</sup> / <sub>64</sub>	0.9843
				0.3770			16.50		0.6496				1.0	1.0000
		9.60		0.3780				<sup>21</sup> / <sub>32</sub>	0.6562			25.50		1.0039

SOURCE: Adapted from Colvin and Stanley, "American Machinists' Handbook," 8th ed., McGraw-Hill, New York, 1945.

## 8.3 GEARING

by George W. Michalec

REFERENCES: Buckingham, "Manual of Gear Design," Industrial Press. Cunningham, Noncircular Gears, *Mach. Des.*, Feb. 19, 1957. Cunningham and Cunningham, Rediscovering the Noncircular Gear, *Mach. Des.*, Nov. 1, 1973. Dudley, "Gear Handbook," McGraw-Hill. Dudley, "Handbook of Practical Gear Design," McGraw-Hill. Michalec, "Precision Gearing: Theory and Practice," Wiley. Shigely, "Engineering Design," McGraw-Hill. AGMA Standards. "Gleason Bevel and Hypoid Gear Design," Gleason Works, Rochester. "Handbook of Gears: Inch and Metric" and "Elements of Metric Gear Technology," Designatronics, New Hyde Park, NY. Adams, "Plastics Gearing: Selection and Application," Marcel Dekker.

### Notation

$a$  = addendum  
 $b$  = dedendum  
 $B$  = backlash, linear measure along pitch circle  
 $c$  = clearance

$C$  = center distance  
 $d$  = pitch diam of pinion  
 $d_b$  = base circle diam of pinion  
 $d_o$  = outside diam of pinion  
 $d_r$  = root diam of pinion  
 $D$  = pitch diameter of gear  
 $D_P$  = pitch diam of pinion  
 $D_G$  = pitch diam of gear  
 $D_o$  = outside diam of gear  
 $D_b$  = base circle diam of gear  
 $D_t$  = throat diam of wormgear  
 $F$  = face width  
 $h_k$  = working depth  
 $h_t$  = whole depth  
 $\text{inv } \phi$  = involute function ( $\tan \phi - \phi$ )

- $l$  = lead (advance of worm or helical gear in 1 rev)
- $l_p(l_G)$  = lead of pinion (gear) in helical gears
- $L$  = lead of worm in one revolution
- $m$  = module
- $m_G$  = gear ratio ( $m_G = N_G/N_P$ )
- $m_p$  = contact ratio (of profiles)
- $M$  = measurement of over pins
- $n_p(n_G)$  = speed of pinion (gear), r/min
- $N_p(N_G)$  = number of teeth in pinion (gear)
- $n_w$  = number of threads in worm
- $p$  = circular pitch
- $p_b$  = base pitch
- $p_n$  = normal circular pitch of helical gear
- $P_d$  = diametral pitch
- $P_{dn}$  = normal diametral pitch
- $R$  = pitch radius
- $R_c$  = radial distance from center of gear to center of measuring pin
- $R_p(R_G)$  = pitch radius of pinion (gear)
- $R_T$  = testing radius when rolled on a variable-center-distance inspection fixture
- $s$  = stress
- $t$  = tooth thickness
- $t_n$  = normal circular tooth thickness
- $T_p(T_G)$  = formative number of teeth in pinion (gear) (in bevel gears)
- $v$  = pitch line velocity
- $X$  = correction factor for profile shift
- $\alpha$  = addendum angle of bevel gear
- $\gamma$  = pitch angle of bevel pinion
- $\gamma_R$  = face angle at root of bevel pinion tooth
- $\gamma_o$  = face angle at tip of bevel pinion tooth
- $\Gamma$  = pitch angle of bevel gear
- $\Gamma_R$  = face angle at root of bevel gear tooth
- $\Gamma_{g_o}$  = face angle at tip of bevel gear tooth
- $\delta$  = dedendum angle of bevel gear
- $\Delta C$  = relatively small change in center distance  $C$
- $\phi$  = pressure angle
- $\phi_n$  = normal pressure angle
- $\psi$  = helix or spiral angle
- $\psi_p(\psi_G)$  = helix angle of teeth in pinion (gear)
- $\Sigma$  = shaft angle of meshed bevel pair

**BASIC GEAR DATA**

**Gear Types** Gears are grouped in accordance with tooth forms, shaft arrangement, pitch, and quality. Tooth forms and shaft arrangements are:

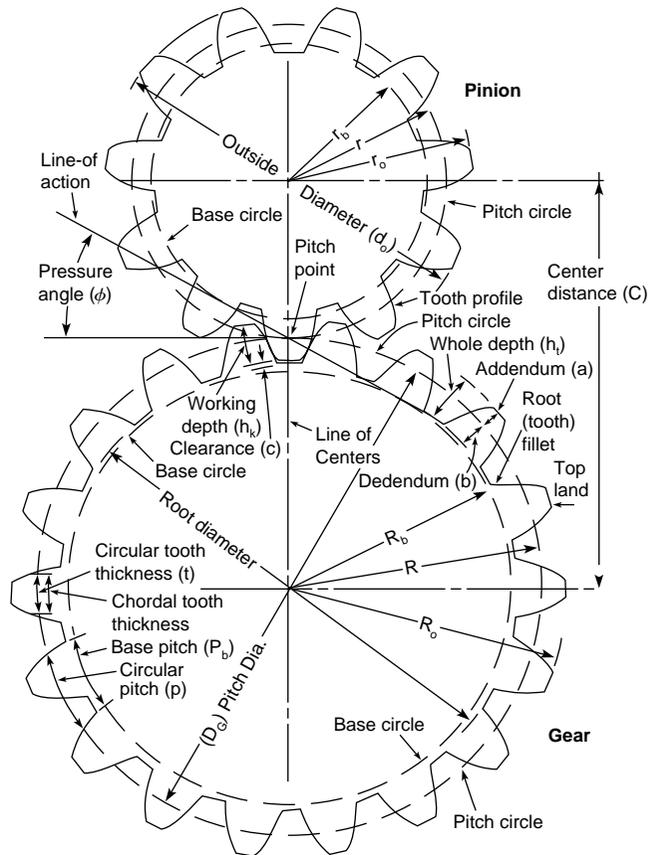
Tooth form	Shaft arrangement
Spur	Parallel
Helical	Parallel or skew
Worm	Skew
Bevel	Intersecting
Hypoid	Skew

**Pitch definitions** (see Fig. 8.3.1). **Diametral pitch**  $P_d$  is the ratio of number of teeth in the gear to the diameter of the pitch circle  $D$  measured in inches,  $P_d = N/D$ . **Circular pitch**  $p$  is the linear measure in inches along the pitch circle between corresponding points of adjacent teeth. From these definitions,  $P_d p = \pi$ . The **base pitch**  $p_b$  is the distance along the line of action between successive involute tooth surfaces. The base and circular pitches are related as  $p_b = p \cos \phi$ , where  $\phi$  = the pressure angle.

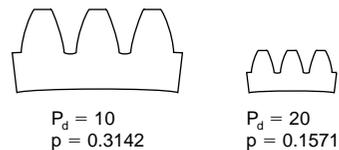
**Pitch circle** is the imaginary circle that rolls without slippage with a pitch circle of a mating gear. The pitch (circle) diameter equals  $D = N/P_d = Np/\pi$ . The basic relation between  $P_d$  and  $p$  is  $P_d p = \pi$ .

**Tooth size** is related to pitch. In terms of diametral pitch  $P_d$ , the relationship is inverse: i.e., large  $P_d$  implies a small tooth, and small  $P_d$

implies a large tooth. Conversely, there is a direct relationship between tooth size and circular pitch  $p$ . A small tooth has a small  $p$ , but a large tooth has a large  $p$ . (See Fig. 8.3.1b.) In terms of  $P_d$ , coarse teeth comprise  $P_d$  less than 20; fine teeth comprise  $P_d$  of 20 and higher. (See Fig. 8.3.1b.) **Quality** of gear teeth is classified as commercial, precision, and ultraprecision.



**Fig. 8.3.1a** Basic gear geometry and nomenclature.



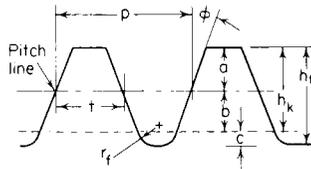
**Fig. 8.3.1b** Comparison of pitch and tooth size.

**Pressure angle**  $\phi$  for all gear types is the acute angle between the common normal to the profiles at the contact point and the common normal to the pitch plane. For spur gears it is simply the acute angle formed by the common tangent between base circles of mating gears and a normal to the line of centers. For standard gears, pressure angles of  $14\frac{1}{2}^\circ$ ,  $20^\circ$ , and  $25^\circ$  have been adopted by ANSI and the gear industry (see Fig. 8.3.1a). The  $20^\circ$  pressure angle is most widely used because of its versatility. The higher pressure angle  $25^\circ$  provides higher strength for highly loaded gears. Although  $14\frac{1}{2}^\circ$  appears in standards, and in past decades was extensively used, it is used much less than  $20^\circ$ . The  $14\frac{1}{2}^\circ$  standard is still used for replacement gears in old design equipment, in applications where backlash is critical, and where advantage can be taken of lower backlash with change in center distance.

The **base circle** (or **base cylinder**) is the circle from which the involute tooth profiles are generated. The relationship between the base-circle and pitch-circle diameter is  $D_b = D \cos \phi$ .

**Tooth proportions** are established by the addendum, dedendum, working depth, clearance, tooth circular thickness, and pressure angle (see Fig. 8.3.1). In addition, gear face width  $F$  establishes thickness of the gear measured parallel to the gear axis.

For involute teeth, proportions have been standardized by ANSI and AGMA into a limited number of systems using a basic rack for specification (see Fig. 8.3.2 and Table 8.3.1). Dimensions for the basic rack are normalized for diametral pitch = 1. Dimensions for a specific pitch are



**Fig. 8.3.2** Basic rack for involute gear systems.  $a$  = addendum;  $b$  = dedendum;  $c$  = clearance;  $h_k$  = working depth;  $h_t$  = whole depth;  $p$  = circular pitch;  $r_f$  = fillet radius;  $t$  = tooth thickness;  $\phi$  = pressure angle.

obtained by dividing by the pitch. Standards for basic involute spur, helical and face gear designs, and noninvolute bevel and wormgear designs are listed in Table 8.3.2.

**Gear ratio** (or **mesh ratio**)  $m_G$  is the ratio of number of teeth in a meshed pair, expressed as a number greater than 1;  $m_G = N_G/N_p$ , where the pinion is the member having the lesser number of teeth. For spur and parallel-shaft helical gears, the base circle ratio must be identical to the gear ratio. The speed ratio of gears is inversely proportionate to their numbers of teeth. Only for standard spur and parallel-shaft helical gears is the pitch diameter ratio equal to the gear ratio and inversely proportionate to the speed ratio.

**Metric Gears—Tooth Proportions and Standards**

Metric gearing not only is based upon different units of length measure but also involves its own unique design standard. This means that metric gears and American-standard-inch diametral-pitch gears are not interchangeable.

In the metric system the *module*  $m$  is analogous to pitch and is defined as

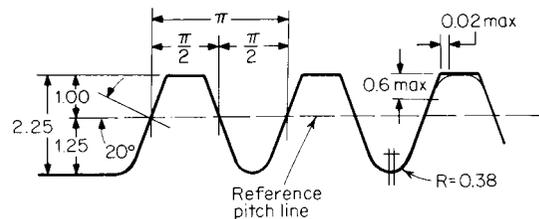
$$m = \frac{D}{N} = \text{mm of pitch diameter per tooth}$$

**Table 8.3.2** Gear System Standards

Gear type	ANSI/AGMA no.	Title
Spur and helical	201.02	Tooth Proportions for Coarse-Pitch Involute Spur Gears
Spur and helical	1003-G93	Tooth Proportions for Fine-Pitch Spur and Helical Gearing
Spur and helical	370.01	Design Manual for Fine-Pitch Gearing
Bevel gears	2005-B88	Design Manual for Bevel Gears (Straight, Zerol, Spiral, and Hypoid)
Worm gearing	6022-C93	Design of General Industrial Coarse-Pitch Cylindrical Worm Gearing
Worm gearing	6030-C87	Design of Industrial Double-Enveloping Worm Gearing
Face gears	203.03	Fine-Pitch on Center-Face Gears for 20-Degree Involute Spur Pinions

Note that, for the module to have proper units, the pitch diameter must be in millimeters.

The **metric module** was developed in a number of versions that differ in minor ways. The German DIN standard is widely used in Europe and other parts of the world. The Japanese have their own version defined in JS standards. Deviations among these and other national standards are minor, differing only as to dedendum size and root radii. The differences have been resolved by the new unified module standard promoted by the International Standards Organization (ISO). This unified version (Fig. 8.3.3) conforms to the new SI in all respects. All major industrial



**Fig. 8.3.3** The ISO basic rack for metric module gears.

countries on the metric system have shifted to this ISO standard, which also is the basis for American metric gearing. Table 8.3.3 lists pertinent current ISO metric standards.

**Table 8.3.1** Tooth Proportions of Basic Rack for Standard Involute Gear Systems

Tooth parameter (of basic rack)	Symbol, Figs. 8.3.1a and 8.3.2	Tooth proportions for various standard systems					
		1	2	3	4	5	6
		Full-depth involute, 14½°	Full-depth involute, 20°	Stub involute, 20°	Coarse-pitch involute spur gears, 20°	Coarse-pitch involute spur gears, 25°	Fine-pitch involute, 20°
1. System sponsors		ANSI and AGMA	ANSI	ANSI and AGMA	AGMA	AGMA	ANSI and AGMA
2. Pressure angle	$\phi$	14½°	20°	20°	20°	25°	20°
3. Addendum	$a$	1/P <sub>d</sub>	1/P <sub>d</sub>	0.8/P <sub>d</sub>	1.000/P <sub>d</sub>	1.000/P <sub>d</sub>	1.000/P <sub>d</sub>
4. Min dedendum	$b$	1.157/P <sub>d</sub>	1.157/P <sub>d</sub>	1/P <sub>d</sub>	1.250/P <sub>d</sub>	1.250/P <sub>d</sub>	1.200/P <sub>d</sub> + 0.002
5. Min whole depth	$h_t$	2.157/P <sub>d</sub>	2.157/P <sub>d</sub>	1.8/P <sub>d</sub>	2.250/P <sub>d</sub>	2.250/P <sub>d</sub>	2.2002/P <sub>d</sub> 0.002 in
6. Working depth	$h_k$	2/P <sub>d</sub>	2/P <sub>d</sub>	1.6/P <sub>d</sub>	2.000/P <sub>d</sub>	2.000/P <sub>d</sub>	2.000/P <sub>d</sub>
7. Min clearance	$h_c$	0.157/P <sub>d</sub>	0.157/P <sub>d</sub>	0.200/P <sub>d</sub>	0.250/P <sub>d</sub>	0.250/P <sub>d</sub>	0.200/P <sub>d</sub> + 0.002 in
8. Basic circular tooth thickness on pitch line	$t$	1.5708/P <sub>d</sub>	1.5708/P <sub>d</sub>	1.5708/P <sub>d</sub>	$\pi/(2P_d)$	$\pi/(2P_d)$	1.5708/P <sub>d</sub>
9. Fillet radius in basic rack	$r_f$	1/3 × clearance	1/2 × clearance	Not standardized	0.300/P <sub>d</sub>	0.300/P <sub>d</sub>	Not standardized
10. Diametral pitch range		Not specified	Not specified	Not specified	19.99 and coarser	19.99 and coarser	20 and finer
11. Governing standard:							
ANSI		B6.1	B6.1	B6.1			
AGMA		201.02		201.02	201.02	201.02	1,003–G93

Table 8.3.3 ISO Metric Gearing Standards

ISO 53: 1974	Cylindrical gears for general and heavy engineering—Basic rack
ISO 54: 1977	Cylindrical gears for general and heavy engineering—Modules and diametral pitches
ISO 677: 1976	Straight bevel gears for general and heavy engineering—Basic rack
ISO 678: 1976	Straight bevel gears for general and heavy engineering—Modules and diametral pitches
ISO 701: 1976	International gear notation—symbols for geometric data
ISO 1122-1: 1983	Glossary of gear terms—Part 1: Geometric definitions
ISO 1328: 1975	Parallel involute gears—ISO system of accuracy
ISO 1340: 1976	Cylindrical gears—Information to be given to the manufacturer by the purchaser in order to obtain the gear required
ISO 1341: 1976	Straight bevel gears—Information to be given to the manufacturer by the purchaser in order to obtain the gear required
ISO 2203: 1973	Technical drawings—Conventional representation of gears

Tooth proportions for standard spur and helical gears are given in terms of the basic rack. Dimensions, in millimeters, are normalized for module  $m = 1$ . Corresponding values for other modules are obtained by multiplying each dimension by the value of the specific module  $m$ . Major tooth parameters are described by this standard:

**Tooth form:** Straight-sided and full-depth, forming the basis of a family of full-depth interchangeable gears.

**Pressure angle:** 20°, conforming to worldwide acceptance.

**Addendum:** Equal to module  $m$ , which corresponds to the American practice of  $1/P_d = \text{addendum}$ .

**Dedendum:** Equal to 12.5 $m$ , which corresponds to the American practice of  $1.25/P_d = \text{dedendum}$ .

**Root radius:** Slightly greater than American standards specifications.

**Tip radius:** A maximum is specified, whereas American standards do not specify. In practice, U.S. manufacturers can specify a tip radius as near zero as possible.

Note that the basic racks for metric and American inch gears are essentially identical, but metric and American standard gears are not interchangeable.

The preferred standard gears of the metric system are not interchangeable with the preferred diametral-pitch sizes. Table 8.3.4 lists commonly used pitches and modules of both systems (preferred values are boldface).

Metric gear use in the United States, although expanding, is still a small percentage of total gearing. Continuing industry conversions and imported equipment replacement gearing are building an increasing demand for metric gearing. The reference list cites a domestic source of stock metric gears of relatively small size, in medium and fine pitches. Large diameter coarse pitch metric gears are made to order by many gear fabricators.

Table 8.3.4 Metric and American Gear Equivalents

Diametral pitch $P_d$	Module $m$	Circular pitch		Circular tooth thickness		Addendum	
		in	mm	in	mm	in	mm
<b>1/2</b>	50.8000	6.2832	159.593	3.1416	79.7965	2.0000	50.8000
0.5080	<b>50</b>	6.1842	157.080	3.0921	78.5398	1.9685	50
0.5644	45	5.5658	141.372	2.7850	70.6858	1.7730	45
0.6048	<b>42</b>	5.1948	131.947	2.5964	65.9734	1.6529	42
0.6513	39	4.8237	122.522	2.4129	61.2610	1.5361	39
0.7056	<b>36</b>	4.4527	113.097	2.2249	56.5487	1.4164	36
0.7697	33.8667	4.1888	106.396	2.0943	53.1977	1.3333	33.8667
0.8467	33	4.0816	103.673	2.0400	51.8363	1.2987	33
0.9407	<b>30</b>	3.7105	94.248	1.8545	47.1239	1.1806	30
	27	3.3395	84.823	1.6693	42.4115	1.0627	27
<b>1</b>	25.4000	3.1416	79.800	1.5708	39.8984	1.0000	25.4001
1.0583	<b>24</b>	2.9685	75.398	1.4847	37.6991	0.9452	24
1.1546	22	2.7210	69.115	1.3600	34.5575	0.8658	22
1.2700	<b>20</b>	2.4737	62.832	1.2368	31.4159	0.7874	20
1.4111	<b>18</b>	2.2263	56.548	1.1132	28.2743	0.7087	18
1.5875	16.9333	2.0944	53.198	1.0472	26.5988	0.6667	16.933
1.8143	<b>16</b>	1.9790	50.267	0.9894	25.1327	0.6299	16
2.1167	<b>14</b>	1.7316	43.983	0.8658	21.9911	0.5512	14
	12.7000	1.5708	39.898	0.7854	19.949	0.5000	12.7000
	<b>12</b>	1.4842	37.699	0.7420	18.8496	0.4724	12
<b>2.5</b>	10.1600	1.2566	31.918	0.6283	15.9593	0.4000	10.1600
2.5400	<b>10</b>	1.2368	31.415	0.6184	15.7080	0.3937	10
2.8222	9	1.1132	28.275	0.5565	14.1372	0.3543	9
<b>3</b>	8.4667	1.0472	26.599	0.5235	13.2995	0.3333	8.4667
3.1416	8.0851	1.0000	25.400	0.5000	12.7000	0.3183	8.0851
3.1750	<b>8</b>	0.9895	25.133	0.4948	12.5664	0.3150	8.00
3.6286	7.2571	0.8976	22.799	0.4488	11.3994	0.2857	7.2571
3.9078	<b>7</b>	0.8658	21.991	0.4329	10.9956	0.2756	7.000
	6.5	0.8039	20.420	0.4020	10.2101	0.2559	6.5
<b>4</b>	6.3500	0.7854	19.949	0.3927	9.9746	0.2500	6.3500
4.2333	<b>6</b>	0.7421	18.850	0.3710	9.4248	0.2362	6.0000
4.6182	5.5	0.6803	17.279	0.3401	8.6394	0.2165	5.5
<b>5</b>	5.0801	0.6283	15.959	0.3142	7.9794	0.2000	5.080
5.0802	5	0.6184	15.707	0.3092	7.8537	0.1968	5.000
5.3474	<b>4.75</b>	0.5875	14.923	0.2938	7.4612	0.1870	4.750
5.6444	<b>4.5</b>	0.5566	14.138	0.2783	7.0688	0.1772	4.500
<b>6</b>	4.2333	0.5236	13.299	0.2618	6.6497	0.1667	4.233
6.3500	<b>4</b>	0.4947	12.565	0.2473	6.2827	0.1575	4.000
6.7733	<b>3.75</b>	0.4638	11.781	0.2319	5.8903	0.1476	3.750
<b>7</b>	3.6286	0.4488	11.399	0.2244	5.6998	0.1429	3.629

Table 8.3.4 Metric and American Gear Equivalents (Continued)

Diametral pitch $P_d$	Module $m$	Circular pitch		Circular tooth thickness		Addendum	
		in	mm	in	mm	in	mm
7.2571	<b>3.5</b>	0.4329	10.996	0.2164	5.4979	0.1378	3.500
7.8154	<b>3.25</b>	0.4020	10.211	0.2010	5.1054	0.1279	3.250
<b>8</b>	3.1750	0.3927	9.974	0.1964	4.9886	0.1250	3.175
8.4667	<b>3</b>	0.3711	9.426	0.1855	4.7130	0.1181	3.000
<b>9</b>	2.8222	0.3491	8.867	0.1745	4.4323	0.1111	2.822
9.2364	<b>2.75</b>	0.3401	8.639	0.1700	4.3193	0.1082	2.750
<b>10</b>	2.5400	0.3142	7.981	0.1571	3.9903	0.1000	2.540
10.1600	<b>2.50</b>	0.3092	7.854	0.1546	3.9268	0.0984	2.500
<b>11</b>	2.3091	0.2856	7.254	0.1428	3.6271	0.0909	2.309
11.2889	<b>2.25</b>	0.2783	7.069	0.1391	3.5344	0.0886	2.250
<b>12</b>	2.1167	0.2618	6.646	0.1309	3.3325	0.0833	2.117
12.7000	<b>2</b>	0.2474	6.284	0.1236	3.1420	0.0787	2.000
<b>13</b>	1.9538	0.2417	6.139	0.1208	3.0696	0.0769	1.954
<b>14</b>	1.8143	0.2244	5.700	0.1122	2.8500	0.0714	1.814
14.5143	<b>1.75</b>	0.2164	5.497	0.1082	2.7489	0.0689	1.750
<b>15</b>	1.6933	0.2094	5.319	0.1047	2.6599	0.0667	1.693
<b>16</b>	1.5875	0.1964	4.986	0.0982	2.4936	0.0625	1.587
16.9333	<b>1.5</b>	0.1855	4.712	0.0927	2.3562	0.0591	1.500
<b>18</b>	1.4111	0.1745	4.432	0.0873	2.2166	0.0556	1.411
<b>20</b>	1.2700	0.1571	3.990	0.0785	1.9949	0.0500	1.270
20.3200	<b>1.25</b>	0.1546	3.927	0.0773	1.9635	0.0492	1.250
22	1.1545	0.1428	3.627	0.0714	1.8136	0.0455	1.155
<b>24</b>	1.0583	0.1309	3.325	0.0655	1.6624	0.0417	1.058
25.4000	<b>1</b>	0.1237	3.142	0.0618	1.5708	0.0394	1.000
28	0.90701	0.1122	2.850	0.0561	1.4249	0.0357	0.9071
28.2222	<b>0.9</b>	0.1113	2.827	0.0556	1.4137	0.0354	0.9000
30	0.84667	0.1047	2.659	0.0524	1.3329	0.0333	0.8467
31.7500	<b>0.8</b>	0.0989	2.513	0.04945	1.2566	0.0315	0.8000
<b>32</b>	0.79375	0.0982	2.494	0.04909	1.2468	0.0313	0.7937
33.8667	<b>0.75</b>	0.0928	2.357	0.04638	1.1781	0.0295	0.7500
36	0.70556	0.0873	2.217	0.04363	1.1083	0.0278	0.7056
36.2857	<b>0.7</b>	0.0865	2.199	0.04325	1.0996	0.0276	0.7000
40	0.63500	0.0785	1.994	0.03927	0.9975	0.0250	0.6350
42.3333	<b>0.6</b>	0.0742	1.885	0.03710	0.9423	0.0236	0.6000
44	0.57727	0.0714	1.814	0.03570	0.9068	0.0227	0.5773
<b>48</b>	0.52917	0.0655	1.661	0.03272	0.8311	0.0208	0.5292
50	0.50800	0.0628	1.595	0.03141	0.7976	0.0200	0.5080
50.8000	<b>0.5</b>	0.06184	1.5707	0.03092	0.7854	0.0197	0.5000
63.5000	<b>0.4</b>	0.04947	1.2565	0.02473	0.6283	0.0157	0.4000
<b>64</b>	0.39688	0.04909	1.2469	0.02454	0.6234	0.0156	0.3969
67.7333	0.375	0.04638	1.1781	0.02319	0.5890	0.0148	0.3750
<b>72</b>	0.35278	0.04363	1.1082	0.02182	0.5541	0.0139	0.3528
72.5714	0.35	0.04329	1.0996	0.02164	0.5498	0.0138	0.3500
78.1538	0.325	0.04020	1.0211	0.02010	0.5105	0.0128	0.3250
<b>80</b>	0.31750	0.03927	0.9975	0.01964	0.4987	0.0125	0.3175
84.6667	<b>0.3</b>	0.03711	0.9426	0.01856	0.4713	0.0118	0.3000
92.3636	0.275	0.03401	0.8639	0.01700	0.4319	0.0108	0.2750
<b>96</b>	0.26458	0.03272	0.8311	0.01636	0.4156	0.0104	0.2646
101.6000	0.25	0.03092	0.7854	0.01546	0.3927	0.00984	0.2500
<b>120</b>	0.21167	0.02618	0.6650	0.01309	0.3325	0.00833	0.2117
125	0.20320	0.02513	0.6383	0.01256	0.3192	0.00800	0.2032
127.0000	<b>0.2</b>	0.02474	0.6284	0.01237	0.3142	0.00787	0.2000
150	0.16933	0.02094	0.5319	0.01047	0.2659	0.00667	0.1693
169.3333	0.15	0.01855	0.4712	0.00928	0.2356	0.00591	0.1500
180	0.14111	0.01745	0.4432	0.00873	0.2216	0.00555	0.1411
<b>200</b>	0.12700	0.01571	0.3990	0.00786	0.1995	0.00500	0.1270
203.2000	0.125	0.01546	0.3927	0.00773	0.1963	0.00492	0.1250

FUNDAMENTAL RELATIONSHIPS OF SPUR AND HELICAL GEARS

**Center distance** is the distance between axes of mating gears and is determined from  $C = (n_G + N_P)/(2P_d)$ , or  $C = (D_G + D_P)/2$ . Deviation from ideal center distance of involute gears is not detrimental to proper (conjugate) gear action which is one of the prime superiority features of the involute tooth form.

**Contact Ratio** Referring to the top part of Fig. 8.3.4 and assuming no tip relief, the pinion engages in the gear at  $a$ , where the outside circle of the gear tooth intersects the line of action  $ac$ . For the usual spur gear and pinion combinations there will be two pairs of teeth theoretically in contact at engagement (a gear tooth and its mating pinion tooth considered as a pair). This will continue until the pair ahead (bottom part of Fig. 8.3.4) disengages at  $c$ , where the outside circle of the pinion intersects the line of action  $ac$ , the movement along the line of action being

*ab*. After disengagement the pair behind will be the only pair in contact until another pair engages, the movement along the line of action for single-pair contact being *bd*. Two pairs are theoretically in contact during the remaining intervals, *ab + dc*.

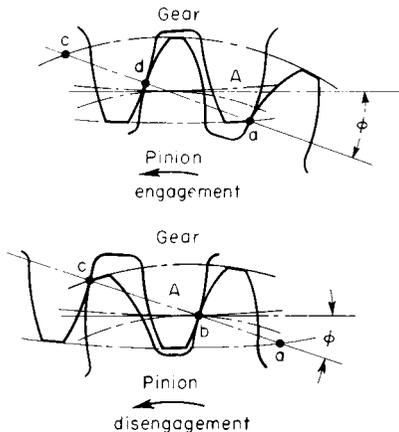


Fig. 8.3.4 Contact conditions at engagement and disengagement.

**Contact ratio** expresses the average number of pairs of teeth theoretically in contact and is obtained numerically by dividing the length of the line of action by the normal pitch. For full-depth teeth, without undercutting, the contact ratio is  $m_p = (\sqrt{D_0^2 - D_b^2} + \sqrt{d_0^2 - d_b^2} - 2C \sin \phi) / (2p \cos \phi)$ . The result will be a mixed number with the integer portion the number of pairs of teeth always in contact and carrying load, and the decimal portion the amount of time an additional pair of teeth are engaged and share load. As an example, for  $m_p$  between 1 and 2:

Load is carried by one pair,  $(2 - m_p)/m_p$  of the time.

Load is carried by two pairs,  $2(m_p - 1)/m_p$  of the time.

In Figs. 8.3.5 to 8.3.7, contact ratios are given for standard generated gears, the lower part of Figs. 8.3.5 and 8.3.6 representing the effect of undercutting.

These charts are applicable to both standard dimetral pitch gears made in accordance with American standards and also standard metric gears that have an addendum of one module.

**Tooth Thickness** For standard gears, the tooth thickness  $t$  of mating gears is equal, where  $t = p/2 = \pi/(2P_d)$  measured linearly along the arc of the pitch circle. The tooth thickness  $t_1$  at any radial point of the tooth (at diameter  $D_1$ ) can be calculated from the known thickness  $t$  at the pitch radius  $D/2$  by the relationship  $t_1 = t(D_1/D) - D_1(\text{inv } \phi_1 - \text{inv } \phi)$ , where  $\text{inv } \phi = \tan \phi - \phi = \text{involute function}$ . Units for  $\phi$  must be radians. Tables of values for  $\text{inv } \phi$  from 0 to  $45^\circ$  can be found in the references (Buckingham and Dudley).

**Over-plus measurements** (spur gears) are another means of deriving tooth thickness. If cylindrical pins are inserted in tooth spaces diametrically opposite one another (or nearest space for an odd number of teeth) (Fig. 8.3.8), the tooth thickness can be derived from the measurement  $M$  as follows:

$$t = D(\pi/N + \text{inv } \phi_1 - \text{inv } \phi - d_w/D_b)$$

$$\cos \phi_1 = (D \cos \phi)/2R_c$$

$$R_c = (M - d_w)/2 \quad \text{for even number of teeth}$$

$$R_c = (M - d_w)/[2 \cos(90^\circ/N)] \quad \text{for odd number of teeth}$$

where  $d_w$  = pin diameter,  $R_c$  = distance from gear center to center of pin, and  $M$  = measurement over pins.

For the reverse situation, the over-pins measurement  $M$  can be found for a given tooth thickness  $t$  at diameter  $D$  and pressure angle  $\phi$  by the following:  $\text{inv } \phi_1 = t/D + \text{inv } \phi + d_w/(D \cos \phi) - \pi/N$ ,  $M = D \cos \phi / \cos \phi_1 + d_w$  (for even number of teeth),  $M = (D \cos \phi / \cos \phi_1) \cos(90^\circ/N) + d_w$  (for odd number of teeth).

Table values of over-pins measures (see Dudley and Van Keuren) facilitate measurements for all standard gears including those with slight departures from standard. (For correlation with tooth thickness and testing radius, see Michalec, *Product Eng.*, May 1957, and "Precision Gearing: Theory and Practice," Wiley.)

**Testing radius  $R_T$**  is another means of determining tooth thickness and refers to the effective pitch radius of the gear when rolled intimately with a master gear of known size calibration. (See Michalec, *Product Eng.*, Nov. 1956, and "Precision Gearing: Theory and Practice," op. cit.) For standard design gears the testing radius equals the pitch radius. The testing radius may be corrected for small departures  $\Delta t$  from ideal tooth thickness by the relationship,  $R_T = R + \Delta t/2 \tan \phi$ , where  $\Delta t = t_1 - t$  and is positive and negative respectively for thicker and thinner tooth thicknesses than standard value  $t$ .

**Backlash  $B$**  is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle. Backlash does not adversely affect proper gear function except for lost mo-

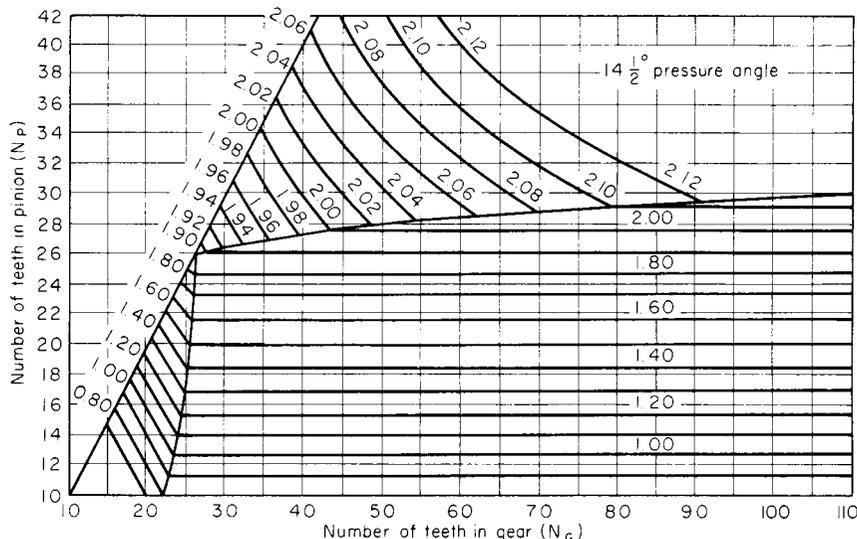


Fig. 8.3.5 Contact ratio, spur gear pairs—full depth, standard generated teeth,  $14\frac{1}{2}^\circ$  pressure angle.

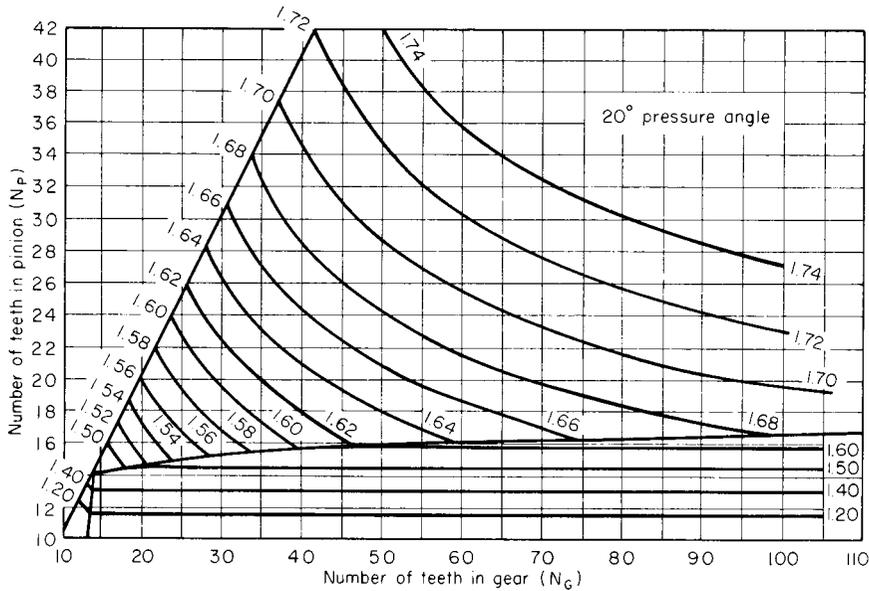


Fig. 8.3.6 Contact ratio, spur gear pairs—full-depth standard generated teeth, 20° pressure angle.

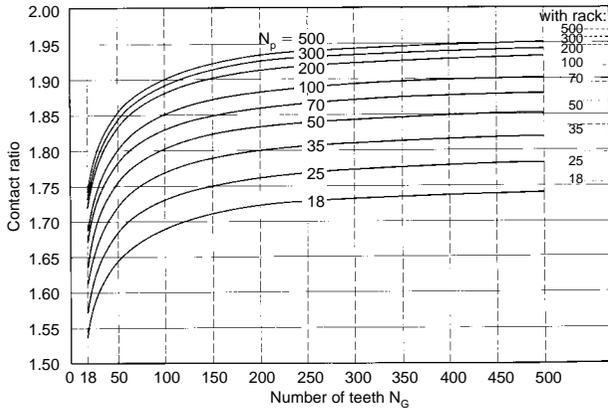


Fig. 8.3.7 Contact ratio for large numbers of teeth—spur gear pairs, full-depth standard teeth, 20° pressure angle. (Data by R. Feeney and T. Wall.)

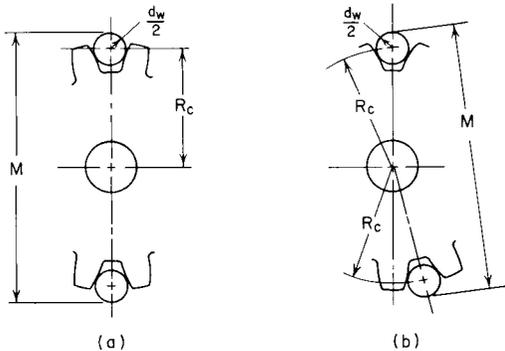


Fig. 8.3.8 Geometry of over-pins measurements (a) for an even number of teeth and (b) for an odd number of teeth.

tion upon reversal of gear rotation. Backlash inevitably occurs because of necessary fabrication tolerances on tooth thickness and center distance plus need for clearance to accommodate lubricant and thermal expansion. Proper backlash can be introduced by a specified amount of tooth thinning or slight increase in center distance. The relationship between small change in center distance  $\Delta C$  and backlash is  $B = 2 \Delta C \tan \phi$  (see Michalec, "Precision Gearing: Theory and Practice").

**Total composite error** (tolerance) is a measure of gear quality in terms of the net sum of irregularity of its testing radius  $R_T$  due to pitch-circle runout and tooth-to-tooth variations (see Michalec, op. cit.).

**Tooth-to-tooth composite error** (tolerance) is the variation of testing radius  $R_T$  between adjacent teeth caused by tooth spacing, thickness, and profile deviations (see Michalec, op. cit.).

**Profile shifted gears** have tooth thicknesses that are significantly different from nominal standard value; excluded are deviations caused by normal allowances and tolerances. They are also known as **modified gears**, **long and short addendum gears**, and **enlarged gears**. They are produced by cutting the teeth with standard cutters at enlarged or reduced outside diameters. The result is a relative shift of the two families of involutes forming the tooth profiles, simultaneously with a shift of the tooth radially outward or inward (see Fig. 8.3.9). Calculation of operating conditions and tooth parameters are

$$C_1 = \frac{(C \cos \phi)}{\cos \phi_1}$$

$$\text{inv } \phi_1 = \text{inv } \phi + \frac{N_p(t'_G + t'_P) - \pi D_P}{D_P(N_p + N_G)}$$

$$t'_G = t + 2X_G \tan \phi$$

$$t'_P = t + 2X_P \tan \phi$$

$$D'_G = (N_G/P_d) + 2X_G$$

$$D'_P = (N_P/P_d) + 2X_P$$

$$D'_o = D'_G + (2/P_d)$$

$$d'_o = D'_P + (2/P_d)$$

where  $\phi$  = standard pressure angle,  $\phi_1$  = operating pressure angle,  $C$  = standard center distance =  $(N_G + N_P)/2P_d$ ,  $C_1$  = operating center distance,  $X_G$  = profile shift correction of gear, and  $X_P$  = profile shift correction of pinion. The quantity  $X$  is positive for enlarged gears and negative for thinned gears.

Table 8.3.5 Metric Spur Gear Design Formulas

To obtain:	From known	Use this formula*
Pitch diameter $D$	Module; diametral pitch	$D = mN$
Circular pitch $p_c$	Module; diametral pitch	$p_c = m\pi = \frac{D}{N}\pi = \frac{\pi}{P}$
Module $m$	Diametral pitch	$m = \frac{25.4}{P}$
No. of teeth $N$	Module and pitch diameter	$N = \frac{D}{m}$
Addendum $a$	Module	$a = m$
Dedendum $b$	Module	$b = 1.25m$
Outside diameter $D_o$	Module and pitch diameter or number of teeth	$D_o = D + 2m = m(N + 2)$
Root diameter $D_r$	Pitch diameter and module	$D_r = D - 2.5m$
Base circle diameter $D_b$	Pitch diameter and pressure angle $\phi$	$D_b = D \cos \phi$
Base pitch $p_b$	Module and pressure angle	$p_b = m\pi \cos \phi$
Tooth thickness at standard pitch diameter $T_{std}$	Module	$T_{std} = \frac{\pi}{2}m$
Center distance $C$	Module and number of teeth	$C = \frac{m(N_1 + N_2)}{2}$
Contact ratio $m_p$	Outside radii, base-circle radii, center distance, pressure angle	$m_p = \frac{\sqrt{1R_o^2 - 1R_b^2} + \sqrt{2R_o^2 - 2R_b^2} - C \sin \phi}{m\pi \cos \phi}$
Backlash (linear) $B$ (along pitch circle)	Change in center distance	$B = 2(\Delta C) \tan \phi$
Backlash (linear) $B$ (along pitch circle)	Change in tooth thickness, $T$	$B = \Delta T$
Backlash (linear) (along line of action) $B_{LA}$	Linear backlash (along pitch circle)	$B_{LA} = B \cos \phi$
Backlash (angular) $B_a$	Linear backlash (along pitch circle)	$B_a = 6,880 \frac{B}{D}$ (arc minutes)
Min. number teeth for no undercutting, $N_c$	Pressure angle	$N_c = \frac{2}{\sin^2 \phi}$

\* All linear dimensions in millimeters.

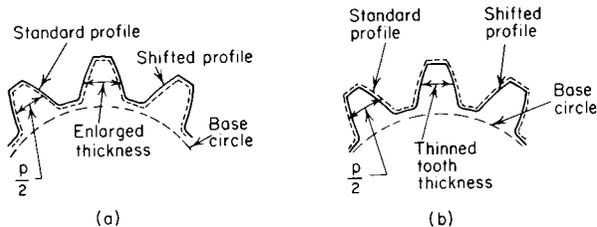


Fig. 8.3.9 Geometry of profile-shifted teeth. (a) Enlarged case; (b) thinned tooth thickness case.

**Metric Module Gear Design Equations** Basic design equations for spur gearing utilizing the metric module are listed in Table 8.3.5. (See Designatronics, "Elements of Metric Gear Technology.")

## HELICAL GEARS

Helical gears divide into two general applications: for driving parallel shafts and for driving skew shafts (mostly at right angles), the latter often referred to as *crossed-axis* helical gears. The helical tooth form may be imagined as consisting of an infinite number of staggered laminar spur gears, resulting in the curved cylindrical helix.

**Pitch of helical gears** is definable in two planes. The diametral and circular pitches measured in the plane of rotation (transverse) are defined as for spur gears. However, pitches measured normal to the tooth are related by the cosine of the helix angle; thus normal diametral pitch =  $P_{dn} = P_d/\cos \psi$ , normal circular pitch =  $p_n = p \cos \psi$ , and  $P_{dn}p_n = \pi$ . **Axial pitch** is the distance between corresponding sides of adjacent teeth measured parallel to the gear axis and is calculated as  $p_a = p \cot \psi$ .

**Pressure angle** of helical gears is definable in the normal and trans-

verse planes by  $\tan \phi_n = \tan \phi \cos \psi$ . The transverse pressure angle, which is effectively the real pressure angle, is always greater than the normal pressure angle.

**Tooth thickness**  $t$  of helical gears can be measured in the plane of rotation, as with spur gears, or normal to the tooth surface  $t_n$ . The relationship of the two thicknesses is  $t_n = t \cos \psi$ .

**Over-Pins Measurement of Helical Gears** Tooth thicknesses  $t$  at diameter  $d$  can be found from a known over-pins measurement  $M$  at known pressure angle  $\phi$ , corresponding to diameter  $D$  as follows:

$$\begin{aligned}
 R_c &= (M - d_w)/2 && \text{for even number of teeth} \\
 R_c &= (M - d_w)/[2 \cos (180/2N)] && \text{for odd number of teeth} \\
 \cos \phi_1 &= (D \cos \phi)/2R_c \\
 \tan \phi_n &= \tan \phi \cos \psi \\
 \cos \psi_b &= \sin \phi_n/\sin \phi \\
 t &= D[\pi/N + \text{inv } \phi_1 - \text{inv } \phi - d_w/(D \cos \phi \cos \psi_b)]
 \end{aligned}$$

**Parallel-shaft helical gears** must conform to the same conditions and requirements as spur gears with parameters (pressure angle and pitch) consistently defined in the transverse plane. Since standard spur gear cutting tools are usually used, normal plane values are standard, resulting in nonstandard transverse pitches and nonstandard pitch diameters and center distances. For parallel shafts, helical gears must have identical helix angles, but must be of opposite hand (left and right helix directions). The commonly used helix angles range from 15 to 35°. To make most advantage of the helical form, the advance of a tooth should be greater than the circular pitch; recommended ratio is 1.5 to 2 with 1.1 minimum. This overlap provides two or more teeth in continual contact with resulting greater smoothness and quietness than spur gears. Because of the helix, the normal component of the tangential pressure on the teeth produces end thrust of the shafts. To remove this objection, gears are made with helixes of opposite hand on each half of the face and are then known as herringbone gears (see Fig. 8.3.10).

**Crossed-axis helical gears**, also called *spiral* or *screw gears* (Fig. 8.3.11), are a simple type of involute gear used for connecting nonpar-

allel, nonintersecting shafts. Contact is point and there is considerably more sliding than with parallel-axis helicals, which limits the load capacity. The individual gear of this mesh is identical in form and specification to a parallel-shaft helical gear. Crossed-axis helicals can connect

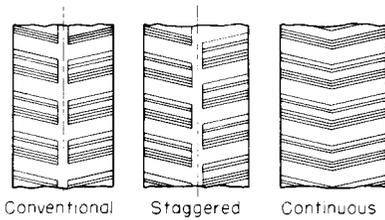


Fig. 8.3.10 Herringbone gears.

any shaft angle  $\Sigma$ , although  $90^\circ$  is prevalent. Usually, the helix angles will be of the same hand, although for some extreme cases it is possible to have opposite hands, particularly if the shaft angle is small.

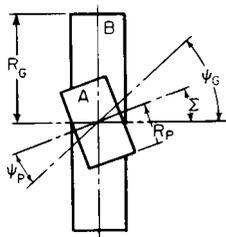


Fig. 8.3.11 Crossed-axis helical gears.

**Helical Gear Calculations** For parallel shafts the center distance is a function of the helix angle as well as the number of teeth, that is,  $C = (N_G + N_P)/(2P_{dn} \cos \psi)$ . This offers a powerful method of gearing shafts at any specified center distance to a specified velocity ratio. For crossed-axis helicals the problem of connecting a pair of shafts for any velocity ratio admits of a number of solutions, since both the pitch radii and the helix angles contribute to establishing the velocity ratio. The formulas given in Tables 8.3.6 and 8.3.7 are of assistance in calculations. The notation used in these tables is as follows:

- $N_P(N_G)$  = number of teeth in pinion (gear)
- $D_P(D_G)$  = pitch diam of pinion (gear)
- $p_P(p_G)$  = circular pitch of pinion (gear)
- $p$  = circular pitch in plane of rotation for both gears
- $P_d$  = diametral pitch in plane of rotation for both gears
- $p_n$  = normal circular pitch for both gears
- $P_{dn}$  = normal diametral pitch for both gears
- $\psi_G$  = tooth helix angle of gear
- $\psi_P$  = tooth helix angle of pinion
- $l_P(l_G)$  = lead of pinion (gear)
- = lead of tooth helix
- $n_P(n_G)$  = r/min of pinion (gear)
- $\Sigma$  = angle between shafts in plan
- $C$  = center distance

Table 8.3.6 Helical Gears on Parallel Shafts

To find:	Formula
Center distance $C$	$\frac{N_G + N_P}{2P_{dn} \cos \psi}$
Pitch diameter $D$	$\frac{N}{P_d} = \frac{N}{P_{dn} \cos \psi}$
Normal diametral pitch $P_{dn}$	$\frac{P_d}{\cos \psi}$
Normal circular pitch $p_n$	$p \cos \psi$
Pressure angle $\phi$	$\tan^{-1} \frac{\tan \phi_n}{\cos \psi}$
Contact ratio $m_p$	$\frac{\sqrt{G D_a^2 - G D_b^2} + \sqrt{P D_a^2 - P D_b^2} + 2C \sin \phi}{2p \cos \phi} + \frac{F \sin \psi}{p_n}$
Velocity ratio $m_G$	$\frac{N_G}{N_P} = \frac{D_G}{D_P}$

Table 8.3.7 Crossed Helical Gears on Skew Shafts

To find:	Formula
Center distance $C$	$\frac{P_n}{2\pi} \left( \frac{N_G}{\cos \psi_G} + \frac{N_P}{\cos \psi_P} \right)$
Pitch diameter $D_G, D_P$	$D_G = \frac{N_G}{P_{dG}} = \frac{N_G}{P_{dn} \cos \psi_G} = \frac{N_G P_n}{\pi \cos \psi_G}$ $D_P = \frac{N_P}{P_{dP}} = \frac{N_P}{P_{dn} \cos \psi_P} = \frac{N_P P_n}{\pi \cos \psi_P}$
Gear ratio $m_G$	$\frac{N_G}{N_P} = \frac{D_G \cos \psi_G}{D_P \cos \psi_P}$
Shaft angle $\Sigma$	$\psi_G + \psi_P$

NONSPUR GEAR TYPES\*

**Bevel gears** are used to connect two intersecting shafts in any given speed ratio. The tooth shapes may be designed in any of the shapes shown in Fig. 8.3.12. A special type of gear known as a **hypoid** was developed by Gleason Works for the automotive industry (see *Jour. SAE*, 18, no. 6). Although similar in appearance to a spiral bevel, it is not a true bevel gear. The basic pitch rolling surfaces are hyperbolas of revolution. Because a "spherical involute" tooth form has a curved crown tooth (the basic tool for generating all bevel gears), Gleason used a straight-sided crown tooth which resulted in bevel gears differing slightly from involute form. Because of the figure 8 shape of the complete theoretical tooth contact path, the tooth form has been called "octoid." Straight-sided bevel gears made by reciprocating cutters are of this type. Later, when curved teeth became widely used (spiral and Zerol), practical limitations of such cutters resulted in introduction of the "spherical" tooth form which is now the basis of all curved tooth bevel gears. (For details see Gleason's publication, "Guide to Bevel Gears.") Gleason Works also developed the generated tooth form

\* In the following text relating to bevel gearing, all tables and figures have been extracted from Gleason Works publications, with permission.

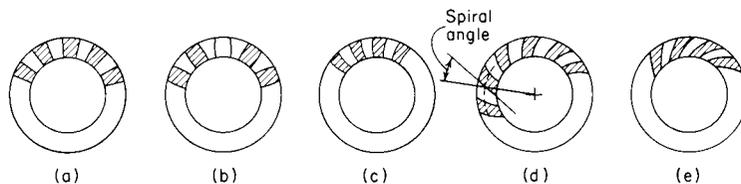


Fig. 8.3.12 Bevel gear types. (a) Old-type straight teeth; (b) modern Coniflex straight teeth (exaggerated crowning); (c) Zerol teeth; (d) spiral teeth; (e) hypoid teeth.

Revecycle and the nongenerated tooth forms Formate and Helixform, used principally for mass production of hypoid gears for the automotive industry.

Referring to Fig. 8.3.13, we see that the pitch surfaces of bevel gears are frustums of cones whose vertices are at the intersection of the axes; the essential elements and definitions follow.

**Addendum angle  $\alpha$ :** The angle between elements of the face cone and pitch cone.

**Back angle:** The angle between an element of the back cone and a plane of rotation. It is equal to the pitch angle.

**Back cone:** The angle of a cone whose elements are tangent to a sphere containing a trace of the pitch circle.

**Back-cone distance:** The distance along an element of the back cone from the apex to the pitch circle.

**Cone distance  $A_o$ :** The distance from the end of the tooth (heel) to the pitch apex.

**Crown:** The sharp corner forming the outside diameter.

**Crown-to-back:** The distance from the outside diameter edge (crown) to the rear of the gear.

**Dedendum angle  $\delta$ :** The angle between elements of the root cone and pitch cone.

**Face angle  $\gamma_o$ :** The angle between an element of the face cone and its axis.

**Face width  $F$ :** The length of teeth along the cone distance.

**Front angle:** The angle between an element of the front cone and a plane of rotation.

**Generating mounting surface, GMS:** The diameter and/or plane of rotation surface or shaft center which is used for locating the gear blank during fabrication of the gear teeth.

**Heel:** The portion of a bevel gear tooth near the outer end.

**Mounting distance, MD:** For assembled bevel gears, the distance from the crossing point of the axes to the registering surface, measured along the gear axis. Ideally, it should be identical to the pitch apex to back.

**Mounting surface, MS:** The diameter and/or plane of rotation surface which is used for locating the gear in the application assembly.

**Octoid:** The mathematical form of the bevel tooth profile. Closely resembles a spherical involute but is fundamentally different.

**Pitch angle  $\Gamma$ :** The angle formed between an element of the pitch cone and the bevel gear axis. It is the half angle of the pitch.

**Pitch apex to back:** The distance along the axis from apex of pitch cone to a locating registering surface on back.

**Registering surface, RS:** The surface in the plane of rotation which locates the gear blank axially in the generating machine and the gear in application. These are usually identical surfaces, but not necessarily so.

**Root angle  $\gamma_R$ :** The angle formed between a tooth root element and the axis of the bevel gear.

**Shaft angle  $\Sigma$ :** The angle between mating bevel-gear axes; also, the sum of the two pitch angles.

**Spiral angle  $\psi$ :** The angle between the tooth trace and an element of the pitch cone, corresponding to helix angle in helical gears. The spiral angle is understood to be at the mean cone distance.

**Toe:** The portion of a bevel tooth near the inner end.

Bevel gears are described by the parameter dimensions at the large end (heel) of the teeth. Pitch, pitch diameter, and tooth dimensions, such as addendum are measurements at this point. At the large end of the gear, the tooth profiles will approximate those generated on a spur gear pitch circle of radius equal to the back cone distance. The formative number of teeth is equal to that contained by a complete spur gear. For pinion and gear, respectively, this is  $T_p = N_p / \cos \gamma$ ;  $T_G = N_G / \cos \Gamma$ , where  $T_p$  and  $T_G$  = formative number teeth and  $N_p$  and  $N_G$  = actual number teeth.

Although bevel gears can connect intersecting shafts at any angle, most applications are for right angles. When such bevels are in a 1 : 1 ratio, they are called **mitre gears**. Bevels connecting shafts other than 90° are called **angular bevel gears**. The speeds of the shafts of bevel gears are

determined by  $n_p/n_G = \sin \Gamma / \sin \gamma$ , where  $n_p(n_G)$  = r/min of pinion (gear), and  $\gamma(\Gamma)$  = pitch angle of pinion (gear).

All standard bevel gear designs in the United States are in accordance with the **Gleason bevel gear system**. This employs a basic pressure angle of 20° with long and short addendums for ratios other than 1 : 1 to avoid undercut pinions and to increase strength.

**20° Straight Bevel Gears for 90° Shaft Angle** Since straight bevel gears are the easiest to produce and offer maximum precision, they are frequently a first choice. Modern straight-bevel-gears generators produce a tooth with localized tooth bearing designated by the Gleason registered tradename Coniflex. These gears, produced with a circular cutter, have a slightly crowned tooth form (see Fig. 8.3.12b). Because of the superiority of Coniflex bevel gears over the earlier reciprocating cutter produced straight bevels and because of their faster production, they are the standards for all bevel gears. The design parameters of Fig. 8.3.13 are calculated by the formulas of Table 8.3.8. Backlash data are given in Table 8.3.9.

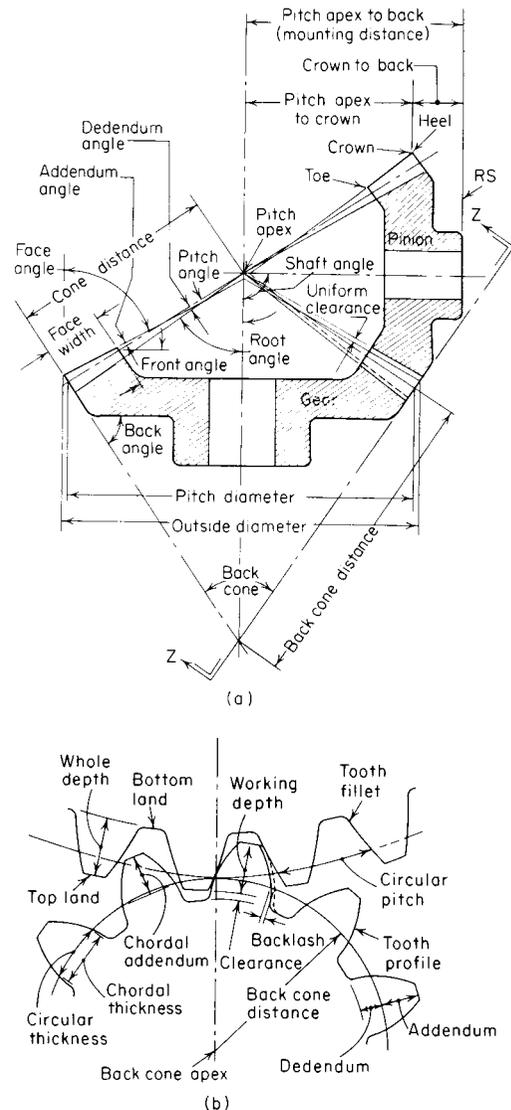


Fig. 8.3.13 Geometry of bevel gear nomenclature. (a) Section through axes; (b) view along axis Z/Z.

**Table 8.3.8 Straight Bevel Gear Dimensions\***  
(All linear dimensions in inches)

1. Number of pinion teeth†	$n$	5. Working depth	$h_k = \frac{2.000}{P_d}$
2. Number of gear teeth†	$N$	6. Whole depth	$h_i = \frac{2.188}{P_d} + 0.002$
3. Diametral pitch	$P_d$	7. Pressure angle	$\phi$
4. Face width	$F$	8. Shaft angle	$\Sigma$

	Pinion	Gear
9. Pitch diameter	$d = \frac{n}{P_d}$	$D = \frac{N}{P_d}$
10. Pitch angle	$\gamma = \tan^{-1} \frac{n}{N}$	$\Gamma = 90^\circ - \gamma$
11. Outer cone distance	$A_o = \frac{D}{2 \sin \Gamma}$	
12. Circular pitch	$p = \frac{3.1416}{P_d}$	
13. Addendum	$a_{OP} = h_k - a_{OG}$	$a_{OG} = \frac{0.540}{P_d} + \frac{0.460}{P_d(N/n)^2}$
14. Dedendum‡	$b_{OP} = \frac{2.188}{P_d} - a_{OP}$	$b_{OG} = \frac{2.188}{P_d} - a_{OG}$
15. Clearance	$c = h_i - h_k$	
16. Dedendum angle	$\delta_p = \tan^{-1} \frac{b_{OP}}{A_o}$	$\delta_G = \tan^{-1} \frac{b_{OG}}{A_o}$
17. Face angle of blank	$\gamma_o = \gamma + \delta_G$	$\Gamma_o = \Gamma + \delta_p$
18. Root angle	$\gamma_R = \gamma - \delta_p$	$\Gamma_R = \Gamma - \delta_G$
19. Outside diameter	$d_o = d + 2a_{OP} \cos \gamma$	$D_o = D + 2a_{OG} \cos \Gamma$
20. Pitch apex to crown	$x_o = \frac{D}{2} - a_{OG} \sin \gamma$	$X_o = \frac{d}{2} - a_{OP} \sin \Gamma$
21. Circular thickness	$t = p - T$	$T = \frac{p}{2} - (a_{OP} - a_{OG}) \tan \phi - \frac{K}{P_d}$ (see Fig. 8.3.14)
22. Backlash	$B$ See Table 8.3.9	
23. Chordal thickness	$t_c = t - \frac{t^2}{6d^2} - \frac{B}{2}$	$T_c = T - \frac{T^2}{6D^2} - \frac{B}{2}$
24. Chordal addendum	$a_{CP} = a_{OP} + \frac{t^2 \cos \gamma}{4d}$	$a_{CG} = a_{OG} + \frac{T^2 \cos \Gamma}{4D}$
25. Tooth angle	$\frac{3,438}{A_o} \left( \frac{t}{2} + b_{OP} \tan \phi \right)$ minutes	$\frac{3,438}{A_o} \left( \frac{T}{2} + b_{OG} \tan \phi \right)$ minutes
26. Limit-point width (L.F.)	$W_{LOP} = (T - 2b_{OP} \tan \phi) - 0.0015$	$W_{LOG} = (t - 2b_{OG} \tan \phi) - 0.0015$
27. Limit-point width (S.E.)	$W_{LIP} = \frac{A_o - F}{A_o} (T - 2b_{OP} \tan \phi) - 0.0015$	$W_{LIG} = \frac{A_o - F}{A_o} (t - 2b_{OG} \tan \phi) - 0.0015$
28. Tool-point width	$W = W_{LIP} - \text{stock allowance}$	$W = W_{LIG} - \text{stock allowance}$

\* Abstracted from "Gleason Straight Bevel Gear Design," Tables 8.3.8 and 8.3.9 and Fig. 8.3.4. Gleason Works, Inc.  
 † Numbers of teeth; ratios with 16 or more teeth in pinion: 15/17 and higher; 14/20 and higher; 13/31 and higher. These can be cut with 20° pressure angle without undercut.  
 ‡ The actual dedendum will be 0.002 in greater than calculated.

**Table 8.3.9 Recommended Normal Backlash for Bevel Gear Meshes\***

$P_d$	Backlash range	$P_d$	Backlash range
1.00–1.25	0.020–0.030	3.50–4.00	0.007–0.009
1.25–1.50	0.018–0.026	4–5	0.006–0.008
1.50–1.75	0.016–0.022	5–6	0.005–0.007
1.75–2.00	0.014–0.018	6–8	0.004–0.006
2.00–2.50	0.012–0.016	8–10	0.003–0.005
2.50–3.00	0.010–0.013	10–12	0.002–0.004
3.00–3.50	0.008–0.011	Finer than 12	0.001–0.003

\* The table gives the recommended normal backlash for gears assembled ready to run. Because of manufacturing tolerances and changes resulting from heat treatment, it is frequently necessary to reduce the theoretical tooth thickness by slightly more than the tabulated backlash in order to obtain the correct backlash in assembly. In case of choice, use the smaller backlash tolerances.

**Angular straight bevel gears** connect shaft angles other than 90° (larger or smaller), and the formulas of Table 8.3.8 are not entirely applicable, as shown in the following:

Item 8, shaft angle, is the specified non-90° shaft angle.

Item 10, pitch angles. Shaft angle  $\Sigma$  less than 90°,  $\tan \gamma = \sin \Sigma / (N/n + \cos \Sigma)$ ; shaft angle  $\Sigma$  greater than 90°,  $\tan \gamma = \sin (180 - \Sigma) / [N/n - \cos (180 - \Sigma)]$ .

For all shaft angles,  $\sin \gamma \sin \Gamma = n/N$ ;  $\Gamma = \Sigma - \gamma$ .

Item 13, addendum, requires calculation of the equivalent 90° bevel gear ratio  $m_{90}$ ,  $m_{90} = [N \cos \gamma / (n \cos \Gamma)]^{1/2}$ . The value  $m_{90}$  is used as the ratio  $N/n$  when applying the formula for addendum. The quantity under the radical is always the absolute value and is therefore always positive.

Item 20, pitch apex to crown,  $x_o = A_o \cos \gamma - a_{op} \sin \gamma$ ,  $X_o = A_o \cos \Gamma - a_{oG} \sin \Gamma$ .

Item 21, circular thickness, except for high ratios,  $K$  may be zero.

**Spiral Bevel Gears for 90° Shaft Angle** The spiral curved teeth produce additional overlapping tooth action which results in smoother gear action, lower noise, and higher load capacity. The spiral angle has been standardized by Gleason at 35°. Design parameters are calculated by formulas of Table 8.3.10.

**Angular Spiral Bevel Gears** Several items deviate from the formulas of Table 8.3.10 in the same manner as angular straight bevel gears. Therefore, the same formulas apply for the deviating items with only the following exception:

Item 21, circular thickness, the value of  $K$  in Fig. 8.3.15 must be determined from the equivalent 90° bevel ratio ( $m_{90}$ ) and the equivalent 90° bevel pinion. The latter is computed as  $n_{90} = n \sin \Gamma_{90} / \cos \gamma$ , where  $\tan \Gamma_{90} = m_{90}$ .

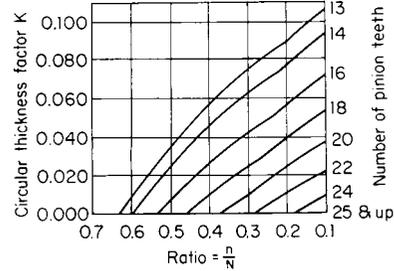


Fig. 8.3.14 Circular thickness factor for straight bevel gears.

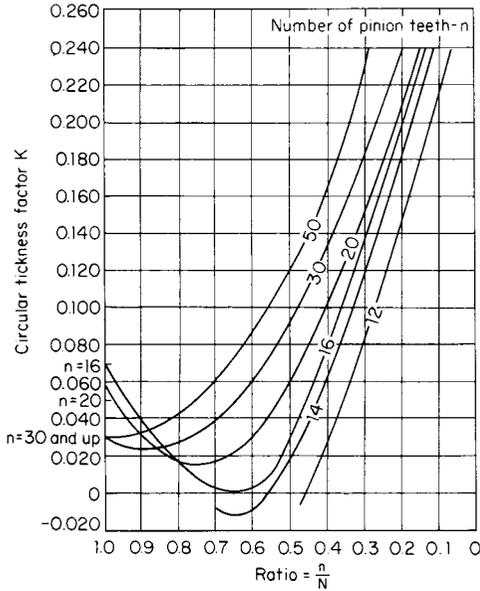
Table 8.3.10 Spiral Bevel Gear Dimensions (All linear dimensions in inches)

1. Number of pinion teeth	$n$	5. Working depth	$h_k = \frac{1.700}{P_d}$
2. Number of gear teeth	$N$	6. Whole depth	$h_i = \frac{1.888}{P_d}$
3. Diametral pitch	$P_d$	7. Pressure angle	$\phi$
4. Face width	$F$	8. Shaft angle	$\Sigma$
	Pinion		Gear
9. Pitch diameter	$d = \frac{n}{P_d}$		$D = \frac{N}{P_d}$
10. Pitch angle	$\gamma = \tan^{-1} \frac{n}{N}$		$\Gamma = 90^\circ - \gamma$
11. Outer cone distance	$A_o = \frac{D}{2 \sin \Gamma}$		
12. Circular pitch	$p = \frac{3.1416}{P_d}$		
13. Addendum	$A_{op} = h_k - a_{oG}$		$a_{oG} = \frac{0.460}{P_d} + \frac{0.390}{P_d(N/n)^2}$
14. Dedendum	$b_{op} = h_i - a_{op}$		$b_{oG} = h_i - a_{oG}$
15. Clearance	$c = h_i - h_k$		
16. Dedendum angle	$\delta_p = \tan^{-1} \frac{b_{op}}{A_o}$		$\delta_G = \tan^{-1} \frac{b_{oG}}{A_o}$
17. Face angle of blank	$\gamma_o = \gamma + \delta_p$		$\Gamma_o = \Gamma + \delta_p$
18. Root angle	$\gamma_r = \gamma - \delta_p$		$\Gamma_r = \Gamma - \delta_p$
19. Outside diameter	$d_o = d + 2a_{op} \cos \gamma$		$D_o = D + 2a_{oG} \cos \Gamma$
20. Pitch apex to crown	$x_o = \frac{D}{2} - a_{op} \sin \gamma$		$X_o = \frac{d}{2} - a_{oG} \sin \Gamma$
21. Circular thickness	$t = p - T$		$T = \frac{p}{2} (a_{op} - a_{oG}) \frac{\tan \phi}{\cos \phi} - \frac{K}{P_d}$ (see Fig. 8.3.15)
22. Backlash	See Table 8.3.9		
23. Hand of spiral	Left or right		Right or left
24. Spiral angle			35°
25. Driving member			Pinion or gear
26. Direction of rotation			Clockwise or counterclockwise

SOURCE: Gleason, "Spiral Bevel Gear System."

The **zerol bevel gear** is a special case of a spiral bevel gear and is limited to special applications. Design and fabrication details can be obtained from Gleason Works.

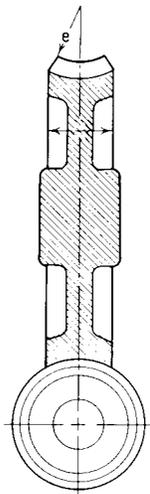
**Hypoid gears** are special and are essentially limited to automotive applications.



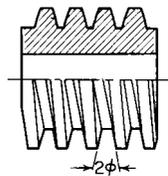
**Fig. 8.3.15** Circular thickness factors for spiral bevel gears with 20° pressure angle and 35° spiral angle. Left-hand pinion driving clockwise or right-hand pinion driving counterclockwise.

**WORMGEARS AND WORMS**

Worm gearing is used for obtaining large speed reductions between nonintersecting shafts making an angle of 90° with each other. If a wormgear such as shown in Fig. 8.3.16 engages a straight worm, as shown in Fig. 8.3.17, the combination is known as **single enveloping worm gearing**. If a wormgear of the kind shown in Fig. 8.3.16 engages a worm as shown in Fig. 8.3.18, the combination is known as **double enveloping worm gearing**.

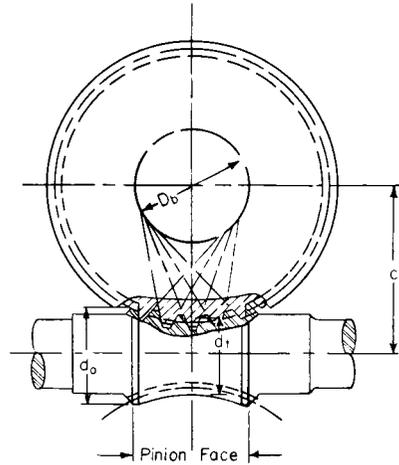


**Fig. 8.3.16** Single enveloping worm gearing.



**Fig. 8.3.17** Straight worm.

With worm gearing, the **velocity ratio** is the ratio between the number of teeth on the wormgear and the number of threads on the worm. Thus, a 30-tooth wormgear meshing with a single threaded worm will have a velocity ratio of 1 : 30; that is, the worm must make 30 rv in order to revolve the wormgear once. For a double threaded worm, there will be 15 rv of the worm to one of the wormgear, etc. High-velocity ratios are thus obtained with relatively small wormgears.



**Fig. 8.3.18** Double enveloping worm gearing.

Tooth proportions of the worm in the central section (Fig. 8.3.17) follow standard rack designs, such as 14½, 20, and 25°. The mating wormgear is cut conjugate for a unique worm size and center distance. The geometry and related design equations for a straight-sided cylindrical worm are best seen from a development of the pitch plane (Fig. 8.3.19).

$$D_w = \text{pitch diameter of worm} = \frac{n_w p_n}{\pi \sin \lambda}$$

$$p_n = p \cos \lambda = \frac{\pi D_w}{n_w} \sin \lambda$$

$$L = \text{lead of worm} = n_w p$$

$$D_g = \text{pitch diameter of wormgear}$$

$$= \frac{N_g}{P_d} = \frac{p N_g}{\pi} = \frac{P_n N_g}{\pi \cos \lambda}$$

$$C = \text{center distance}$$

$$= \frac{D_w + D_g}{2} = \frac{P_n}{2\pi} \left( \frac{N_g}{\cos \lambda} + \frac{n_w}{\sin \lambda} \right)$$

where  $n_w$  = number of threads in worm;  $N_g$  = number of teeth in wormgear;  $Z$  = velocity ratio =  $N_g/n_w$ .

The pitch diameter of the wormgear is established by the number of teeth, which in turn comes from the desired gear ratio. The pitch diameter of the worm is somewhat arbitrary. The lead must match the wormgear's circular pitch, which can be satisfied by an infinite number of worm diameters; but for a fixed lead value, each worm diameter has a unique lead angle. AGMA offers a design formula that provides near optimized geometry:

$$D_w = \frac{C^{0.875}}{2.2}$$

where  $C$  = center distance. Wormgear face width is also somewhat arbitrary. Generally it will be ⅓ to ⅔ of the worm's outside diameter.

**Worm mesh nonreversibility**, a unique feature of some designs, occurs because of the large amount of sliding in this type of gearing. For a given coefficient of friction there is a critical value of lead angle below

which the mesh is nonreversible. This is generally  $10^\circ$  and lower but is related to the materials and lubricant. Most single thread worm meshes are in this category. This locking feature can be a disadvantage or in some designs can be put to advantage.

**Double enveloping worm gearing** is special in both design and fabrication. Application is primarily where a high load capacity in small space is desired. Currently, there is only one source of manufacture in the United States: Cone Drive Division of Ex-Cello Corp. For design details and load ratings consult publications of Cone Drive and AGMA Standards.

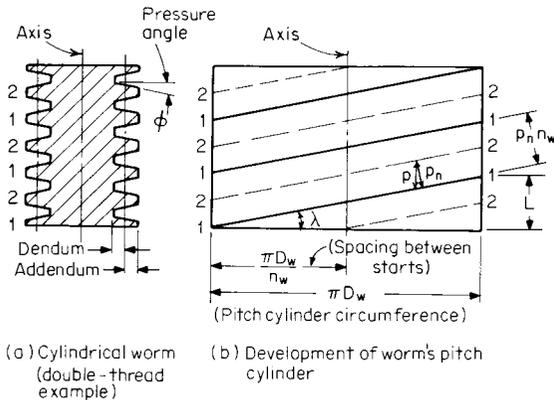


Fig. 8.3.19 Cylindrical worm geometry and design parameters.

#### Other Gear Types

Gears for special purposes include the following (details are to be found in the references):

**Spiroid** (Illinois Tool Works) gears, used to connect skew shafts, resemble a hypoid-type bevel gear but in performance are more like worm meshes. They offer very high ratios and a large contact ratio resulting in high strength. The **Helicon** (Illinois Tool Works) gear is a variation in which the pinion is not tapered, and ratios under 10:1 are feasible.

**Beveloid** (Vincor Corp.) gears are tapered involute gears which can couple intersecting shafts, skew shafts, and parallel shafts.

**Face gears** have teeth cut on the rotating face plane of the gear and mate with standard involute spur gears. They can connect intersecting or nonparallel, nonintersecting shafts.

**Noncircular gears or function gears** are used for special motions or as elements of analog computers. They can be made with elliptical, logarithmic, spiral, and other functions. See Cunningham references; also, Cunningham Industries, Inc., Stamford, CT.

#### DESIGN STANDARDS

In addition to the ANSI and AGMA standards on basic tooth proportions, the AGMA sponsors a large number of national standards dealing with gear design, specification, and inspection. (Consult AGMA, 1500 King St., Arlington, VA 22314, for details.) Helpful general references are AGMA, "Gear Handbook," 390.03 and ANSI/AGMA, "Gear Classification and Inspection Handbook," 2000-A88, which establish a system of quality classes for all gear sizes and pitches, ranging from crude coarse commercial gears to the highest orders of fine and coarse ultra-precision gears.

There are 13 quality classes, numbered from 3 through 15 in ascending quality. Tolerances are given for key functional parameters: runout, pitch, profile, lead, total composite error, tooth-to-tooth composite error, and tooth thickness. Also, tooth thickness tolerances and recommended mesh backlash are included. These are related to diametral pitch and pitch diameter in recognition of fabrication achievability. Data are available for spur, helical, herringbone, bevel, and worm gear-

ing; and spur and helical racks. Special sections cover gear applications and suggested quality number; gear materials and treatments; and standard procedure for identifying quality, material, and other pertinent parameters. These data are too extensive for inclusion in this handbook, and the reader is referred to the cited AGMA references.

#### STRENGTH AND DURABILITY

Gear teeth fail in two classical manners: tooth breakage and surface fatigue pitting. Instrument gears and other small, lightly loaded gears are designed primarily for tooth-bending beam strength since minimizing size is the priority. Power gears, usually larger, are designed for both strengths, with surface durability often more critical. Expressions for calculating the beam and surface stresses started with the Lewis-Buckingham formulas and now extend to the latest AGMA formulas.

The Lewis formula for analysis of beam strength, now relegated to historical reference, serves to illustrate the fundamentals that current formulas utilize. In the Lewis formula, a tooth layout shows the load assumed to be at the tip (Fig. 8.3.20). From this Lewis demonstrated that the beam strength  $W_b = FSY/P_d$ , where  $F$  = face width;  $S$  = allowable stress;  $Y$  = Lewis form factor;  $P_d$  = diametral pitch. The form factor  $Y$  is derived from the layout as  $Y = 2P_d/3$ . The value of  $Y$  varies with tooth design (form and pressure angle) and number of teeth. In the case of a helical gear tooth, there is a thrust force  $W_{th}$  in the axial direction that arises and must be considered as a component of bearing load. See Fig. 8.3.21*b*. Buckingham modified the Lewis formula to include dynamic effects on beam strength and developed equations for evaluating surface stresses. Further modifications were made by other investigators, and have resulted in the most recent AGMA rating formulas which are the basis of most gear designs in the United States.

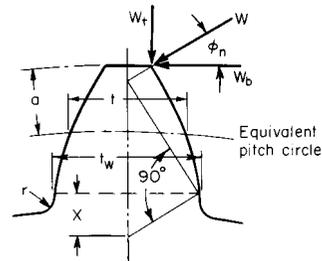


Fig. 8.3.20 Layout for beam strength (Lewis formula).

#### AGMA Strength and Durability Rating Formulas

For many decades the AGMA Gear Rating Committee has developed and provided tooth beam strength and surface durability (pitting resistance) formulas suitable for modern gear design. Over the years, the formulas have gone through a continual evolution of revision and improvement. The intent is to provide a common basis for rating various gear types for differing applications and thus have a uniformity of practice within the gear industry. This has been accomplished via a series of standards, many of which have been adopted by ANSI.

The latest standards for rating bending beam strength and pitting resistance are ANSI/AGMA 2001-C95, "Fundamental Rating Factors and Calculation Methods for Involute, Spur, and Helical Gear Teeth" (available in English and metric units) and AGMA 908-B89, "Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical, and Herringbone Gear Teeth." These standards have replaced AGMA 218.01 with improved formulas and details.

The rating formulas in Tables 8.3.11 and 8.3.12 are abstracted from ANSI/AGMA 2001-B88, "Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth," with permission.

**Overload factor  $K_o$**  is intended to account for an occasional load in excess of the nominal design load  $W_t$ . It can be established from experience with the particular application. Otherwise use  $K_o = 1$ .

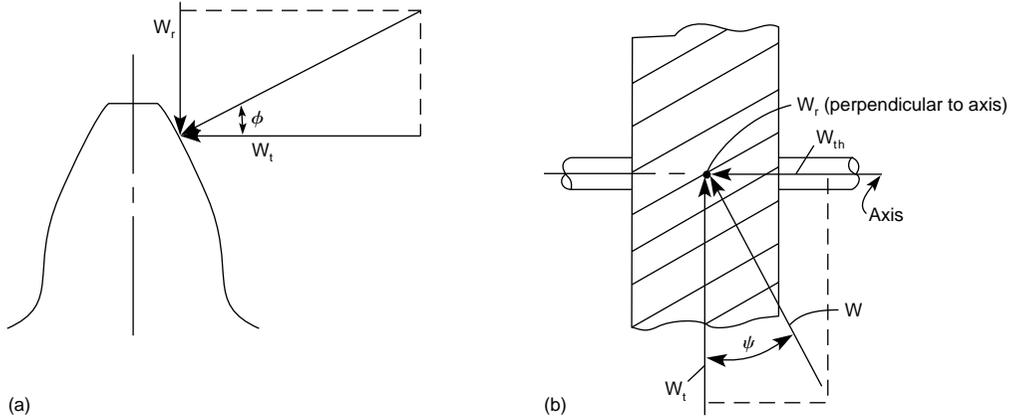


Fig. 8.3.21 Forces on spur and helical teeth. (a) Spur gear; (b) helical gear.

**Table 8.3.11 AGMA Pitting Resistance Formula for Spur and Helical Gears**  
(See Note 1 below.)

$$s_c = C_p \sqrt{W_t K_o K_v K_s \frac{K_m C_f}{dF} \frac{I}{I}}$$

where  $s_c$  = contact stress number, lb/in<sup>2</sup>  
 $C_p$  = elastic coefficient,\* (lb/in<sup>2</sup>)<sup>0.5</sup> (see text and Table 8.3.13)  
 $W_t$  = transmitted tangential load, lb  
 $K_o$  = overload factor (see text)  
 $K_v$  = dynamic factor (see Fig. 8.3.22)  
 $K_s$  = size factor (see text)  
 $K_m$  = load distribution factor (see text and Table 8.3.14)  
 $C_f$  = surface condition factor for pitting resistance (see text)  
 $F$  = net face width of narrowest member, in  
 $I$  = geometry factor for pitting resistance (see text and Figs. 8.3.23 and 8.3.24)  
 $d$  = operating pitch diameter of pinion, in

$$= \frac{2C}{m_G + 1} \quad \text{for external gears}$$

$$= \frac{2C}{m_G - 1} \quad \text{for internal gears}$$

where  $C$  = operating center distance, in  
 $m_G$  = gear ratio (never less than 1.0)

**Allowable contact stress number  $s_{ac}$**

$$s_c \leq \frac{s_{ac} Z_N C_H}{S_H K_T K_R}$$

where  $s_{ac}$  = allowable contact stress number, lb/in<sup>2</sup> (see Tables 8.3.15 and 8.3.16; Fig. 8.3.34)  
 $Z_N$  = stress cycle factor for pitting resistance (see Fig. 8.3.35)  
 $C_H$  = hardness ratio factor for pitting resistance (see text and Figs. 8.3.36 and 8.3.37)  
 $S_H$  = safety factor for pitting (see text)  
 $K_T$  = temperature factor (see text)  
 $K_R$  = reliability factor (see Table 8.3.19)

\* **Elastic coefficient  $C_p$**  can be calculated from the following equation when the paired materials in the pinion-gear set are not listed in Table 8.3.13:

$$C_p = \sqrt{\frac{1}{\pi[(1 - \mu_p^2)/E_p + (1 - \mu_G^2)/E_G]}}$$

where  $\mu_p(\mu_G)$  = Poisson's ratio for pinion (gear)  
 $E_p(E_G)$  = modulus of elasticity for pinion (gear), lb/in<sup>2</sup>

**Note 1:** If the rating is calculated on the basis of uniform load, the transmitted tangential load is

$$W_t = \frac{33,000P}{v_i} = \frac{2T}{d} = \frac{126,000P}{n_p d}$$

where  $P$  = transmitted power, hp  
 $T$  = transmitted pinion torque, lb · in  
 $v_i$  = pitch line velocity at operating pitch diameter, ft/min =  $\frac{\pi n_p d}{12}$

**Table 8.3.12 AGMA Bending Strength Fundamental Formula for Spur and Helical Gears**  
(See Note 1 in Table 8.3.11.)

$$s_t = W_t K_o K_v K_s \frac{P_d K_m K_B}{F J}$$

where  $s_t$  = bending stress number, lb/in<sup>2</sup>  
 $K_B$  = rim thickness factor (see Fig. 8.3.38)  
 $J$  = geometry factor for bending strength (see text and Figs. 8.3.25 to 8.3.31)  
 $P_d$  = transverse diametral pitch, in<sup>-1</sup>;  
 $P_{dn}$  for helical gears

$$P_d = \frac{\pi}{p_x \tan \psi_s} = P_{dn} \cos \psi_s \quad \text{for helical gears}$$

where  $P_{dn}$  = normal diametral pitch, in<sup>-1</sup>  
 $p_x$  = axial pitch, in  
 $\psi_s$  = helix angle at standard pitch diameter

$$\psi_s = \arcsin \frac{\pi}{p_x P_{dn}}$$

**Allowable bending stress numbers  $s_{at}$**

$$s_t \leq \frac{s_{at} Y_N}{S_F K_T K_R}$$

where  $s_{at}$  = allowable bending stress number, lb/in<sup>2</sup> (see Tables 8.3.17 and 8.3.18 and Figs. 8.3.39 to 8.3.41)  
 $Y_N$  = stress cycle factor for bending strength (see Fig. 8.3.42)  
 $S_F$  = safety factor for bending strength (see text)

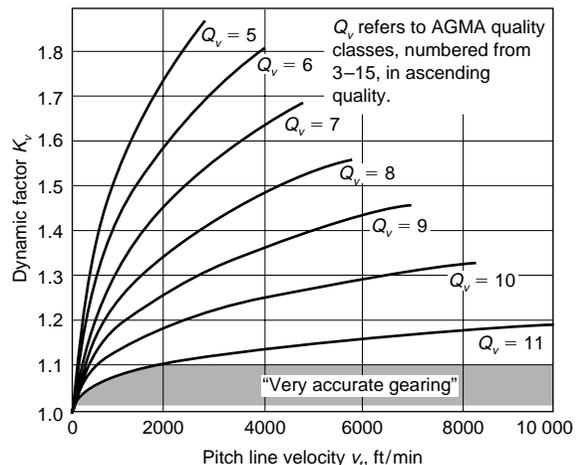


Fig. 8.3.22 Dynamic factor  $K_v$ . (Source: ANSI/AGMA 2001-C95, with permission.)

Table 8.3.13 Elastic Coefficient  $C_p$ 

Pinion material	Pinion modulus of elasticity $E_p$ , lb/in <sup>2</sup> (MPa)	Gear material and modulus of elasticity $E_G$ , lb/in <sup>2</sup> (MPa)					
		Steel $30 \times 10^6$ ( $2 \times 10^5$ )	Malleable iron $25 \times 10^6$ ( $1.7 \times 10^5$ )	Nodular iron $24 \times 10^6$ ( $1.7 \times 10^5$ )	Cast iron $22 \times 10^6$ ( $1.5 \times 10^5$ )	Aluminum bronze $17.5 \times 10^6$ ( $1.2 \times 10^5$ )	Tin bronze $16 \times 10^6$ ( $1.1 \times 10^5$ )
Steel	$30 \times 10^6$ ( $2 \times 10^5$ )	2,300 (191)	2,180 (181)	2,160 (179)	2,100 (174)	1,950 (162)	1,900 (158)
Malleable iron	$25 \times 10^6$ ( $1.7 \times 10^5$ )	2,180 (181)	2,090 (174)	2,070 (172)	2,020 (168)	1,900 (158)	1,850 (154)
Nodular iron	$24 \times 10^6$ ( $1.7 \times 10^5$ )	2,160 (179)	2,070 (172)	2,050 (170)	2,000 (166)	1,880 (156)	1,830 (152)
Cast iron	$22 \times 10^6$ ( $1.5 \times 10^5$ )	2,100 (174)	2,020 (168)	2,000 (166)	1,960 (163)	1,850 (154)	1,800 (149)
Aluminum bronze	$17.5 \times 10^6$ ( $1.2 \times 10^5$ )	1,950 (162)	1,900 (158)	1,880 (156)	1,850 (154)	1,750 (145)	1,700 (141)
Tin bronze	$16 \times 10^6$ ( $1.1 \times 10^5$ )	1,900 (158)	1,850 (154)	1,830 (152)	1,800 (149)	1,700 (141)	1,650 (137)

Poisson's ratio = 0.30.

SOURCE: ANSI/AGMA 2001-B88; with permission.

**Size factor  $K_s$**  is intended to factor in material nonuniformity due to tooth size, diameter, face width, etc. AGMA has not established factors for general gearing; use  $K_s = 1$  unless there is information to warrant using a larger value.

**Load distribution factor  $K_m$**  reflects the nonuniform loading along the lines of contact due to gear errors, installation errors, and deflections. Analytical and empirical methods for evaluating this factor are presented in ANSI/AGMA 2001-C95 but are too extensive to include here. Alternately, if appropriate for the application,  $K_m$  can be extrapolated from values given in Table 8.3.14.

**Surface condition factor  $C_f$**  is affected by the manufacturing method (cutting, shaving, grinding, shotpeening, etc.). Standard factors have not been established by AGMA. Use  $C_f = 1$  unless experience can establish confidence for a larger value.

**Geometry factors  $I$  and  $J$**  relate to the shape of the tooth at the point of contact, the most heavily loaded point. AGMA 908-B89 (Information Sheet, Geometry Factors for Determining the Pitting Resistance and Bending Strength for Spur, Helical and Herringbone Gear Teeth) presents detailed procedures for calculating these factors. The standard also includes a collection of tabular values for a wide range of gear tooth designs, but they are too voluminous to be reproduced here in their entirety. Earlier compact graphs of  $I$  and  $J$  values in AGMA 218.01 are still valid. They are presented here along with several curves from AGMA 610-E88, which are based upon 218.01. See Figs. 8.3.23 to 8.3.31.

**Allowable contact stress  $s_{ac}$**  and **allowable bending stress  $s_{at}$**  are obtainable from Tables 8.3.15 to 8.3.18. Contact stress hardness specification applies to the start of active profile at the center of the face width, and for bending stress at the root diameter in the center of the tooth space and face width. The lower stress values are for general design purposes;

upper values are for high-quality materials and high-quality control. (See ANSI/AGMA 2001-C95, tables 7 through 10, regarding detailed metallurgical specifications; stress grades 1, 2, and 3; and type A and B hardness patterns.)

For **reversing loads**, allowable bending stress values,  $s_{at}$  are to be reduced to 70 percent. If the rim thickness cannot adequately support the load, an additional derating factor  $K_B$  is to be applied. See Fig. 8.3.38.

**Hardness ratio factor  $C_H$**  applies when the pinion is substantially harder than the gear, and it results in work hardening of the gear and increasing its capacity. Factor  $C_H$  applies to only the gear, not the pinion. See Figs. 8.3.36 and 8.3.37.

**Safety factors  $S_H$  and  $S_F$**  are defined by AGMA as factors beyond  $K_O$  and  $K_R$ ; they are used in connection with extraordinary risks, human or economic. The values of these factors are left to the designer's judgment as she or he assesses all design inputs and the consequences of possible failure.

**Temperature factor  $K_T = 1$**  when gears operate with oil temperature not exceeding 250°F.

**Reliability factor  $K_R$**  accounts for statistical distribution of material failures. Typically, material strength ratings (Tables 8.3.13, and 8.3.15 to 8.3.18; Fig. 8.3.34 and 8.3.42) are based on probability of one failure in 100 at  $10^7$  cycles. Table 8.3.19 lists reliability factors that may be used to modify the allowable stresses and the probability of failure.

#### Strength and Durability of Bevel, Worm, and Other Gear Types

For bevel gears, consult the referenced Gleason publications; for worm gearing refer to AGMA standards; for other special types, refer to Dudley, "Gear Handbook."

Table 8.3.14 Load-Distribution Factor  $K_m$  for Spur Gears\*

Characteristics of support	Face width, in			
	0–2	6	9	16 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full face contact exists			Over 2.2	

\* An approximate guide only. See ANSI/AGMA 2001-C95 for derivation of more exact values.  
SOURCE: Darle W. Dudley, "Gear Handbook," McGraw-Hill, New York, 1962.

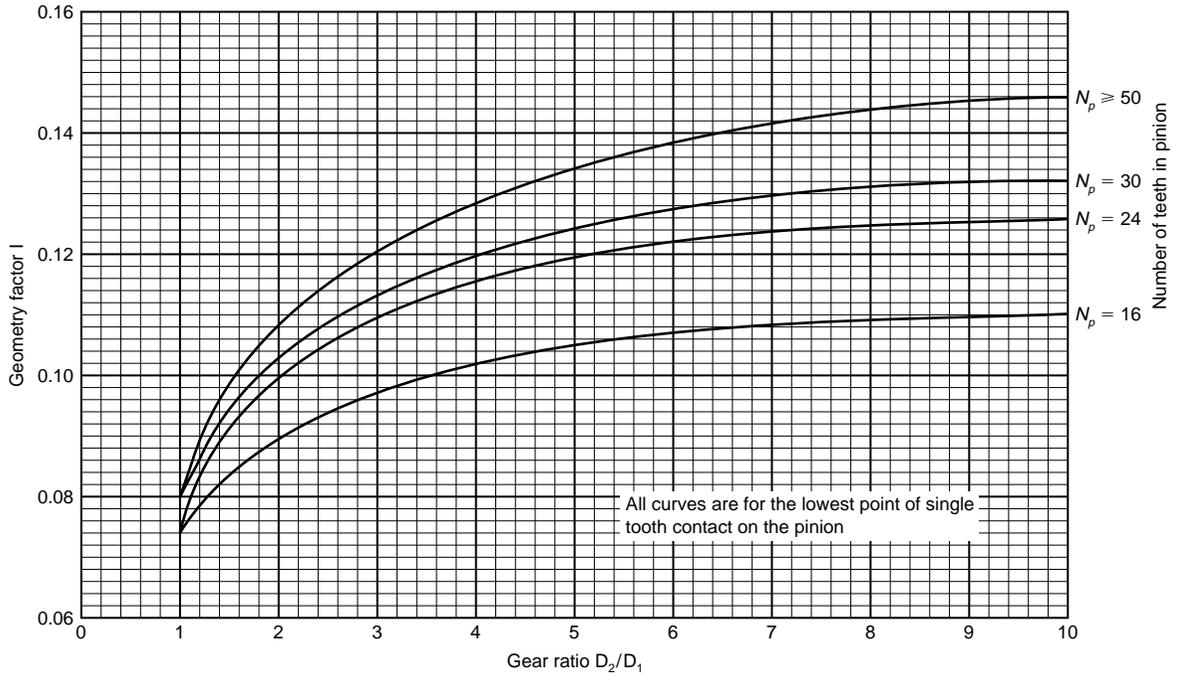


Fig. 8.3.23 Geometry factor  $I$  for  $20^\circ$  full-depth standard spur gears. (Source: ANSI/AGMA 2018-01, with permission.)

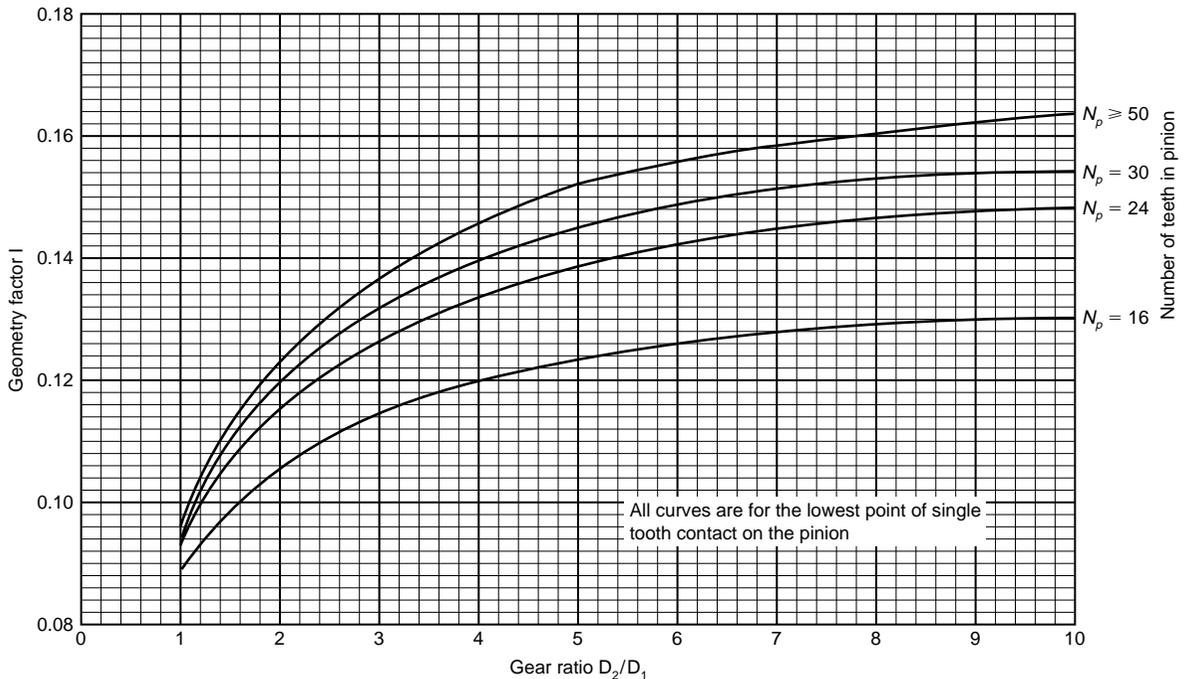


Fig. 8.3.24 Geometry factor  $I$  for  $25^\circ$  full-depth standard spur gears. (Source: ANSI/AGMA 2018-01, with permission.)

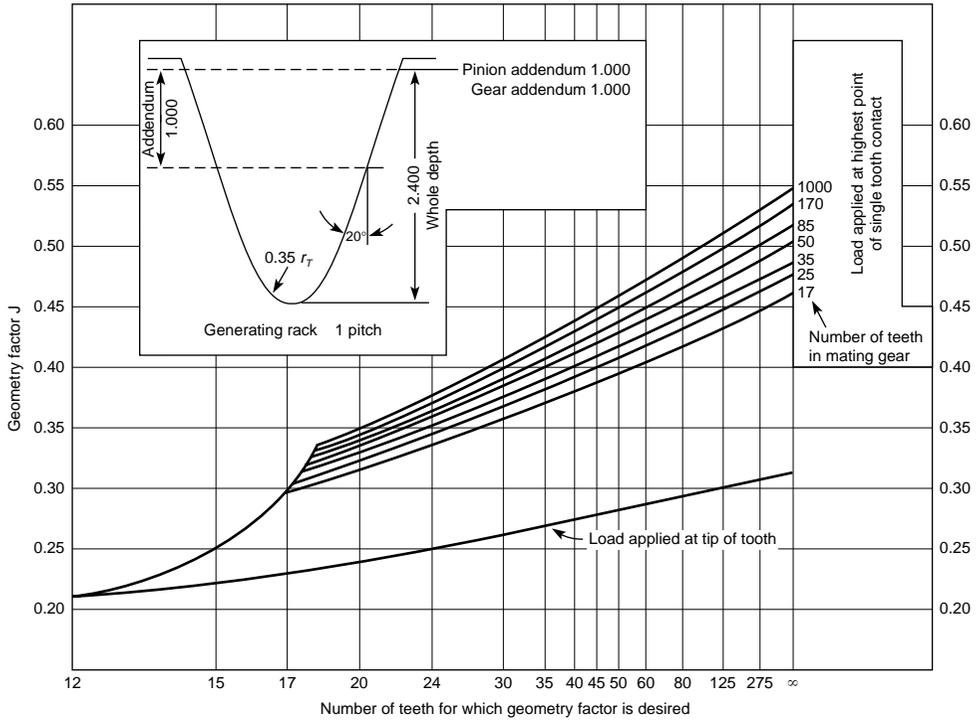


Fig. 8.3.25 Geometry factor  $J$  for 20° standard addendum spur gears. (Source: ANSI/AGMA 2018-01, with permission.)

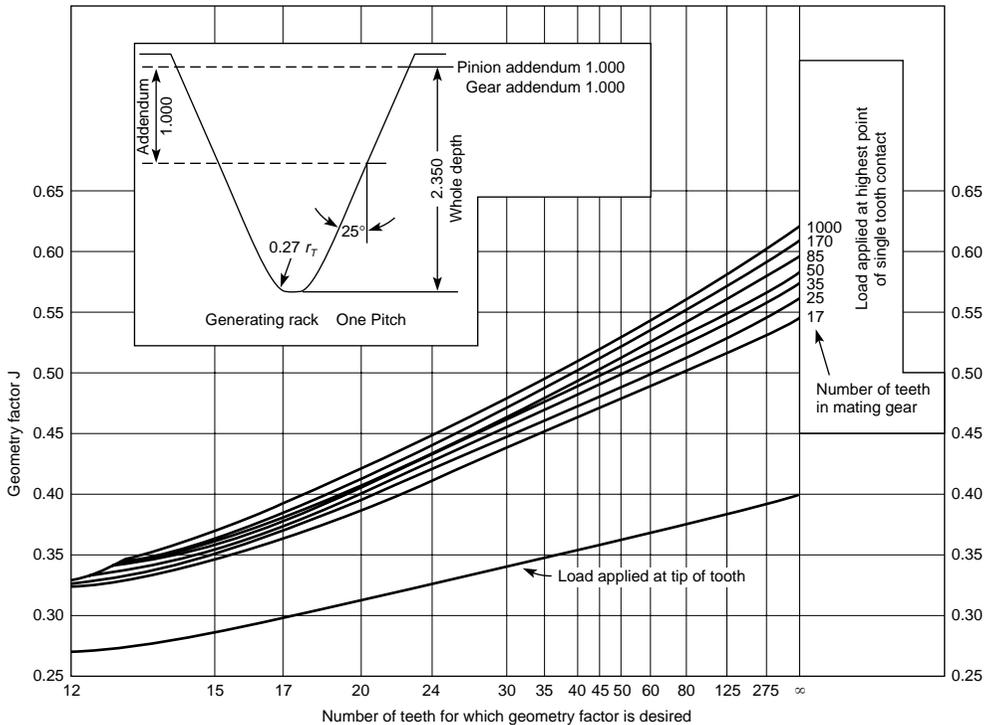
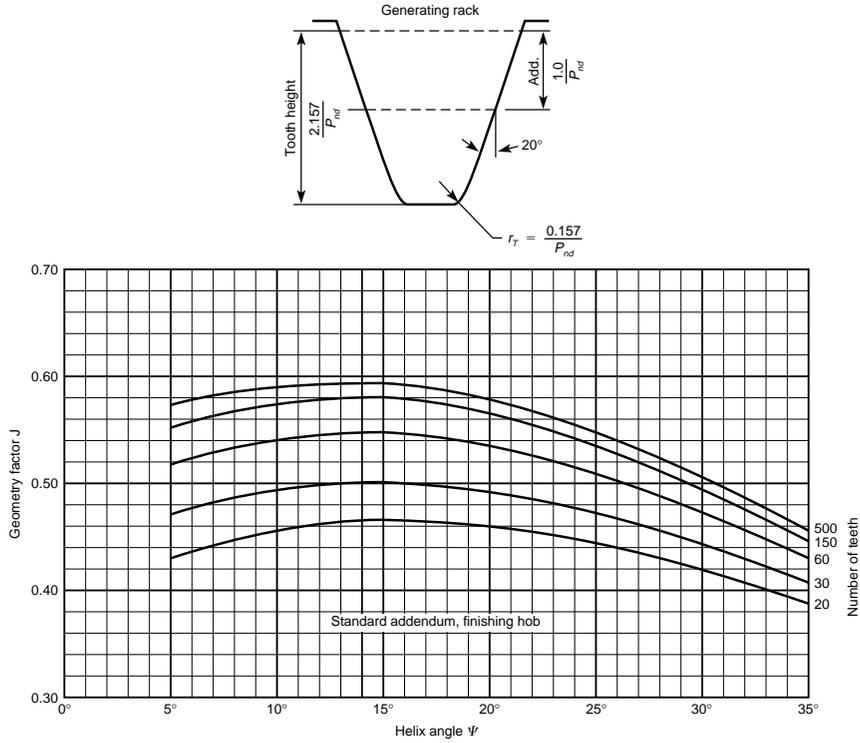
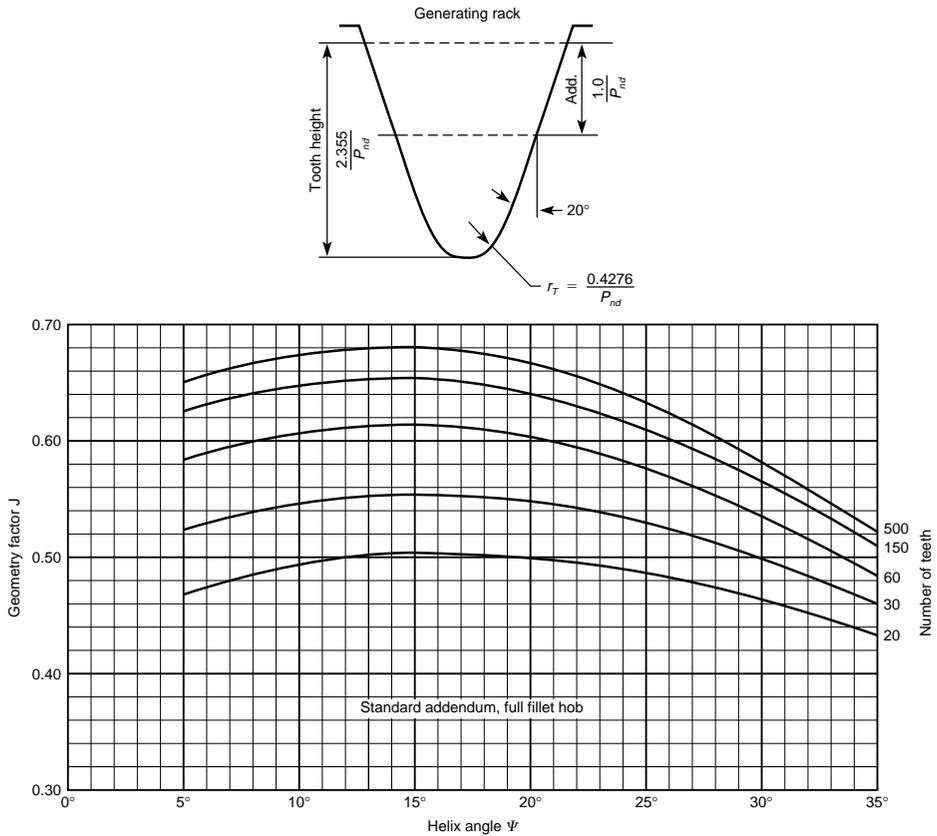


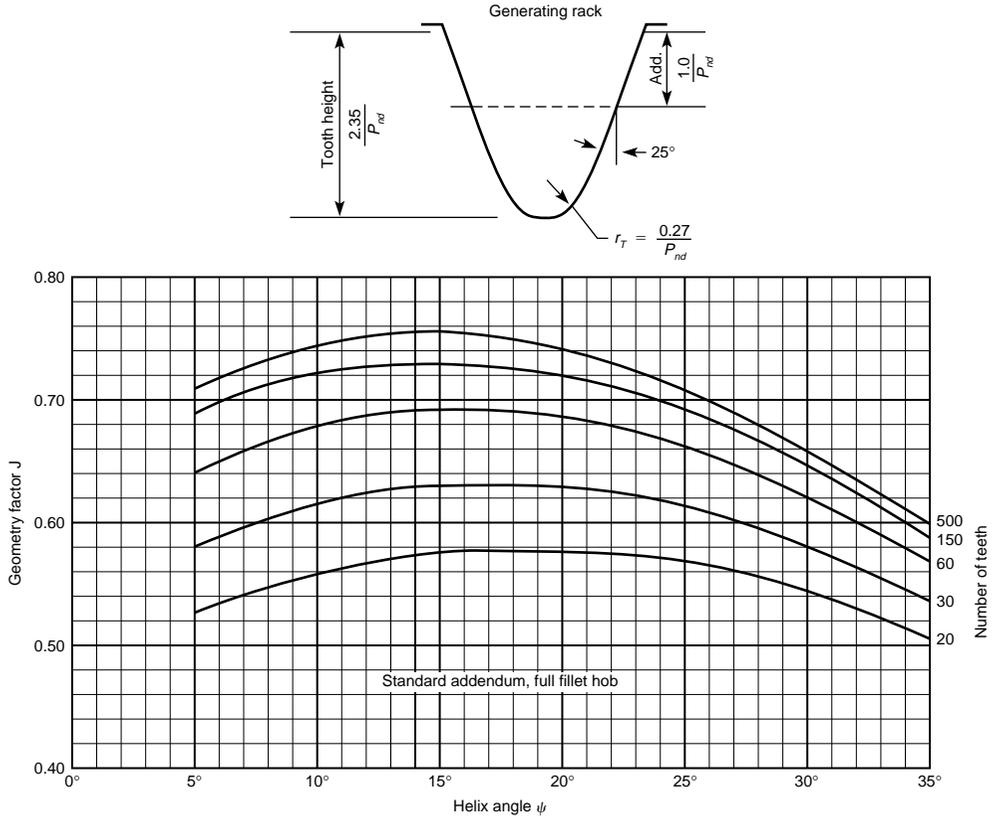
Fig. 8.3.26 Geometry factor  $J$  for 25° standard addendum spur gears. (Source: ANSI/AGMA 2018-01, with permission.)



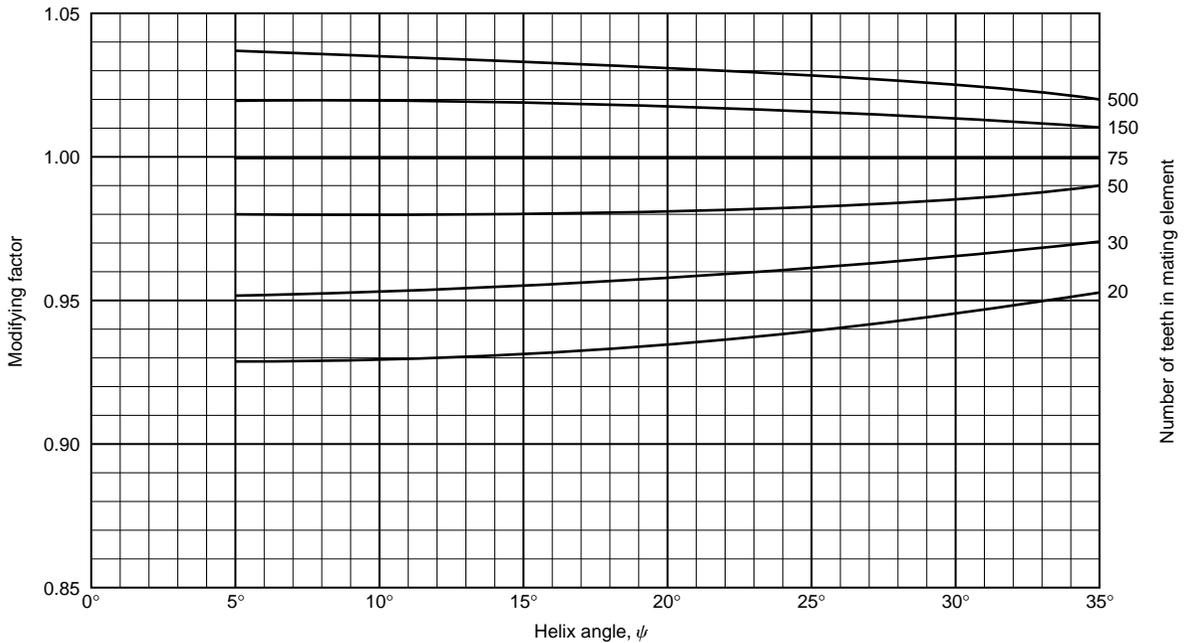
**Fig. 8.3.27** Geometry factor  $J$  for 20° normal pressure angle helical gears. (Standard addendum, finishing hob.) (Source: ANSI/AGMA 2018-01, with permission.)



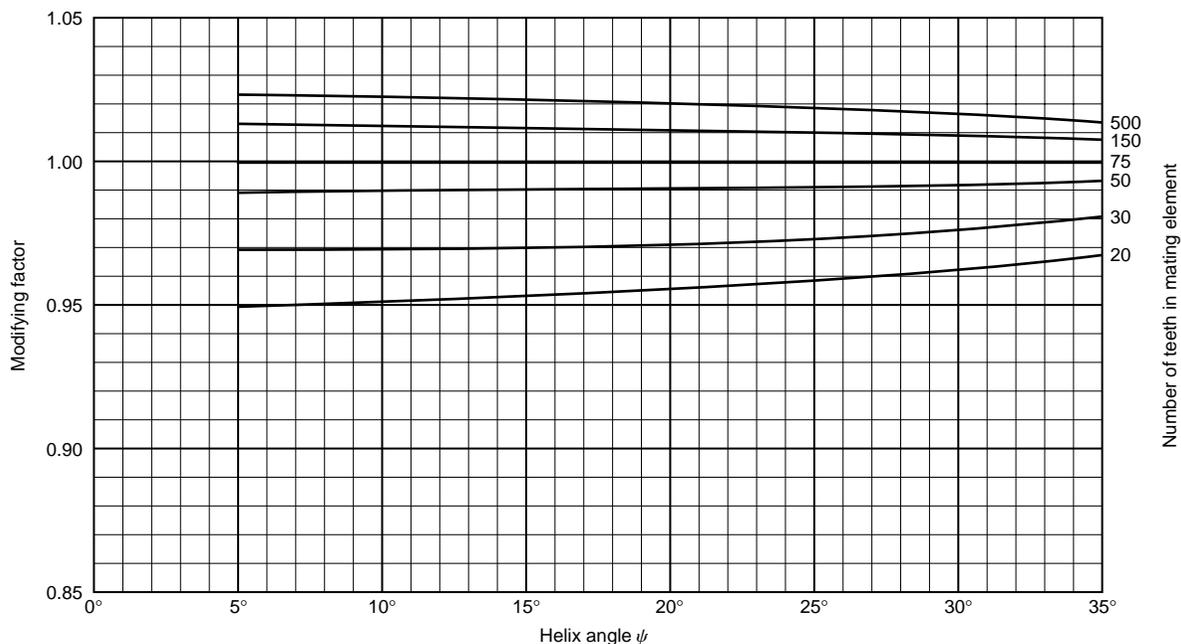
**Fig. 8.3.28** Geometry factor  $J$  for 20° normal pressure angle helical gears. (Standard addendum, full fillet hob.) (Source: ANSI/AGMA 2018-01, with permission.)



**Fig. 8.3.29** Geometry factor  $J$  for  $25^\circ$  normal pressure angle helical gears. (Standard addendum, full fillet hob.)  
 (Source: ANSI/AGMA 2018-01, with permission.)



**Fig. 8.3.30** Factor  $J$  multipliers for  $20^\circ$  normal pressure angle helical gears. The modifying factor can be applied to the  $J$  factor when other than 75 teeth are used in the mating element. (Source: ANSI/AGMA 2018-01, with permission.)



**Fig. 8.3.31** Factor  $J$  multipliers for 25° normal pressure angle helical gears. The modifying factor can be applied to the  $J$  factor when other than 75 teeth are used in the mating element. (Source: ANSI/AGMA 6010-E88, with permission.)

**Table 8.3.15** Allowable Contact Stress Number  $s_{ac}$  for Steel Gears

Material designation	Heat treatment	Minimum surface hardness*	Allowable contact stress number $s_{ac}$ , lb/in <sup>2</sup>		
			Grade 1	Grade 2	Grade 3
Steel	Through-hardened*	Fig. 8.3.34	Fig. 8.3.34	Fig. 8.3.34	—
	Flame-* or induction-hardened*	50 HRC 54 HRC	170,000 175,000	190,000 195,000	— —
	Carburized and hardened*	Table 9 Note 1	180,000	225,000	275,000
	Nitrided* (through-hardened steels)	83.5 HR15N 84.5 HR15N	150,000 155,000	163,000 168,000	175,000 180,000
2.5% Chrome (no aluminum)	Nitrided*	87.5 HR15N	155,000	172,000	189,000
Nitralloy 135M	Nitrided*	90.0 HR15N	170,000	183,000	195,000
Nitralloy N	Nitrided*	90.0 HR15N	172,000	188,000	205,000
2.5% Chrome (no aluminum)	Nitrided*	90.0 HR15N	176,000	196,000	216,000

Note 1: Table 9 and Tables 7, 8, and 10 cited in Tables 8.3.16 to 8.3.18 are in ANSI/AGMA 2001-95.

\* The allowable-stress numbers indicated may be used with the case depths shown in Figs. 8.3.32 and 8.3.33.

SOURCE: Abstracted from ANSI/AGMA 2001-C95, with permission.

**Table 8.3.16** Allowable Contact Stress Number  $s_{ac}$  for Iron and Bronze Gears

Material	Material designation*	Heat treatment	Typical minimum surface hardness	Allowable contact stress number $s_{ac}$ , lb/in <sup>2</sup>
ASTM A48 gray cast iron	Class 20	As cast	—	50,000–60,000
	Class 30	As cast	174 HB	65,000–75,000
	Class 40	As cast	201 HB	75,000–85,000
ASTM A536 ductile (nodular) iron	Grade 60-40-18	Annealed	140 HB	77,000–92,000
	Grade 80-55-06	Quenched & tempered	179 HB	77,000–92,000
	Grade 100-70-03	Quenched & tempered	229 HB	92,000–112,000
	Grade 120-90-02	Quenched & tempered	269 HB	103,000–126,000
Bronze	—	Sand-cast	Minimum tensile strength 40,000 lb/in <sup>2</sup>	30,000
	ASTM B-148 Alloy 954	Heat-treated	Minimum tensile strength 90,000 lb/in <sup>2</sup>	65,000

\* See ANSI/AGMA 2004-B89, "Gear Materials and Heat Treatment Manual."

SOURCE: Abstracted from ANSI/AGMA 2001-C95, with permission.

Table 8.3.17 Allowable Bending Stress Number  $s_{at}$  for Steel Gears

Material designation	Heat treatment	Minimum surface hardness	Allowable bending stress number $s_{at}$ , lb/in <sup>2</sup>		
			Grade 1	Grade 2	Grade 3
Steel	Through-hardened	Fig. 8.3.39	Fig. 8.3.39	Fig. 8.3.39	—
	Flame* or induction-hardened* with type A pattern	Table 8§	45,000	55,000	—
	Flame* or induction hardened* with type B pattern	Table 8§ Note 1	22,000	22,000	—
	Carburized and hardened*	Table 9§ Note 2	55,000	65,000 or 70,000†	75,000
	Nitrided*‡ (through-hardened steels)	83.5 HR15N	Fig. 8.3.40	Fig. 8.3.40	—
Nitralloy 135M, nitralloy N, and 2.5% Chrome (no aluminum)	Nitrided‡	87.5 HR15N	Fig. 8.3.41	Fig. 8.3.41	Fig. 8.3.41

Note 1: See Table 8 in ANSI/AGMA 2001-C95.

Note 2: See Table 9 in ANSI/AGMA 2001-C95.

\* The allowable-stress numbers indicated may be used with the case depths shown in Figs. 8.3.32 and 8.3.33.

† If bainite and microcracks are limited to grade 3 levels, 70,000 lb/in<sup>2</sup> may be used.

‡ The overload capacity of nitrided gears is low. Since the shape of the effective  $S-N$  curve is flat, the sensitivity to shock should be investigated before one proceeds with the design.

§ The tabular material is too extensive to record here. Refer to ANSI/AGMA 2001-C95, tables 7 to 10.

SOURCE: Abstracted from ANSI/AGMA 2001-C95, with permission.

Table 8.3.18 Allowable Bending Stress Number  $s_{at}$  for Iron and Bronze Gears

Material	Material designation*	Heat treatment	Typical minimum surface hardness	Allowable bending stress number $s_{at}$ , lb/in <sup>2</sup>
ASTM A48 gray cast iron	Class 20	As cast	—	5,000
	Class 30	As cast	174 HB	8,500
	Class 40	As cast	201 HB	13,000
ASTM A536 ductile (nodular) iron	Grade 60-40-18	Annealed	140 HB	22,000–33,000
	Grade 80-55-06	Quenched & tempered	179 HB	22,000–33,000
	Grade 100-70-03	Quenched & tempered	229 HB	27,000–40,000
	Grade 120-90-02	Quenched & tempered	269 HB	31,000–44,000
Bronze	ASTM B-148 Alloy 954	Sand-cast	Minimum tensile strength 40,000 lb/in <sup>2</sup>	5,700
		Heat-treated	Minimum tensile strength 90,000 lb/in <sup>2</sup>	23,600

\* See ANSI/AGMA 2004-B89, "Gear Materials and Heat Treatment Manual."

SOURCE: Abstracted from ANSI/AGMA 2001-C95, with permission.

Table 8.3.19 Reliability Factors  $K_R$ 

Requirements of application	$K_R$ *
Fewer than one failure in 10,000	1.50
Fewer than one failure in 1,000	1.25
Fewer than one failure in 100	1.00
Fewer than one failure in 10	0.85†
Fewer than one failure in 2	0.70†,‡

\* Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of  $K_R$  is selected for bending.

† At this value, plastic flow might occur rather than pitting.

‡ From test data extrapolation.

SOURCE: Abstracted from ANSI/AGMA 2001-C95, with permission.

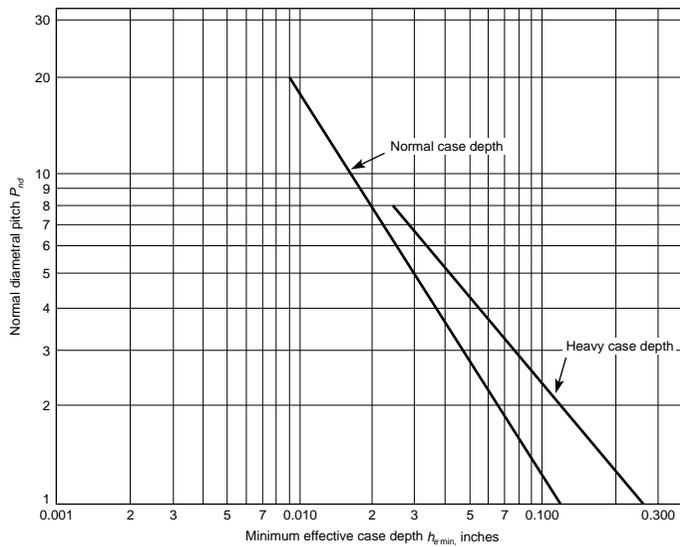
## GEAR MATERIALS

(See Tables 8.3.15 to 8.3.18.)

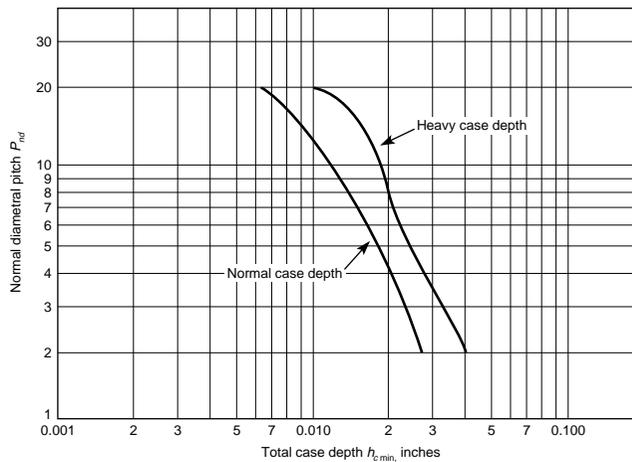
### Metals

Plain carbon steels are most widely used as the most economical; similarly, cast iron is used for large units or intricate body shapes. Heat-treated carbon and alloy steels are used for the more severe load- and wear-resistant applications. Pinions are usually made harder to equalize wear. Strongest and most wear-resistant gears are a combination of heat-treated high-alloy steel cores with case-hardened teeth. (See Dudley, "Gear Handbook," chap. 10.) Bronze is particularly recommended for wormgears and crossed helical gears. Stainless steels are limited to special corrosion-resistant environment applications. Aluminum alloys are used for light-duty instrument gears and airborne lightweight requirements.

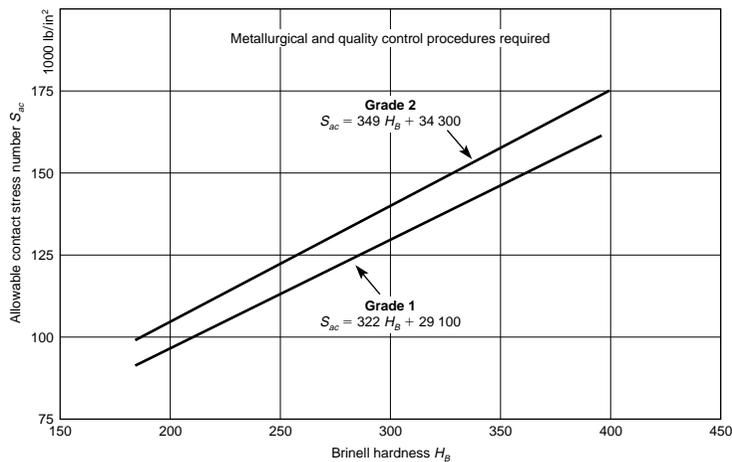
Sintered powdered metals technology offers commercial high-quality gearing of high strength at very economical production costs. Die-cast gears for light-duty special applications are suitable for many products.



**Fig. 8.3.32** Minimum effective case depth for carburized gears  $h_{e,min}$ . Effective case depth is defined as depth of case with minimum hardness of 50 RC. Total case depth to core carbon is approximately 1.5 times the effective case depth. (Source: ANSI/AGMA 2001-C95, with permission.)



**Fig. 8.3.33** Minimum total case depth for nitrided gears  $h_{c,min}$ . (Source: ANSI/AGMA 2001-C95, with permission.)



**Fig. 8.3.34** Allowable contact stress number for through-hardened steel gears  $s_{ac}$ . (Source: ANSI/AGMA 2001-C95, with permission.)

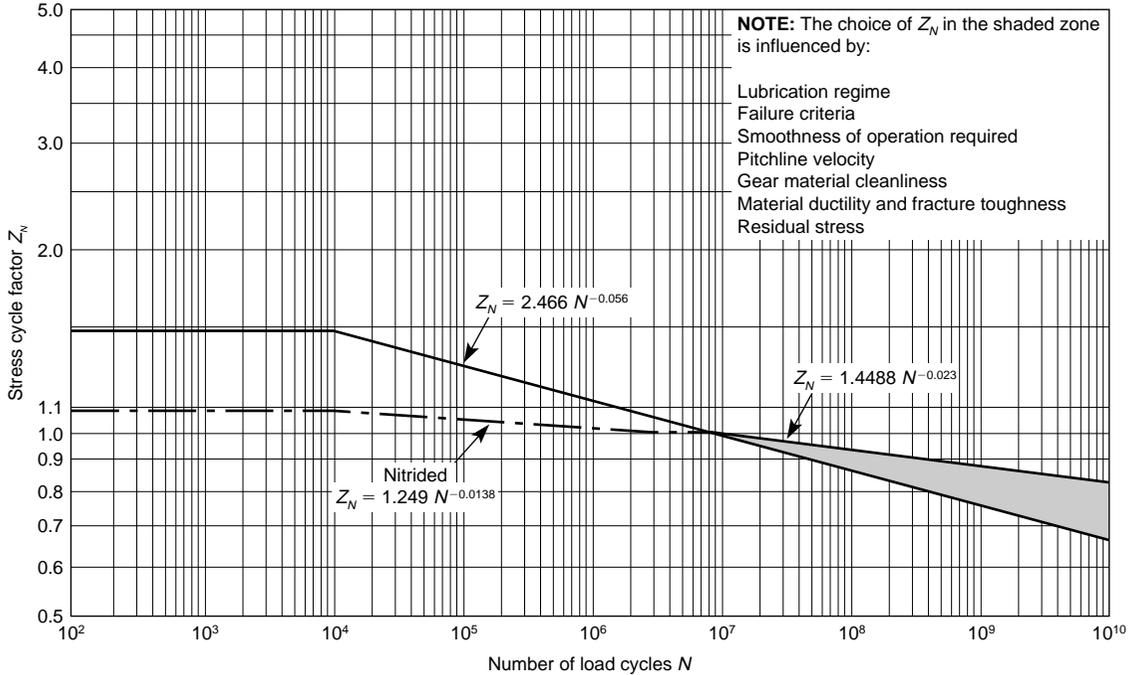


Fig. 8.3.35 Pitting resistance stress cycle factor  $Z_N$ . (Source: ANSI/AGMA 2001-C95, with permission.)

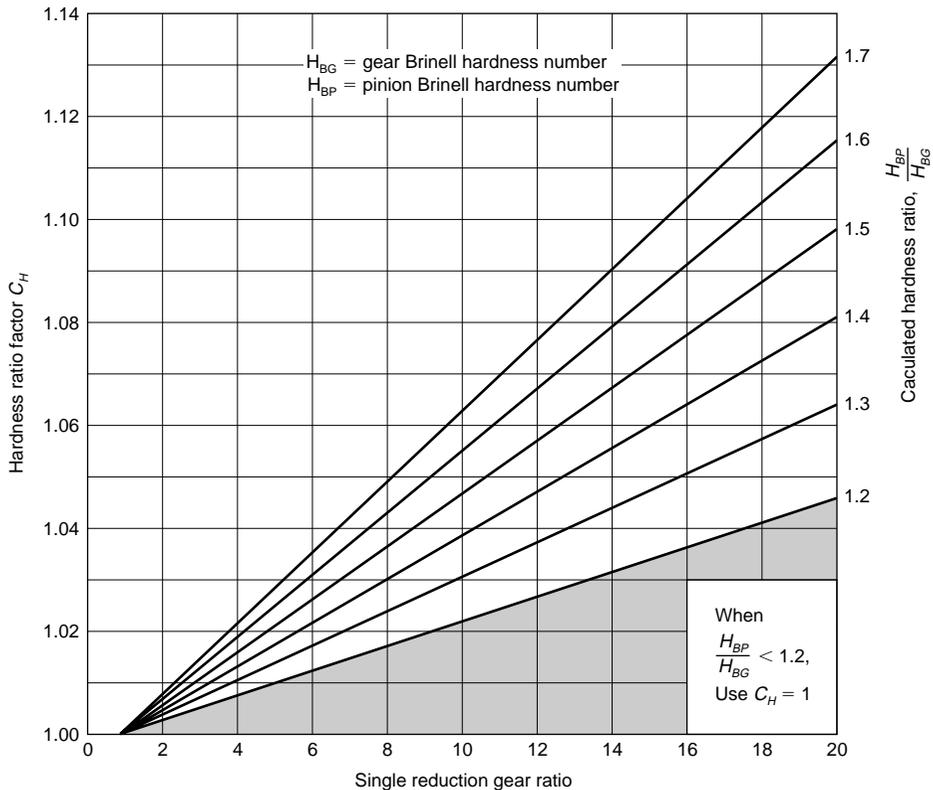


Fig. 8.3.36 Hardness ratio factor  $C_H$  (through-hardened). (Source: ANSI/AGMA 2001-C95, with permission.)

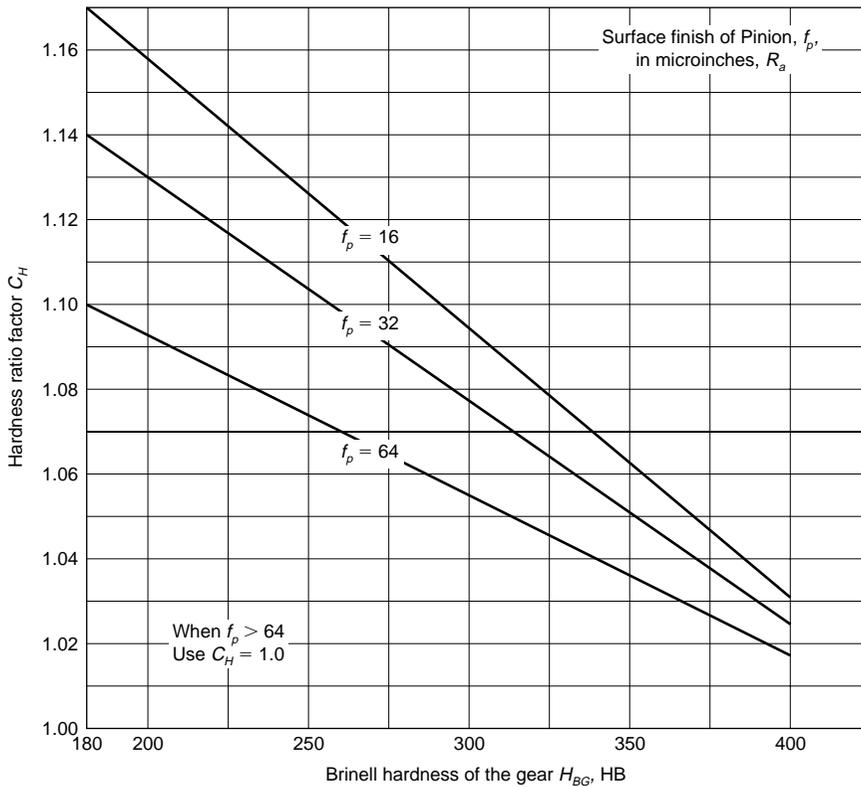


Fig. 8.3.37 Hardness ratio factor  $C_H$  (surface-hardened pinions). (Source: ANSI/AGMA 2001-C95, with permission.)

For power gear applications, heat treatment is an important part of complete and proper design and specification. Heat treatment descriptions and specification tolerances are given in the reference cited below.

Precision gears of the small device and instrument types often require protective coatings, particularly for aircraft, marine, space, and military applications. There is a wide choice of chemical and electroplate coatings offering a variety of properties and protection.

For pertinent properties and details of the above special materials and protective coatings see Michalec, "Precision Gearing," chap. 9, Wiley.

### Plastics

In recent decades, various forms of nonmetallic gears have displaced metal gears in particular applications. Most plastics can be hobbled or shaped by the same methods used for metallic gears. However, high-strength composite plastics suitable for good-quality gear molding have become available, along with the development of economical high-speed injection molding machines and improved methods for producing accurate gear molds.

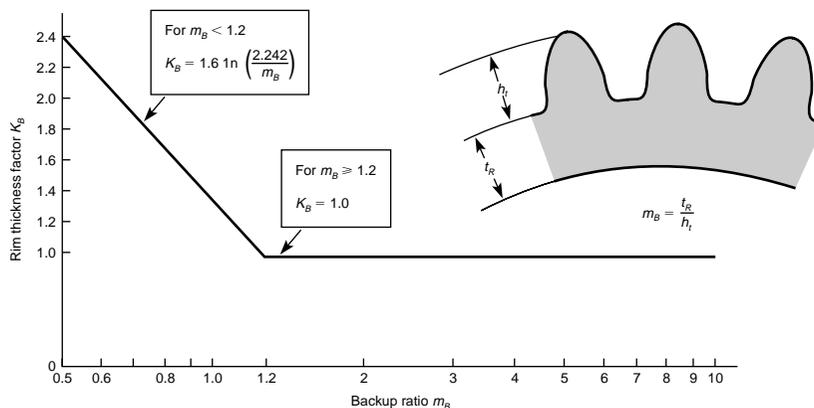


Fig. 8.3.38 Rim thickness factor  $K_B$ . (Source: ANSI/AGMA 2001-C95, with permission.)

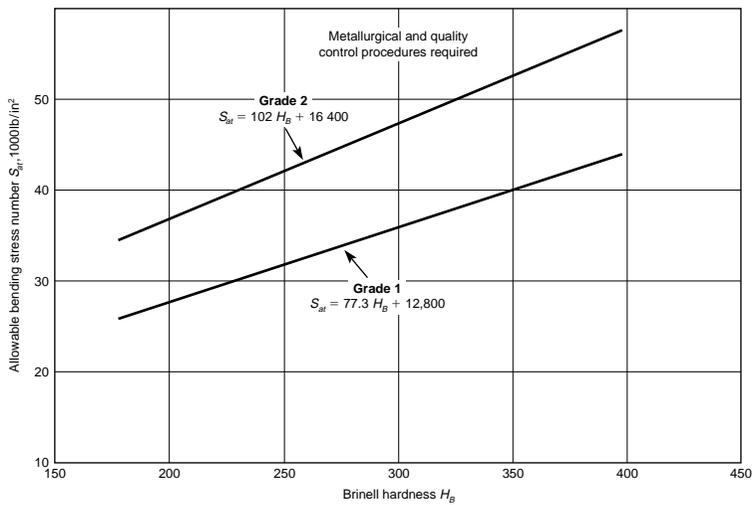


Fig. 8.3.39 Allowable bending stress numbers for through-hardened steel gears  $s_{at}$ . (Source: ANSI/AGMA 2001-C95, with permission.)

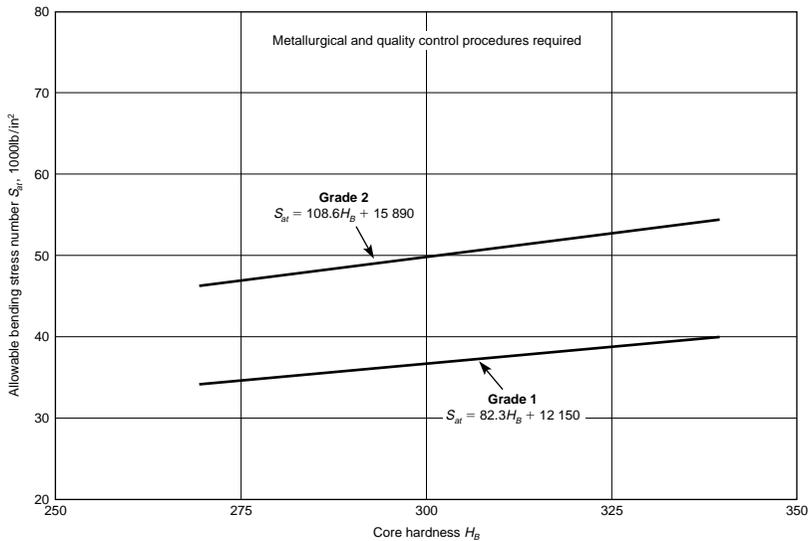


Fig. 8.3.40 Allowable bending stress numbers  $s_{at}$  for nitrided through-hardened steel gears (i.e., AISI 4140 and 4340). (Source: ANSI/AGMA 2001-C95, with permission.)

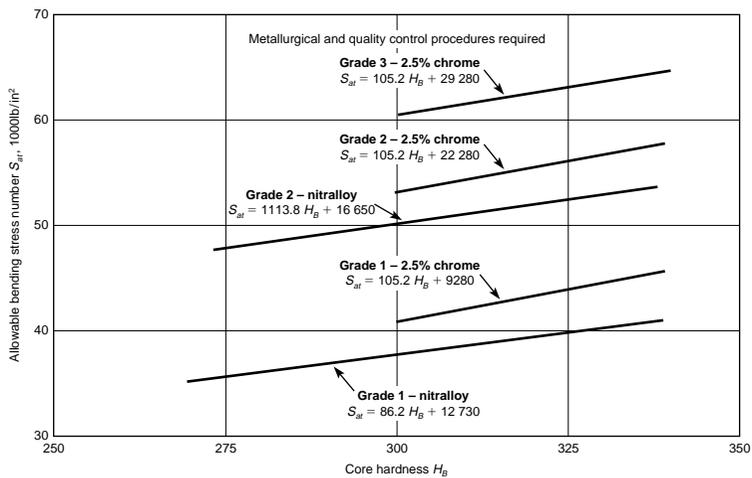


Fig. 8.3.41 Allowable bending stress numbers for nitrided steel gears  $s_{at}$ . (Source: ANSI/AGMA 2001-C95, with permission.)

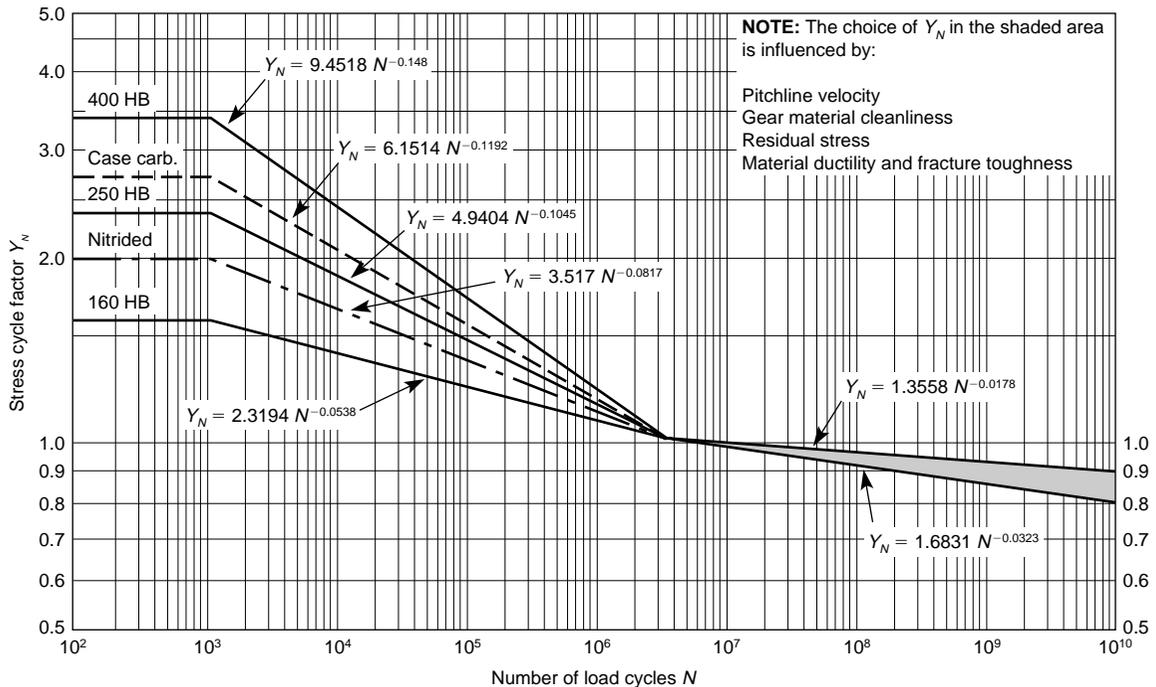


Fig. 8.3.42 Bending strength stress cycle factor  $Y_N$ . (Source: ANSI/AGMA 2001-C95, with permission.)

The most significant features and advantages of plastic gear materials are:

- Cost-effectiveness of injection molding process
- Wide choice of characteristics: mechanical strength, density, friction, corrosion resistance, etc.
- One-step production; no preliminary or secondary operations
- Uniformity of parts
- Ability to integrate special shapes, etc., into gear body
- Elimination of machining operations
- Capability to mold with metallic insert hubs, if required, for more precise bore diameter or body stability
- Capability to mold with internal solid lubricants
- Ability to operate without lubrication
- Quietness of operation
- Consistent with trend toward greater use of plastic housings and components

Plastic gears do have limitations relative to metal gears. The most significant are:

- Less load-carrying capacity
- Cannot be molded to as high accuracy as machined metal gears
- Much larger coefficient of expansion compared to metals
- Less environmentally stable with regard to temperature and water absorption
- Can be negatively affected by some chemicals and lubricants
- Initial high cost in mold manufacture to achieve proper tooth geometry accuracies
- Narrower range of temperature operation, generally less than 250°F and not lower than 0°F

For further information about plastic gear materials and achievable precision, consult the cited reference: Michalec, "Precision Gearing: Theory and Practice."

For a comprehensive presentation of gear molding practices, design, plastic materials, and strength and durability of plastic gears, consult the cited reference: Designatronics, "Handbook of Gears: Inch and Metric." pp. T131–T158.

## GEAR LUBRICATION

Proper lubrication is important to prevention of premature wear of tooth surfaces. In the basic action of involute tooth profiles there is a significant sliding component along with rolling action. In worm gearing sliding is the predominant consideration. Thus, a lubricant is essential for all gearing subject to measurable loadings, and even for lightly or negligibly loaded instrument gearing it is needed to reduce friction. Excellent oils and greases are available for high unit load, high speed gearing. See Secs. 6.11 and 8.4; also consult lubricant suppliers for recommendations and latest available high-quality lubricants with special-purpose additives.

General and specific information for lubrication of gearing is found in ANSI/AGMA 9005-D94, "Industrial Gear Lubrication," which covers open and enclosed gearing of all types. AGMA lists a family of lubricants in accordance with viscosities, numbered 1 through 13, with a cross-reference to equivalent ISO grades. See Table 8.3.20. For AGMA's lubrication recommendations for open and closed gearing related to pitch line speed and various types of lubrication systems, refer to Tables 8.3.21 to 8.3.23. For worm gearing and other information, see ANSI/AGMA 9005-D94.

Information in Table 8.3.24 may be used as a quick guide to gear lubricants and their sources for general-purpose instrument and medium-size gearing. Lubricant suppliers should be consulted for specific high-demand applications.

Often gear performance can be enhanced by special additives to the oil. For this purpose, colloidal additives of graphite, molybdenum disulfide ( $\text{MoS}_2$ ), and Teflon are very effective. These additives are particularly helpful to reduce friction and prevent wear; they are also very beneficial to reduce the rate of wear once it has begun, and thus they prolong gear life. The colloidal additive combines chemically with the metal surface material, resulting in a tenacious layer of combined material interposed between the base metals of the meshing teeth. The size of the colloidal additives is on the order of  $2 \mu\text{m}$ , sufficiently fine not to interfere with the proper operation of the lubricant system filters. Table 8.3.25 lists some commercial colloidal additives and their sources of supply.

Plastic gears are often operated without any external lubrication, and

**Table 8.3.20 Viscosity Ranges for AGMA Lubricants**

Rust- and oxidation-inhibited gear oils, AGMA lubricant no.	Viscosity range <sup>a</sup> mm <sup>2</sup> /s (cSt) at 40°C	Equivalent ISO grade <sup>a</sup>	Extreme pressure gear lubricants, <sup>b</sup> AGMA lubricant no.	Synthetic gear oils, <sup>c</sup> AGMA lubricant no.
0	28.8–35.2	32		0 S
1	41.4–50.6	46		1 S
2	61.2–74.8	68	2 EP	2 S
3	90–110	100	3 EP	3 S
4	135–165	150	4 EP	4 S
5	198–242	220	5 EP	5 S
6	288–352	320	6 EP	6 S
7, 7 Comp <sup>d</sup>	414–506	460	7 EP	7 S
8, 8 Comp <sup>d</sup>	612–748	680	8 EP	8 S
8A Comp <sup>d</sup>	900–1,100	1,000	8A EP	—
9	1,350–1,650	1,500	9 EP	9 S
10	2,880–3,520	—	10 EP	10 S
11	4,140–5,060	—	11 EP	11 S
12	6,120–7,480	—	12 EP	12 S
13	190–220 cSt at 100°C (212°F) <sup>e</sup>	—	13 EP	13 S
Residual compounds, <sup>f</sup> AGMA lubricant no.	Viscosity ranges <sup>e</sup> cSt at 100°C (212°F)			
14R	428.5–857.0			
15R	857.0–1,714.0			

<sup>a</sup> Per ISO 3448, “Industrial Liquid Lubricants—ISO Viscosity Classification,” also ASTM D2422 and British Standards Institution B.S. 4231.

<sup>b</sup> Extreme-pressure lubricants should be used only when recommended by the gear manufacturer.

<sup>c</sup> Synthetic gear oils 9S to 13S are available but not yet in wide use.

<sup>d</sup> Oils marked *Comp* are compounded with 3% to 10% fatty or synthetic fatty oils.

<sup>e</sup> Viscosities of AGMA lubricant no. 13 and above are specified at 100°C (210°F) since measurement of viscosities of these heavy lubricants at 40°C (100°F) would not be practical.

<sup>f</sup> Residual compounds—diluent type, commonly known as solvent cutbacks—are heavy oils containing a volatile, nonflammable diluent for ease of application. The diluent evaporates, leaving a thick film of lubricant on the gear teeth. Viscosities listed are for the base compound without diluent.

CAUTION: These lubricants may require special handling and storage procedures. Diluent can be toxic or irritating to the skin. Do not use these lubricants without proper ventilation. Consult lubricant supplier’s instructions.

SOURCE: Abstracted from ANSI/AGMA 9005-D94, with permission.

**Table 8.3.21 AGMA Lubricant Number Guidelines for Open Gearing (Continuous Method of Application)<sup>a,b</sup>**

Ambient temperature <sup>c</sup> °C (°F)	Character of operation	Pressure lubrication		Splash lubrication		Idler immersion
		Pitch line velocity		Pitch line velocity		Pitch line velocity
		Under 5 m/s (1,000 ft/min)	Over 5 m/s (1,000 ft/min)	Under 5 m/s (1,000 ft/min)	5–10 m/s (1,000–3,000 ft/min)	Up to 1.5 m/s (300 ft/min)
–10 to 15 <sup>d</sup> (15–60)	Continuous	5 or 5 EP	4 or 4 EP	5 or 5 EP	4 or 4 EP	8–9 8 EP–9 EP
	Reversing or frequent “start/stop”	5 or 5 EP	4 or 4 EP	7 or 7 EP	6 or 6 EP	8–9 8 EP–9 EP
10 to 50 <sup>d</sup> (50–125)	Continuous	7 or 7 EP	6 or 6 EP	7 or 7 EP	8–9 <sup>f</sup> 8 EP–9 EP	11 or 11 EP
	Reversing or frequent “start/stop”	7 or 7 EP	6 or 6 EP	9–10 <sup>e</sup> 9 EP–10 EP		

<sup>a</sup> AGMA lubricant numbers listed above refer to gear lubricants shown in Table 8.3.20. Physical and performance specifications are shown in Tables 1 and 2 of ANSI/ASMA 9005-D94. Although both R & O and EP oils are listed, the EP is preferred. Synthetic oils in the corresponding viscosity grades may be substituted where deemed acceptable by the gear manufacturer.

<sup>b</sup> Does not apply to worm gearing.

<sup>c</sup> Temperature in vicinity of the operating gears.

<sup>d</sup> When ambient temperatures approach the lower end of the given range, lubrication systems must be equipped with suitable heating units for proper circulation of lubricant and prevention of channelling. Check with lubricant and pump suppliers.

<sup>e</sup> When ambient temperature remains between 30°C (90°F) and 50°C (125°F) at all times, use 10 or 10 EP.

<sup>f</sup> When ambient temperature remains between 30°C (90°F) and 50°C (125°F) at all times, use 9 or 9 EP.

SOURCE: Abstracted from ANSI/AGMA 9005-D94, with permission.

**Table 8.3.22 AGMA Lubricant Number Guidelines for Open Gearing Intermittent Applications<sup>a,b,c</sup>**

Gear pitch line velocity does not exceed 7.5 m/s (1,500 ft/min)

Ambient temperature, <sup>d</sup> °C (°F)	Intermittent spray systems <sup>e</sup>			Gravity feed or forced-drip method <sup>g</sup>	
	R&O or EP lubricant	Synthetic lubricant	Residual compound <sup>f</sup>	R&O or EP lubricant	Synthetic lubricant
–10–15 (15–60)	11 or 11 EP	11 S	14 R	11 or 11 EP	11 S
5–40 (40–100)	12 or 12 EP	12 S	15 R	12 or 12 EP	12 S
20–50 (70–125)	13 or 13 EP	13 S	15 R	13 or 13 EP	13 S

<sup>a</sup> AGMA viscosity number guidelines listed above refer to gear oils shown in Table 8.3.20.

<sup>b</sup> Does not apply to worm gearing.

<sup>c</sup> Feeder must be capable of handling lubricant selected.

<sup>d</sup> Ambient temperature is temperature in vicinity of gears.

<sup>e</sup> Special compounds and certain greases are sometimes used in mechanical spray systems to lubricate open gearing. Consult gear manufacturer and spray system manufacturer before proceeding.

<sup>f</sup> Diluents must be used to facilitate flow through applicators.

<sup>g</sup> EP oils are preferred, but may not be available in some grades.

SOURCE: Abstracted from ANSI/AGMA 9005-D94, with permission.

**Table 8.3.23 AGMA Lubricant Number Guidelines for Enclosed Helical, Herringbone, Straight Bevel, Spiral Bevel, and Spur Gear Drives<sup>a</sup>**

Pitch line velocity <sup>b,c</sup> of final reduction stage	AGMA lubricant numbers, <sup>a,d,e</sup> ambient temperature °C (°F) <sup>f,g</sup>			
	− 40 to − 10 (− 40 to + 14)	− 10 to + 10 (14 to 50)	10 to 35 (50 to 95)	35 to 55 (95 to 131)
Less than 5 m/s (1,000 ft/min) <sup>h</sup>	3 S	4	6	8
5–15 m/s (1,000–3,000 ft/min)	3 S	3	5	7
15–25 m/s (3,000–5,000 ft/min)	2 S	2	4	6
Above 25 m/s (5,000 ft/min) <sup>h</sup>	0 S	0	2	3

<sup>a</sup> AGMA lubricant numbers listed above refer to R&O and synthetic gear oil shown in Table 8.3.20. Physical and performance specifications are shown in Tables 1 and 3 of ANSI/AGMA 9005-D94. EP or synthetic gear lubricants in the corresponding viscosity grades may be substituted where deemed acceptable by the gear drive manufacturer.

<sup>b</sup> Special considerations may be necessary at speeds above 40 m/s (8,000 ft/min). Consult gear drive manufacturer for specific recommendations.

<sup>c</sup> Pitch line velocity replaces previous standards' center distance as the gear drive parameter for lubricant selection.

<sup>d</sup> Variations in operating conditions such as surface roughness, temperature rise, loading, speed, etc., may necessitate use of a lubricant of one grade higher or lower. Contact gear drive manufacturer for specific recommendations.

<sup>e</sup> Drives incorporating wet clutches or overrunning clutches as backstopping devices should be referred to the gear manufacturer, as certain types of lubricants may adversely affect clutch performance.

<sup>f</sup> For ambient temperatures outside the ranges shown, consult the gear manufacturer.

<sup>g</sup> Pour point of lubricant selected should be at least 5°C (9°F) lower than the expected minimum ambient starting temperature. If the ambient starting temperature approaches lubricant pour point, oil sump heaters may be required to facilitate starting and ensure proper lubrication (see 5.1.6 in ANSI/AGMA 9005-D94).

<sup>h</sup> At the extreme upper and lower pitch line velocity ranges, special consideration should be given to all drive components, including bearing and seals, to ensure their proper performance.

SOURCE: Abstracted from ANSI/AGMA 9005-D94, with permission.

**Table 8.3.24 Typical Gear Lubricants**

Lubricant type	Military specification	Useful temp range, °F	Commercial source and specification (a partial listing)		Remarks and applications
			Source	Identification	
<b>Oils</b>					
Petroleum	MIL-L-644B	− 10 to 250	Exxon Corporation Franklin Oil and Gas Co. Royal Lubricants Co. Texaco	#4035 or Unvis P-48 L-499B Royco 380 1692 Low Temp. Oil	Good general-purpose lubricant for all quality gears having a narrow range of operating temperature
Diester	MIL-L-6085A	− 67 to 350	Anderson Oil Co. Eclipse Pioneer Div., Bendix Shell Oil Co. E. F. Houghton and Co.	Windsor Lube I-245X Pioneer P-10 AeroShell Fluid 12 Cosmolubric 270	General-purpose, low-starting torque, and stable over a wide temperature range. Particularly suited for precision instrument gears and small machinery gears
Diester	MIL-L-7808C	− 67 to 400	Sinclair Refining Co. Socony Mobil Oil Co. Bray Oil Co.	Aircraft Turbo S Oil Avrex S Turbo 251 Brayco 880	Suitable for oil spay or mist system at high temperature. Particularly suitable for high-speed power gears
Silicone		− 75 to 350	Exxon Corporation Dow-Corning Corp.	Exxon Turbine Oil 15 DC200	Rated for low-starting torque and lightly loaded instrument gears
Silicone		− 100 to 600	General Electric Co.	Versilube 81644	Best load carrier of silicone oils with widest temperature range. Applicable to power gears requiring wide temperature ranges
<b>Greases</b>					
Diester oil-lithium soap	MIL-G-7421A	− 100 to 200	Royal Lubricants Co. Texaco	Royco 21 Low Temp. No. 1888	For moderately loaded gears requiring starting torques at low temperatures
Diester oil-lithium soap	MIL-G-3278A	− 67 to 250	Exxon Corporation Shell Oil Co. Sinclair Refining Co. Bray Oil Co.	Beacon 325 AeroShell Grease II Sinclair 3278 Grease Braycote 678	General-purpose light grease for precision instrument gears, and generally lightly loaded gears
Petroleum oil-sodium soap	MIL-L-3545	− 20 to 300	Exxon Corporation Standard Oil Co. of Calif.	Andok 260 RPM Aviation Grease #2	A high-temperature lubricant for high speed and high loads
Mineral oil-sodium soap	—	− 25 to 250	Exxon Corporation	Andok C	Stiff grease that channels readily. Suitable for high speeds and highly loaded gears

Table 8.3.25 Solid Oil Additives

Lubricant type	Temperature range, °F	Source	Identification	Remarks
Colloidal graphite	Up to 1,000	Acheson Colloids Co.	SLA 1275	Good load capacity, excellent temperature resistance
Colloidal MoS <sub>2</sub>	Up to 750	Acheson Colloids Co.	SLA 1286	Good antiwear
Colloidal Teflon	Up to 575	Acheson Colloids Co.	SLA 1612	Low coefficient of friction

they will provide long service life if the plastic chosen is correct for the application. Plastics manufacturers and their publications can be consulted for guidance. Alternatively, many plastic gear materials can be molded with internal solid lubricants, such as MoS<sub>2</sub>, Teflon, and graphite.

#### GEAR INSPECTION AND QUALITY CONTROL

Gear performance is not only related to the design, but also depends upon obtaining the specified quality. Details of gear inspection and control of subtle problems relating to quality are given in Michalec, "Precision Gearing," Chap. 11.

#### COMPUTER MODELING AND CALCULATIONS

A feature of the latest AGMA rating standards is that the graphs, including those presented here, are accompanied by equations which allow

application of computer-aided design. Gear design equations and strength and durability rating equations have been computer modeled by many gear manufacturers, users, and university researchers. Numerous software programs, including integrated CAD/CAM, are available from these places, and from computer system suppliers and specialty software houses. It is not necessary for gear designers, purchasers, and fabricators to create their own computer programs.

With regard to gear tooth strength and durability ratings, many custom gear house designers and fabricators offer their own computer modeling which incorporates modifications of AGMA formulas based upon experiences from a wide range of applications.

The following organizations offer software programs for design and gear ratings according to methods outlined in AGMA publications: Fairfield Manufacturing Company Gear Software; Geartech Software, Inc.; PC Gears; Universal Technical Systems, Inc. For details and current listings, refer to AGMA's latest "Catalog of Technical Publications."

## 8.4 FLUID FILM BEARINGS

by Vittorio (Rino) Castelli

REFERENCES: "General Conference on Lubrication and Lubricants," ASME. Fuller, "Theory and Practice of Lubrication for Engineers," 2d ed., Wiley. Booser, "Handbook of Lubrication. Theory and Design," vol. 2, CRC Press. Barwell, "Bearing Systems, Principles and Practice," Oxford Univ. Press. Cameron, "Principles of Lubrication," Longmans Greene. "Proceedings," Second International Symposium on Gas Lubrication, ASME. Gross, "Fluid-Film Lubrication," Wiley. Gunter, "Dynamic Stability of Rotor-Bearing Systems," NASA SP-113, Government Printing Office.

Plain bearings, according to their function, may be

**Journal bearings**, cylindrical, carrying a rotating shaft and a radial load

**Thrust bearings**, the function of which is to prevent axial motion of a rotating shaft

**Guide bearings**, to guide a machine element in its translational motion, usually without rotation of the element

In exceptional cases of design, or with a complete **failure of lubrication**, a bearing may run dry. The coefficient of friction is then between 0.25 and 0.40, depending on the materials of the rubbing surfaces. With the **bearing barely greasy**, or when the bearing is well lubricated but the speed of rotation is very slow, boundary lubrication takes place. The coefficient of friction may vary from 0.08 to 0.14. This condition occurs also in any bearing when the shaft is starting from rest if the bearing is not equipped with an oil lift.

**Semifluid**, or **mixed**, lubrication exists between the journal and bearing when the conditions are not such as to form a load-carrying fluid film and thus separate the surfaces. Semifluid lubrication takes place at comparatively low speed, with intermittent or oscillating motion, heavy load, insufficient oil supply to the bearing (wick or waste-lubrication, drop-feed lubrication). Semifluid lubrication may also exist in thrust bearings with fixed parallel-thrust collars, in guide bearings of machine tools, in bearings with copious lubrication where the shaft is bent or the bearing is misaligned, or where the bearing surface is interrupted by improperly arranged oil grooves. The coefficient of friction in such bearings may range from 0.02 to 0.08 (Fuller, Mixed Friction Conditions in Lubrication. *Lubrication Eng.*. 1954).

Fluid or **complete lubrication**, when the rubbing surfaces are completely separated by a fluid film, provides the lowest friction losses and prevents wear. A certain amount of oil must be fed to the oil film in order to compensate for end leakage and maintain its carrying capacity. Such lubrication can be provided under pressure from a pump or gravity tank, by automatic lubricating devices in self-contained bearings (oil rings or oil disks), or by submersion in an oil bath (thrust bearings for vertical shafts).

#### Notation

$R$  = radius of bearing, length  
 $r$  = radius of journal, length  
 $c = mr = R - r$  = radial clearance, length  
 $W$  = bearing load, force  
 $\mu$  = viscosity = force  $\times$  time/length<sup>2</sup>  
 $Z$  = viscosity, centipoise (cP); 1 cP =  $1.45 \times 10^{-7}$  lb  $\cdot$  s/in<sup>2</sup> (0.001 N  $\cdot$  s/m<sup>2</sup>)  
 $\beta$  = angle between load and entering edge of oil film  
 $\eta$  = coefficient for side leakage of oil  
 $\nu$  = kinematic viscosity =  $\mu/\rho$ , length<sup>2</sup>/time  
 $R_e$  = Reynolds number =  $umr/\nu$   
 $P_a$  = absolute ambient pressure, force/area  
 $P$  =  $W/(ld)$  = unit pressure, lb/in<sup>2</sup>  
 $N$  = speed of journal, r/min  
 $m$  = clearance ratio (diametral clearance/diameter)  
 $F$  = friction force, force  
 $A$  = operating characteristic of plain cylindrical bearing  
 $P'$  = alternate operating characteristic of plain cylindrical bearing  
 $h_0$  = minimum film thickness, length  
 $\varepsilon$  = eccentricity ratio, or ratio of eccentricity to radial clearance  
 $e$  = eccentricity = distance between journal and bearing centers, length  
 $f$  = coefficient of friction  
 $f'$  = friction factor =  $F/(\pi r l \rho u^2)$   
 $l$  = length of bearing, length  
 $d = 2r$  = diameter of journal, length